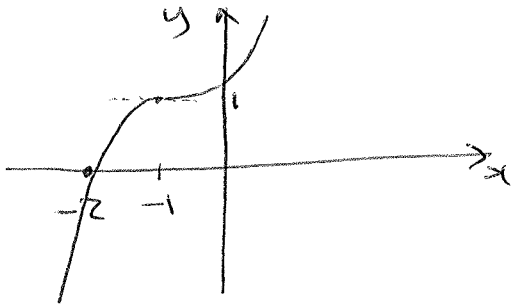
1. $y = x^3 + 3x^2 + 3x + 2$

$= (x+1)^3 + 1.$

So this is the $y = x^3$ shifted up by 1 & to the left by 1.

It has a zero when $(x+1)^3 = -1 \Rightarrow x+1 = -1 \Rightarrow x = -2$



2. $y = x \sin^2 x$. This is a slightly different example of an envelope curve.

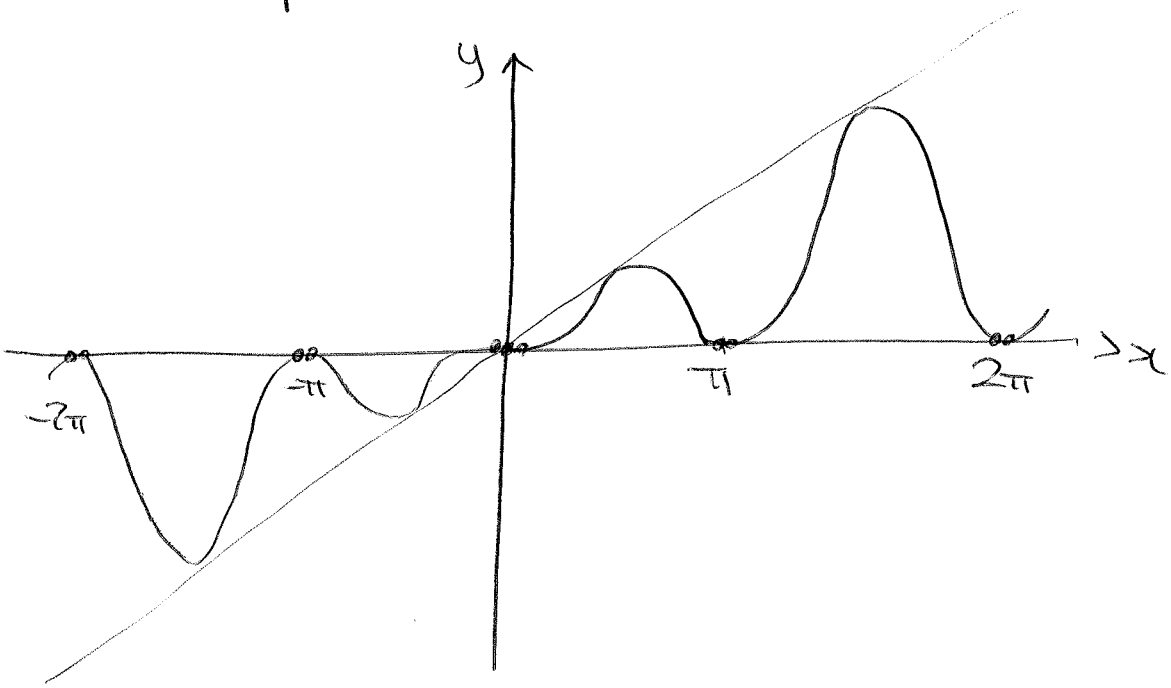
As $0 \leq \sin^2 x \leq 1$ then

$0 \leq x \sin^2 x \leq x$ when x is positive.

and $x \leq x \sin^2 x \leq 0$ when x is negative.

There are double zeros at $x = \pm n\pi$, $n = 1, 2, \ldots$

There is a triple zero at $x = 0$.

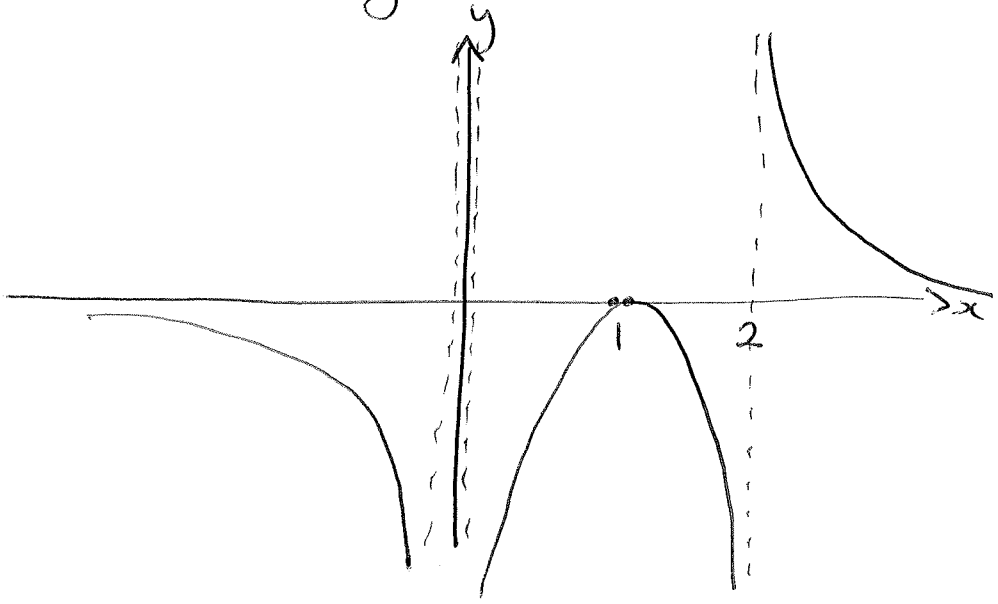


3. $y = \dfrac{(x-1)^2}{x^2(x-2)}$.

Zeros @ $x = 1, 1$

Poles @ $x = 0, 0, 2$

When $x \to \infty$, then $y \approx 1/x$.



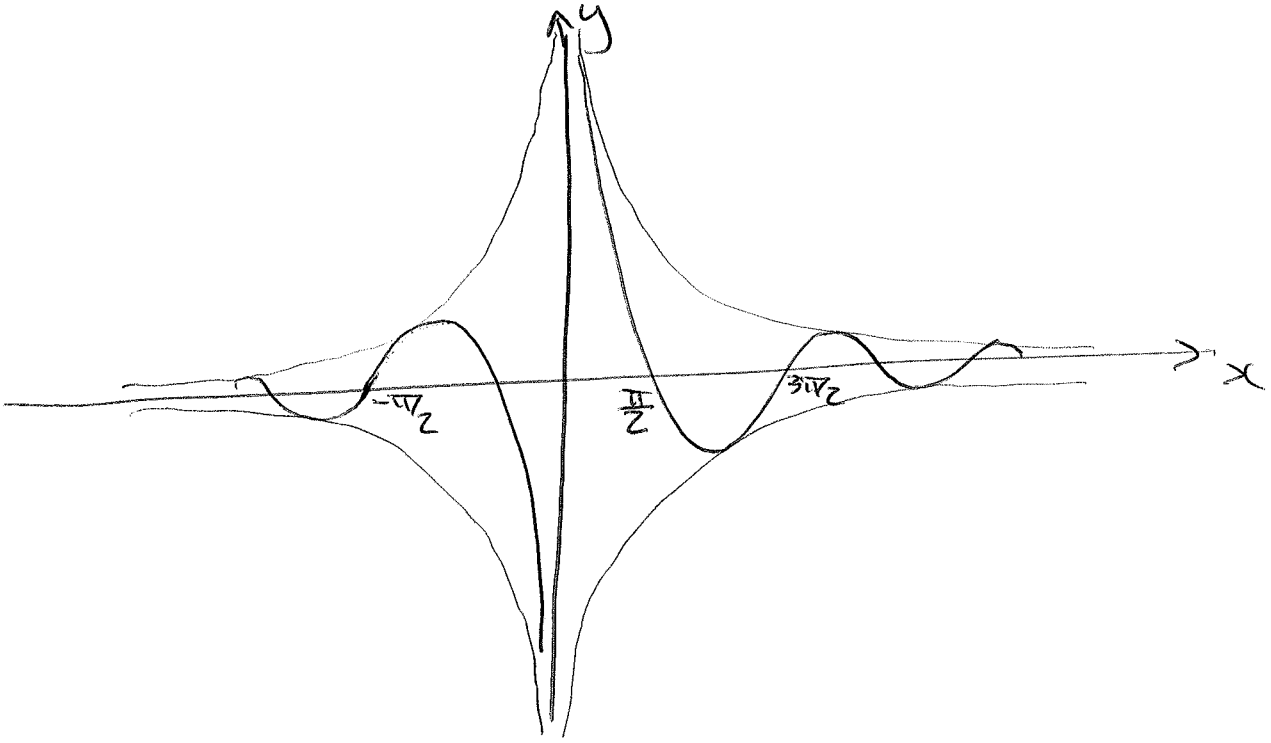
4. $y = \dfrac{\cos x}{x}$.

We have zeros at $x = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2 \cdots$

Pole at $x = 0$.

Odd function

Envelope is $\pm\frac{1}{x}$.

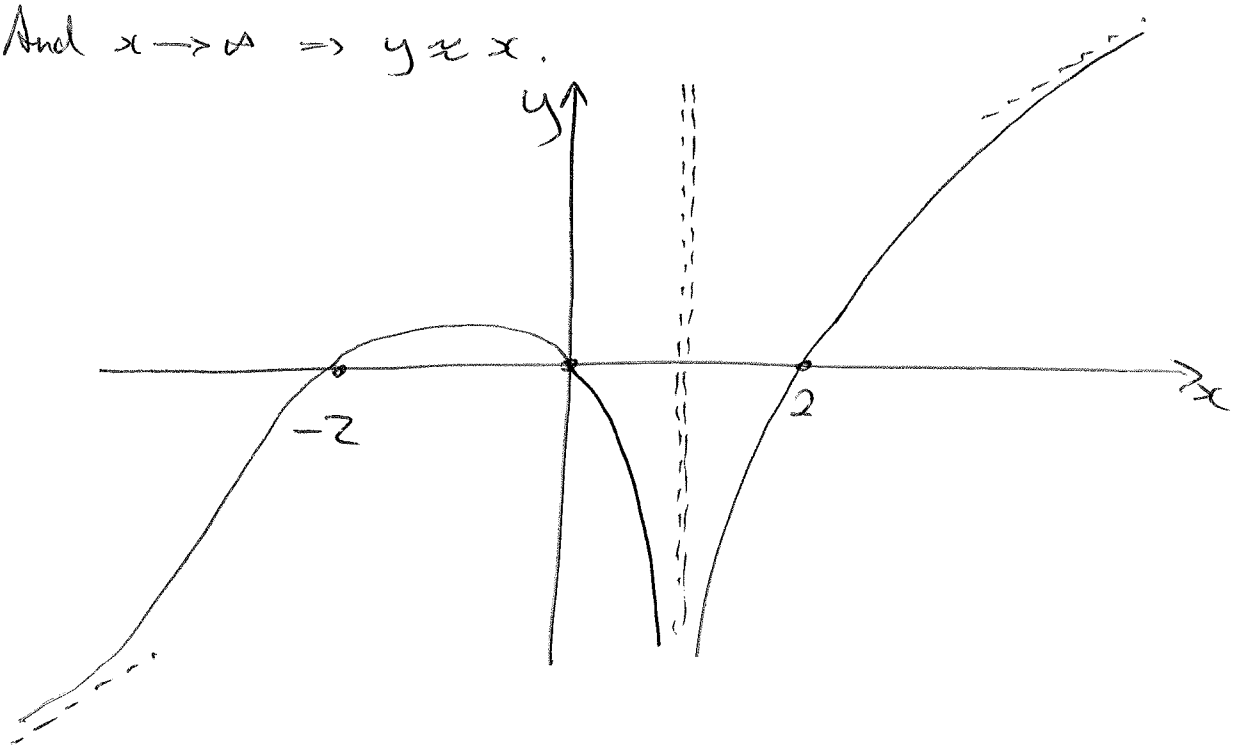


5. $y = \dfrac{x^3 - 4x}{(x-1)^2} = \dfrac{x(x-2)(x+2)}{(x-1)^2}$

So zeros @ $x = 0, 2, -2$

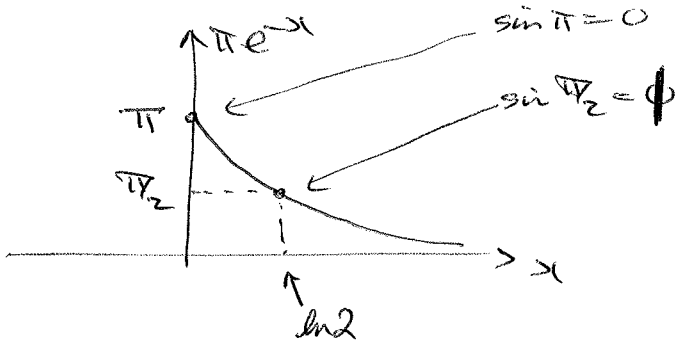
poles @ $x = +1, +1$

And $x \to \infty \Rightarrow y \propto x$.

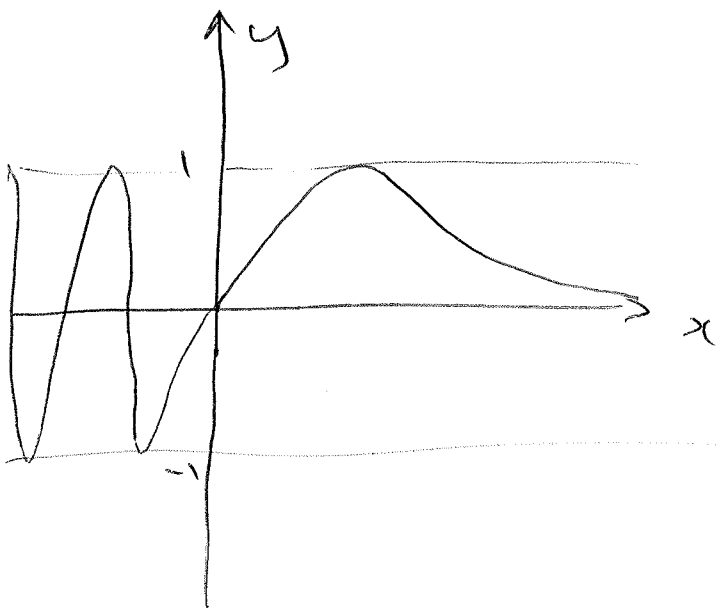


6. $y = \sin(\pi e^{-x})$.

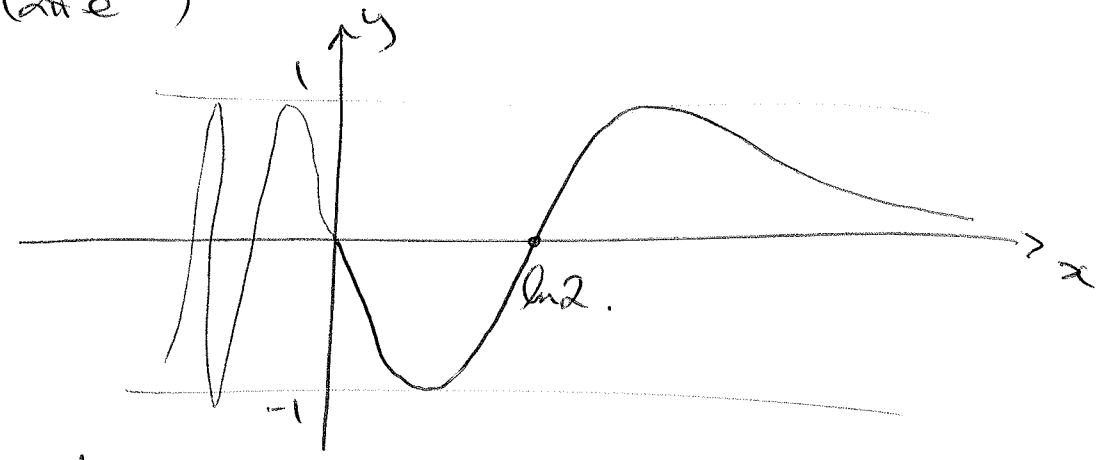
Nb.:



$\Rightarrow$



7. $y = \sin(2\pi e^{-x})$

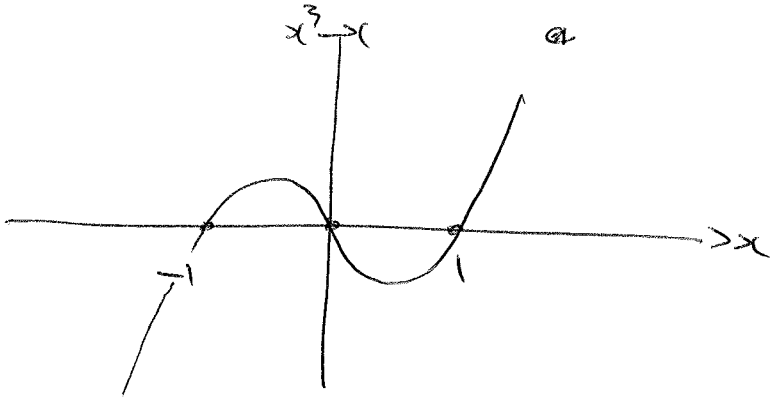


The only positive zero occurs when $2\pi e^{-x} = \pi$

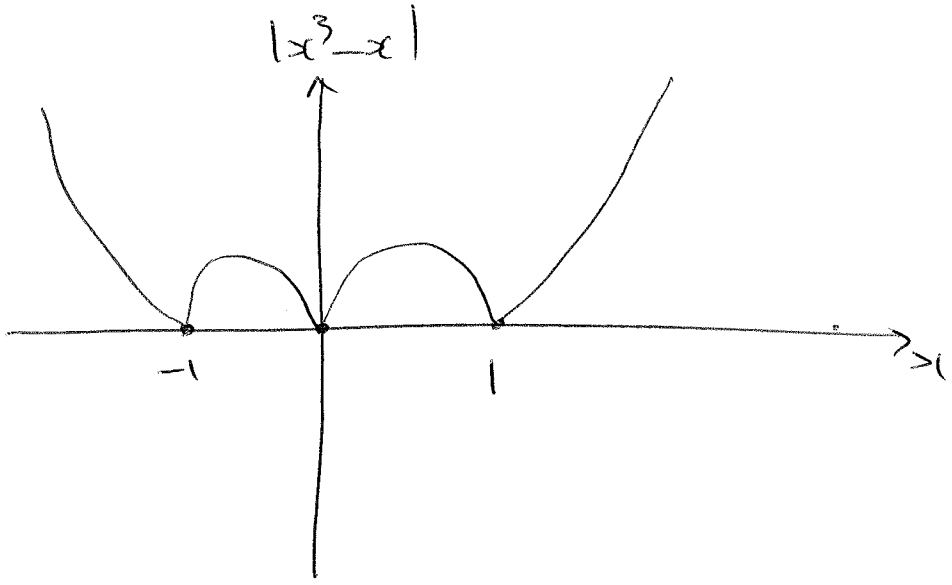
$\Rightarrow x = \ln 2.$

8. $y = |x^3 - x|$

First note that $x^3 - x = x(x-1)(x+1)$ — zeros at $x = -1, 0, 1$

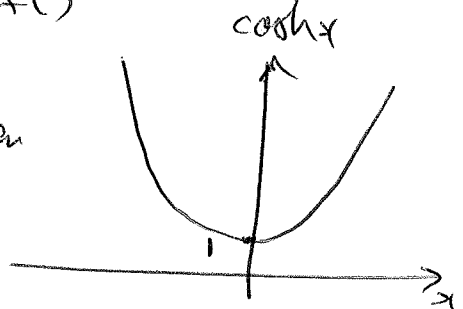


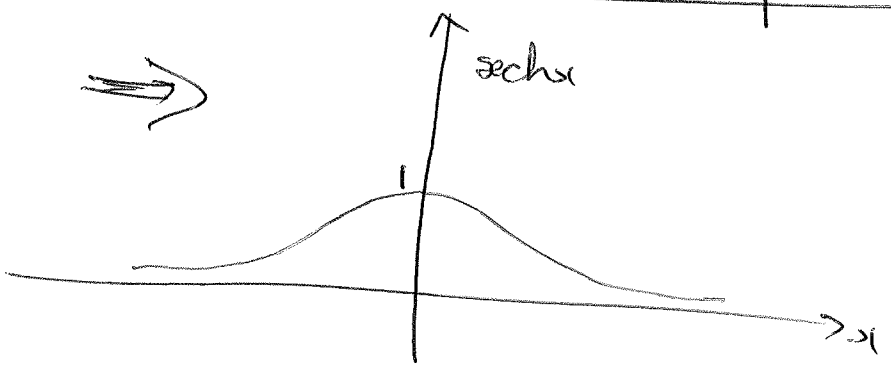
Hence



9. $y = \text{sech}^2(x-1) + \text{sech}^2(x+1)$

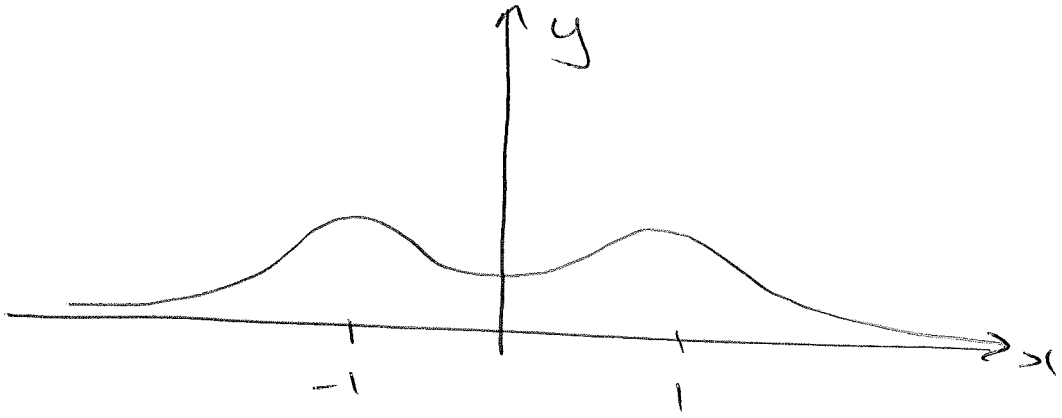
As $\text{sech}\, x = \frac{1}{\cosh x}$ then





For sketching purposes $\text{sech}^2 x$ is roughly the same as $\text{sech}\, x$.

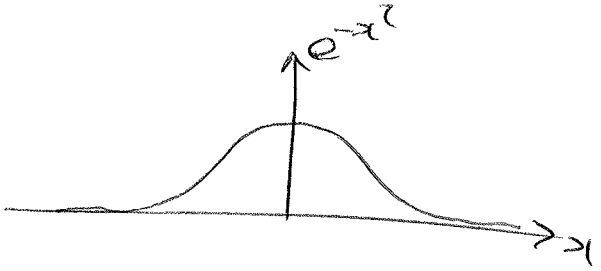
Hence



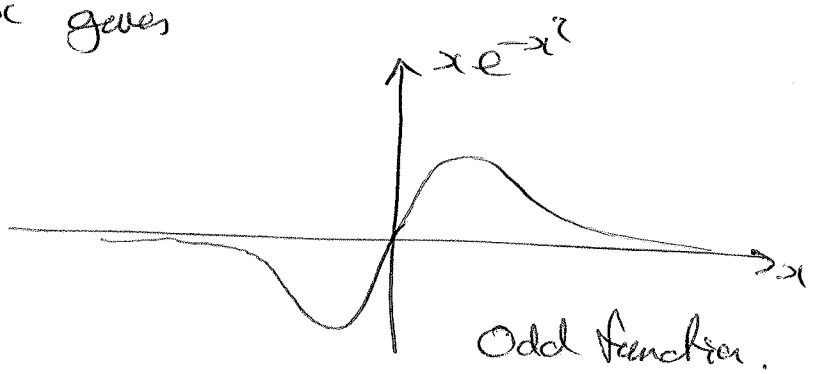
is the sum of two sech^2 functions, one of which is centred @ $x = -1$ & the other at $x = 1$.

10. $y = xe^{-x^2}$.

Now e^{-x^2} is:

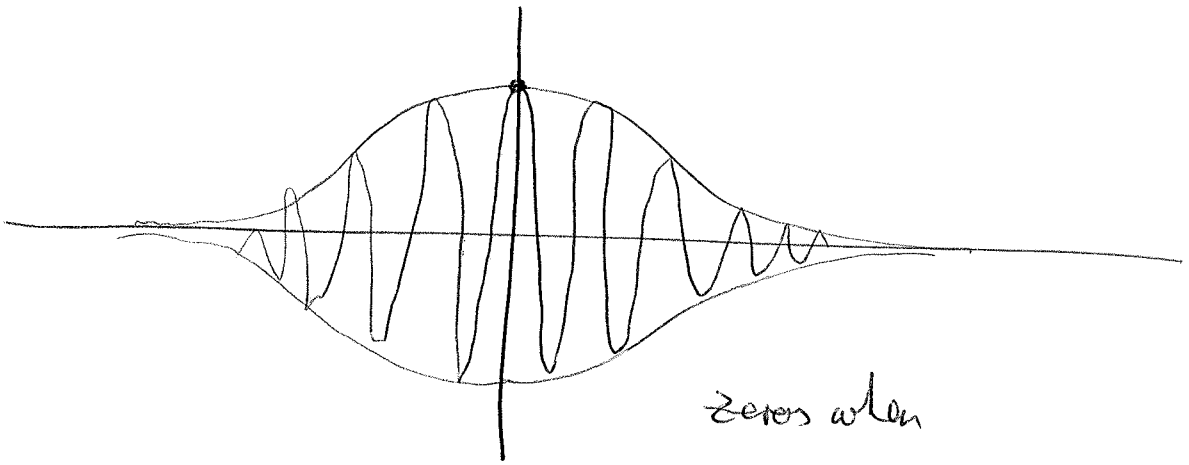


Multiplication by x gives



Odd function.

11. $y = e^{-x^2}\cos(100\pi x)$. This is an even function.



zeros when

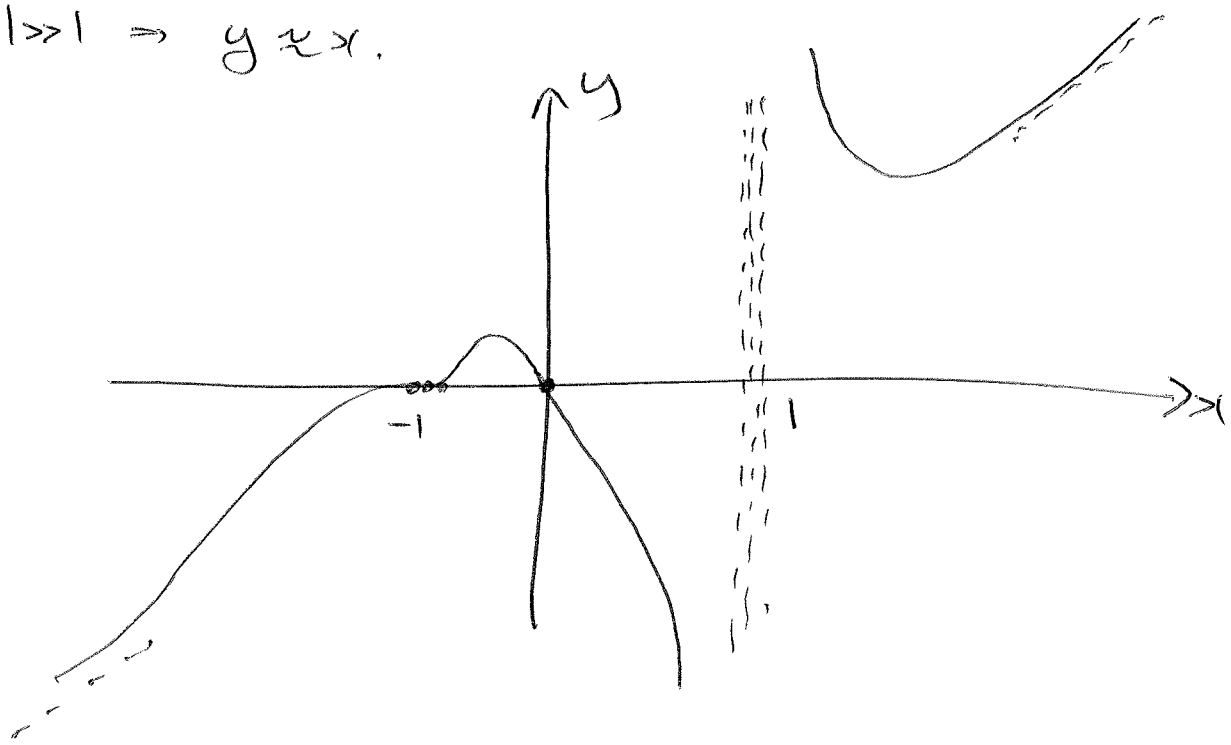
$100\pi x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

12. $y = \left(\frac{x+1}{x-1}\right)^3 x$.

zeros at $x = 0, -1, -1, -1$

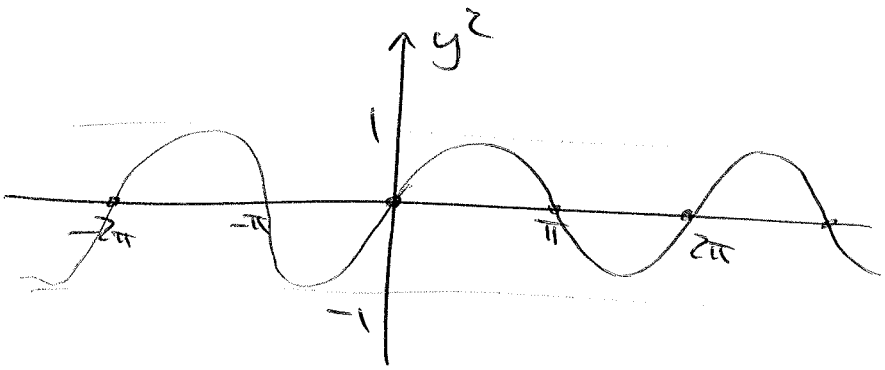
poles at $x = 1, 1, 1$

$|x| \gg 1 \Rightarrow y \approx x$.

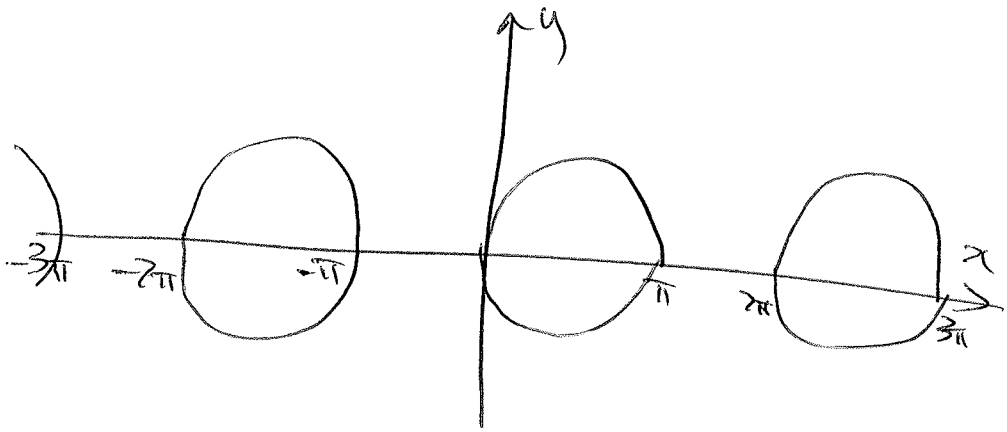


13. $y^2 = \sin x$

First draw $\sin x$:

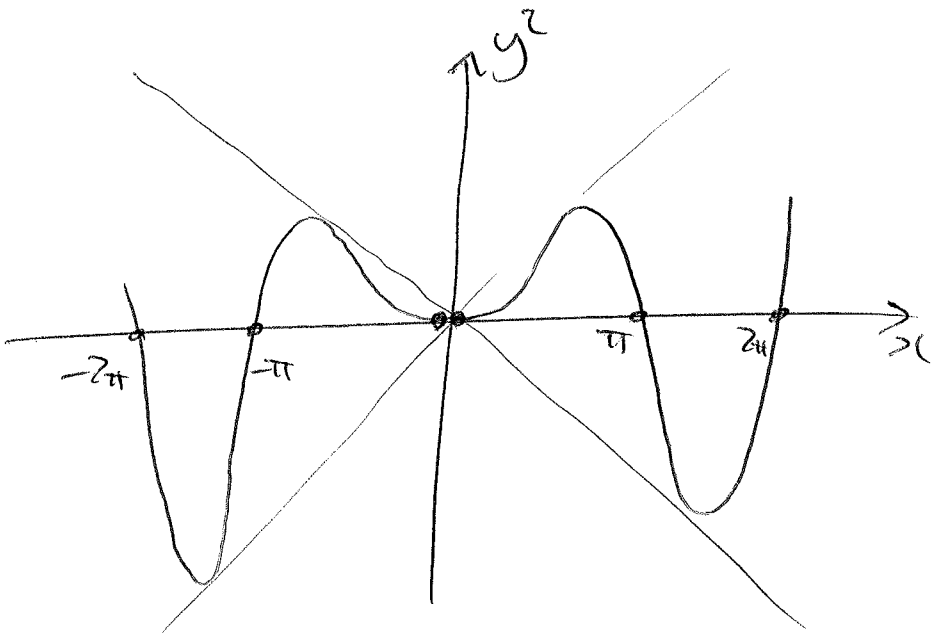


Then take square roots:

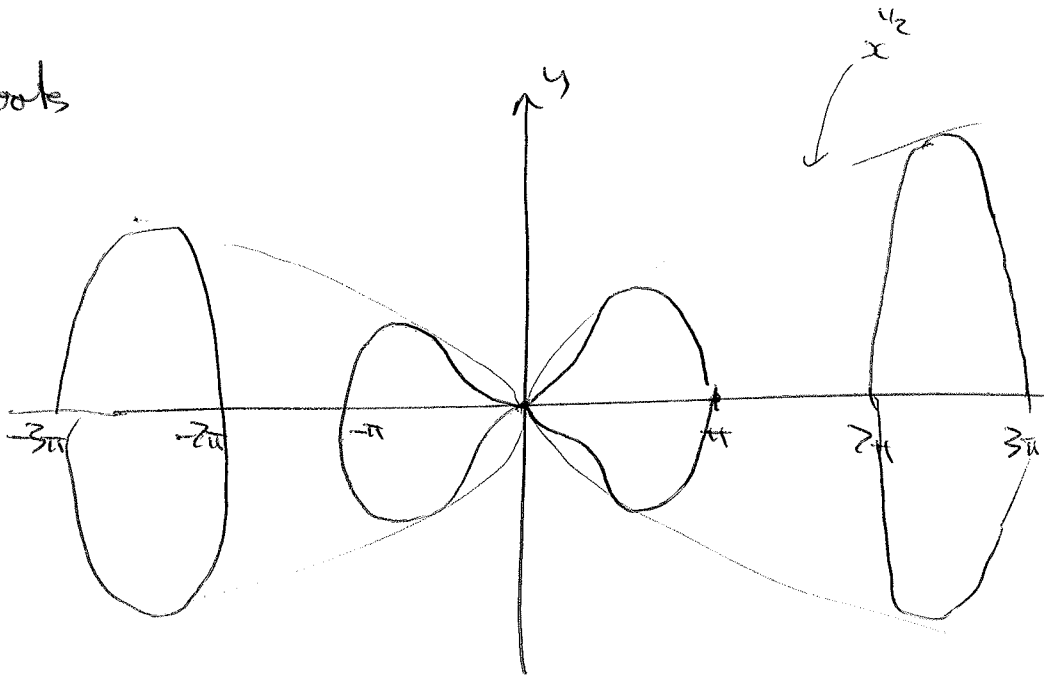


14. $y^2 = x \sin x$

First draw $x \sin x$



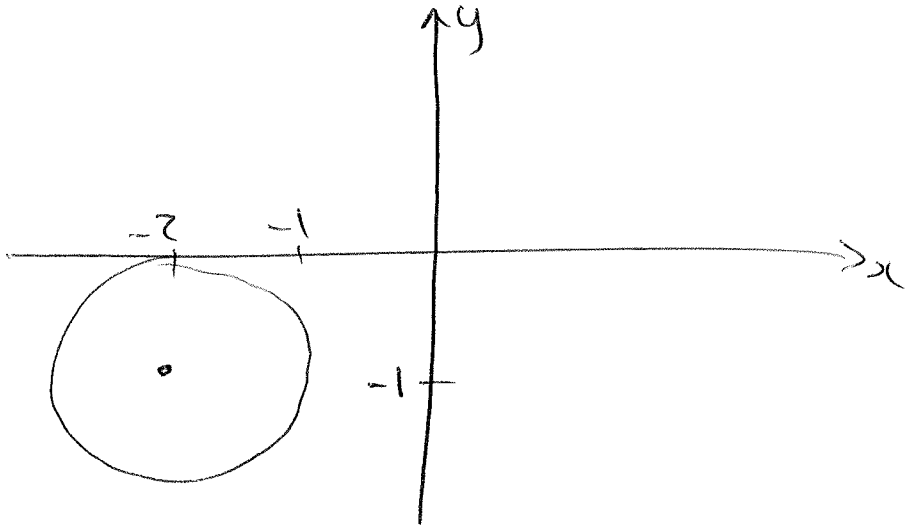
Then take square roots



15. $x^2+y^2+4x+2y+4=0$

Complete the squares: $(x+2)^2+(y+1)^2=1$

$\Rightarrow$ circle of radius 1 centred at $x=-2, y=-1$.



16. $y=\dfrac{(x-3)^2(x^2-4)x}{(x+3)(x^2-1)}$

Zeros @ $x=3,3,2,-2,0$

Poles @ $x=-3,1,-1$

$|x|\gg 1 \Rightarrow y \propto x^2$