For Langmuir adsorption of proteins, why does concentration of protein at $R_{\text{max}}/2 = K_D$?

Consider a protein, $P$ binding with a surface immobilised antibody $A$, forming a surface bound complex $AP$. We measure a response $R$, proportional to concentration of $AP$.

$$R \propto [AP]_t$$

The measured response is proportional to both the rate of association and dissociation:

$$\frac{dR}{dt} = k_{\text{ass}} [A]_t [P]_t - k_{\text{diss}} [AP]_t$$

**At Half Coverage**

At $R_{\text{max}}/2$, have exactly half surface sites covered, so we can write:

$$[A]_t = [A]_o / 2$$

and:

$$[AP]_t = [A]_o / 2 \quad \text{Where} \ [A]_o \ \text{is the concentration of antibodies on the surface.}$$

So we can write:

$$\frac{dR}{dt} = \frac{k_{\text{ass}} [A]_o [P]_t}{2} - \frac{k_{\text{diss}} [A]_o}{2}$$
At equilibrium, (Langmuir values are all at equilibrium for a given concentration of protein (above)).

\[
\frac{dR}{dt} = 0 \quad \text{So:} \quad \frac{k_{\text{ass}}[A]_{\text{ass}}[P]_t}{2} = \frac{k_{\text{dis}}[A]_o}{2}
\]

which simplifies to:

\[
k_{\text{ass}}[P]_t = k_{\text{dis}}
\]

and so:

\[
[P]_t = \frac{k_{\text{dis}}}{k_{\text{ass}}} = K_D
\]