ME10304 – Mathematics 1 – Feedback on the 2020/2021 exam.

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General comments.

Due to the pandemic arrangements, namely a 2-hour closed-book paper being sat as a 24-hour open-book paper with what I thought was only a very incremental increase in the difficulty, I fully expected a substantial increase in the average mark from previous years, and this did happen! The average mark was roughly 82% with a standard deviation of 14.0%.

The great majority of scripts were well-organised and clear. A few legends even underlined the solutions or shaded them in nicely – this is an enormous help to the marker. With two exceptions, the questions were answered in numerical order; this too helps me greatly with the marking – many thanks. The primary negatives were the following.

- A dark scan, possibly due to poor light or the use of 2H pencil, which makes it difficult to read.
- A sideways scan. With Inspera this is very difficult to rotate, and I have forwarded my views on this to Inspera. I also had a few pages which were upside down!
- Writing in a two-column format. This isn't an issue when marking paper scripts, but produces a huge amount of scrolling when marking online.
- Small handwriting, especially with lined paper.
- Loads of crossings-out. With an online exam over such a long period of time, crossings-out shouldn't be there!
- Scruffy writing with information not in a logical sequence.

None of the above are a problem for paper scripts (although I expect scruffy handwriting in the usual two-hour sprint), but they do make life difficult when marking online.

So my general recommendation for online exam scripts is to use (if possible) a plain white paper with as dark a pen as possible to maximise contrast, to be generous with the size of the writing, to take care with the orientation of the scan, to attend to the lighting conditions and to write in a single column.

Overall, I was extremely happy with the content of the scripts. I usually find a large number of small errors, but this year there were some categories of such errors which were entirely absent. Whether this is due to having 24 hours to write, or whether my upgraded notes have served you well, I really don't know. Because of this very positive context, I will concentrate mainly on the negative aspects for each question, below.

Question 1.	**Curve sketching.** Average mark: 8.1/10. Quite a few students spent a lot of time writing huge amounts when it isn't necessary. Some tried to find minima and maxima — again this is not necessary for this question. An astonishingly large number didn't write on the axis the value of x at which the pole or zero occurs; some marks were deducted for this. Frequently the salient features weren't mentioned.
(a)	Perhaps the worst subquestion! Many of the curves looked like what a graphical calculator will produce with only one double zero, and therefore they were very much like a squashed Macdonalds logo. Often double roots at $\pm n\pi$ weren't mentioned at all or shown. Sometimes the envelope wasn't shown.
(b)	Very good indeed.
(c)	I really needed to see some workings for this so that I could be assured that it wasn't a sketch from a graphical calculator. This equation can be arranged in the form for an ellipse, although the theory used in the lectures also works well. Being a y^2-formula, the solution is of $\pm$function-of-x form.
(d)	Quite a variety of responses to this, almost all of which were correct. Those that weren't didn't have the zeros in the right place.
Question 2.	**Complex numbers.** Average mark: 9.0/10.
(a)	Excellent. Sometimes the arithmetic went astray. Some people didn't find the conjugate or the modulus.
(b)	Very good. Some students took half a page to do this even though it is a one-liner! Here is the glacially slow version which I saw on a dozen scripts: $z = 2e^{j\pi/4} = a + bj \quad \Rightarrow \quad r = 2 \text{ and } \theta = \pi/4 \quad \Rightarrow \quad a^2 + b^2 = 4.$ Also $b/a = \tan\theta = 1 \quad \Rightarrow \quad b = a \quad \Rightarrow \quad 2a^2 = 4 \quad \Rightarrow \quad a = \sqrt{2}$ $\Rightarrow \quad b = \sqrt{2} \quad \Rightarrow \quad z = \sqrt{2} + \sqrt{2}j.$ The smart approach is $z = 2e^{j\pi/4} = 2\Big(\cos(\pi/4) + j\sin(\pi/4)\Big) = 2\Big(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\Big) = \sqrt{2} + \sqrt{2}j.$ Both methods are rigorous and perfectly correct, although the first can yield real and imaginary components with incorrect signs.
(c)	Very good. It's always necessary to find the argument, rather than leaving it as an arctan, for which there are two possible values.
(d)	Very good.

Question 3.	**Differentiation.** Average mark: 8.9/10.
(a)	All three parts of this were answered quite well. Given that this was an online paper, I wasn't at all keen just to see the answers with no workings. Software is available these days, and I needed to see some sort of proof that this had been worked by the student in order to give full marks.
(b)	Also answered well. Quite a few people didn't use the $y'' \neq 0$ criterion to identify which was the maximum and which was the minimum. Even more people assumed that $y''(0) = 0$ corresponds to an inflexion point — it doesn't in general, but we need a nonzero value of $y'''(0)$ to confirm it to be. The sketch should also depict $x = 0$ very clearly as an inflexion point, either by making it very obvious on the sketch or by inserting our three black disks. Otherwise I assumed that this information hadn't been used.
Question 4.	**Partial differentiation.** Average mark: 8.1/10.
(a)	This part was done very well in general. The final results for both of the derivatives can be simplified into a very compact form and they should have been. A very small number of students obtained the correct expression for f_y by referring to f_x and appealing to symmetry — you are proper LEGENDS, for this particular one is not straightforward (because an extra minus sign is needed) and I didn't teach it. Unfortunately, a couple of other students also appealed to symmetry but did so incorrectly.
(b)	The number of critical points was four. It was very difficult to read those scripts which hadn't used the tabular form I introduced in the lectures and recommended for the exam.
Question 5.	**Integration 1.** Average mark: 8.3/10.
(a)	Very good. Some students just gave the numerical answer — I needed to see the workings in order to award marks.
(b)	This was answered well despite being a tricky integral that needed two substitutions. Some persisted in writing the original limits (i.e. the x-limits) during and after the substitutions, and formally this is incorrect. The return back to x then gives an ∞/∞ limit as $x \to \infty$. Never write ∞/∞ in your maths. On this occasion the assumed limit of 1 was correct, but it might not have been. However, if the limits had been changed after each substitution, then the problem becomes very much easier and quicker to solve.
(c)	Some used partial fractions, some identified it as an f'/f integral. Either is fine, but the solution does need the modulus signs and the arbitrary constant for full marks.
Question 6.	**Integration 2.** Average mark: 8.3/10.
(a)	Generally very good. I did have a typo on the exam paper, and those two who solved it as a Cartesian problem were dealt with fairly. Others saw my intent. About a dozen students differentiated the r part of the function rather than integrated — oops. Other missed out the extra r that is needed for integrations in polar coordinates.
(b)	V was usually correct. Unlike in previous years, no-one wrote the initial area integral incorrectly, but this too was a lengthy tour-de-force. Impressively well done in general. A couple of students presumably used an integration package on a calculator and even admitted to it. I needed to see workings....
Question 7.	**Series.** Average mark: 7.6/10.
(a)	Generally very well done. A small minority really messed this up, and did so in a bewhildering variety of ways. I couldn't have asked this question in previous years.
(b)	Well done. Don't forget that the arbitary constant has to be shown to be zero.
(c)	Two applications of l'Hôpital's rule with an intermediate cancellation. Impressively negotiated.

Question 8.	**Vectors 1.** Average mark: 9.4/10.
	The best question. Generally a walk in the park. Occasionally the cross product went astray.
Question 9.	**Vectors 2.** Average mark: 7.1/10.
	Generally parts (a) and (b) were answered very well indeed. In parts (c) and (d) many people got the right answer but not by using the method that I specified in the question — no marks.
Question 10.	**Miscellaneous.** Average mark: 7.3/10.
	Part (a) did need the asymptotes, $y \sim \pm\pi/2$, to be shown clearly. I also needed to see the working for Part (b). Part (c) was usually answered well, and in Part (d) the constant of integration needed to be quoted and evaluated. Part (e) was an extra two marks for those who got to the end of part (d) correctly. I did find a few students who provided the correct number (and sometimes the exact indefinite integral of $\tan^{-1} x$) without doing Part (d)), but note the phrasing of Part (e) on the paper, namely *Integrate the power series found in part (d) and use this* $\cdots$.