ME10304 – Mathematics 1 – Feedback on the 2021/2022 exam.

Unit convenor: Dr D Andrew S Rees

General comments.

Last year's ME10304 exam was an open-book affair with 24 hours in which to complete upload it. By contrast this year's 2-hour open-book exam was a much less sedate event, and therefore the average mark, 67%, was substantially lower that last year's. This year's was also just slightly lower than recent for pre-covid exams. I designed this exam to be as close to being of the same standard as were pre-covid exams, but with the exception of the last part of Q2 which was somewhat unusual and a rare novelty. Congratulations to the two students with 100%.

With a few exceptions, the questions were answered in numerical order; this helps me greatly with the marking – many thanks.

Overall, I was very happy with the content of the scripts. It was gratifying to see evidence of those few places in the unit where I had decided to teach slightly differently from previous years. Because of this positive context, I will concentrate mainly on the negative aspects for each question, below.

Question 1.	**Curve sketching.** Average mark: 7.5/10. Quite a few students spent a lot of time writing huge amounts of text when it isn't necessary. Some tried to find minima and maxima — again this is not necessary for this question. An astonishingly large number didn't write on the axis the value of x at which the pole or zero occurs; some marks were deducted for this. Frequently the salient features weren't mentioned.
(a)	The only issue was that some people sketched $x^3 - x^2$ rather than $x^2 - x^3$. The roots were found correctly, but in those few cases the large-x behaviour had been assumed incorrectly.
(b)	Very good indeed. I would have liked to have seen a very short proof that $\sinh x + \cosh x = e^x$ rather than haveing nothing at all. A few claimed that $\infty - \infty = 0$, which is hardly ever correct.
(c)	I really needed to see a statement about the zeros and their multiplicity. So $x = 0$ is a triple zero and $x = n\pi$ is a double zero for all noninteger values of n.
(d)	I would expect to see the large-x behaviour and a list of all roots and poles and their multiplicity, no more and no less. Then one may 'join the dots'. In this case many thought the $y \to 1$ as $x \to \infty$ whereas it is $y \to 1$. A large number of students referred to the pole at $y = 2$; this is incorrect for it is an asymptote.

Question 2.	**Complex numbers.** Average mark: 5.7/10.
(a)	Excellent. Sometimes the arithmetic went astray. The correct argument for $11 - 2j$ is either -0.179844 (see a sketch of the complex plane) or 6.103332. The former is for those who, like computers and calculators, prefer their arguments to lie between $-\pi$ and π. The latter for those who prefer them to lie between 0 and 2π.
(b)	Not as good as part (a). There was fair bit of confusion about the correct argument for this complex number. It should have been 2.060754. Some subtracted π from this to get -1.080839 but this is incorrect even though the tangent of the argument is the same. Again, a quick sketch of the complex plane helps.
(c)	Many fell foul of this part. I asked for an analytical value for $\sin 36°$, but some merely found it numerically using the calculator. It was clear that some had found the analytical form from elsewhere, but didn't present the required derivtion. The application of de Moivre's theorem for $n = 5$ and then with $\theta = \pi/5$, yields a quadratic equation to solve for $\sin^2 \pi/5$.

Question 3.	**Differentiation.** Average mark: 7.9/10.
(a)	All three parts of this were answered quite well. Given that this was an online paper, I wasn't at all keen just to see the answers with no workings. Software is available these days, and I needed to see some sort of proof that this had been worked out by the student in order to give full marks. Apart from part (a) which can easily be done as a one-liner, I needed to see some workings in order to give marks.
(b)	Also answered well in general. Quite a few people didn't use the $y'' \neq 0$ criterion to identify which was the maximum and which was the minimum.

Question 4.	**Partial differentiation.** Average mark: 7.7/10.
(a)	This part was done very well in general. The final result for $\partial f/\partial y$ may be simplified into a very compact form and ideally it should have been.
(b)	There were three critical points. Some thought that there were four and included an incorrect case, $(x, y) = (0, 0)$, which comes from the two possible ways that $z_x = 0$. This means that $z_y \neq 0$ and $(0, 0)$ isn't a critical point.
	Many used the tabular form that I introduced in the lectures — many thanks. It makes it much easier to see what was going on.

Question 5.	**Integration 1.** Average mark: 6.7/10.
(a)	Integration by parts three times. Very good. Some students merely quoted the numerical answer — I need to see the workings in order to award marks. A few differentiated both terms in the integrand rather than integrating one and differentiating the other. The very great majority who used the new integration by parts technique got it right and within only a few lines. By contrast, those who used the traditional method and still got it right (many congratulations, an astonishing feat, but so wasteful of time) typically took between a page and two full pages to complete the integration.
(b)	There are two potential substitutions but this was answered quite well.
(c)	This partial fractions questions was dogged by a large variety of issues from algebraic slips to the incorrect writing down of the question on the paper/iPad. For these reasons I would have expected slightly better marks.
	The variable of integration was written incorrectly on the paper as t, rather than as s. Many saw my intent — thank you. Some thought it was a test, and merely integrated this function fo s with respect to t — I had to give the full marks. One or two did the latter but still showed that they could handle partical fractions, not that this was necessary when integrating with respect to t. Some hedged their bets and did it both ways!

Question 6.	**Integration 2.** Average mark: 6.9/10.
(a)	Generally very good. The chief error was the omission of the extra r when integrating in polar coordinates. Remember the Cornish pasty example!
(b)	V was usually correct. The solutions for A were surprisingly good. The chief error was of the following form: $(a + b)^{1/2} = a^{1/2} + b^{1/2}$ when attempting find the integral.

Question 7.	**Series.** Average mark: 5.2/10.
(a)	Often very well done. Quite a few people only quoted the final Taylor's series solution without workings. I cannot tell if this was a quotation from the formula book or the result of workings which didn't appear on the script. I needed to see some proof that the general Taylor's series expansion (which could be quoted without proof) had been used. Some provided full workings using $(e^x - e^{-x})/2$ in place of $\sinh x$. Fair enough, but it is quick to memorise the differentiation and integration rules for $\sinh x$ and $\cosh x$ because they are even easier than for $\sin x$ and $\cos x$. Occasionally the summation counter went astray.
(b)	Usually well done. Don't forget that the arbitary constant has to be shown to be zero. Occasionally the summation counter went astray.
(c)	Frequently answered incorrectly. Many answered that the radius of convergence is 1 because it was reckoned that $$\frac{(n+1)^2}{(2n+1)(2n+2)}$$ tends to towards 1 as $n \to \infty$, rather than to $\frac{1}{4}$.
(c)	Two applications of l'Hôpital's rule but somehow the 4 in $4\cosh x$ wasn't seen even if the student had written it down!

Question 8.	**Vectors 1.** Average mark: 9.0/10.
	The best question on the exam. Generally a walk in the park. Only very occasionally did the cross product go astray. Curiously part (a), a simple summation, was the next most frequent error.

Question 9.	**Vectors 2.** Average mark: 5.8/10.
	A variety of errors here.
Question 10.	**Miscellaneous.** Average mark: 4.9.
	The least favourite question. Again, zeros, poles and large-$\lvert x \rvert$ asymptotes were often not stated. Some didn't even show these features on the sketch. The rather long derivative calculation was often done well. The integrand needed to be decomposed before the application of partial fractions, but this was often missed out. Some didn't apply the limits.

Miscellaneous issues

- Some students admitted to using their graphical calculators. I applaud the honesty but I needed to see your workings.

- Two terrible glitches: $\dfrac{y}{x^2+y^2} = \dfrac{y}{x^2} + \dfrac{1}{y}$. $\qquad \left[-\dfrac{6}{16}e^{-2t}\right]_0^\infty = -\dfrac{3}{16}$.

- For partial differentiation the notation is $\dfrac{\partial f}{\partial x}$, and not $\dfrac{\delta f}{\delta x}$.

- Three errors here:

$$\int_1^3 \frac{(x-2)^2}{x(x+1)} = \int_1^3 \left[\frac{4}{x} - \frac{9}{x+1}\right] dx = \Big[4\ln x - 9\ln x + 1\Big]_1^3 = 0.1561.$$

  I reckon that the correct final answer from the incorrect application of Partial Fractions must have come from somewhere else.

- I found several instances of a definite integral with a positive integrand taking a negative value. Hmmmm, that always suggests a mistake somewhere!

- In Q10, note that the reciprocal of $y(x)$ is $1/y(x)$, and it is not the result of reflecting the $y(x)$ curve in the line, $y = x$.

- An unusual notation: -0.17956^{rad}. The superscript is never needed especially in calculus. Angles are always in radians. Angles in degrees is the exception. To see this, first sketch the graph of $\sin\theta$ where θ is in degrees. Now try to estimate the slope at $\theta = 0$ by drawing a suitable straight line on the graph and estimating its slope. It comes to roughly $1/90$ rather than 1.

- And these? $\dfrac{2\infty^2}{\infty^2} = 2$ and $\infty - \infty = 0$. Aaaarghhh!

- Terminology. $\dfrac{dy}{dx}$ of $\sinh x$ — **NO!** $\quad \dfrac{d}{dx}$ of sinh — **YES!** That said, $\dfrac{d\sinh x}{dx}$ is better again.

- It is so much better and more compact to write $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!}{n!} x^n$ as $\displaystyle\sum_{n=0}^{\infty} (n+1)x^n$.

- Note that $\displaystyle\int_0^1 x^3 \left(1+9x^4\right)^{1/2} dx \neq \int_0^1 x^3 \left(1+3x^2\right) dx$.

- Don't write *assytote*, *asymtope*, *assymtope*, *verticle* or *assymetric*. All of these should have been *asymptote* in Q1 and Q10.

- Note that *MAXIMA* is a plural while *MINIMA* is singular. Astonishingly the majority of the class got this wrong! In Q3 I was often given: *The value at $x = 1$ is a maxima*, which is incorrect. It is a maximum.

- For Q7 some wrote $\displaystyle\sum_{k=0}^{-2} \binom{-2}{k} (-x)^k$. I can see what you were trying to do, but this doesn't follow from the previous analysis. I am not sure that it even makes sense since summation counters increase.

- Both of the following are ambiguous and certainly don't follow BODMAS (or its friends). $\dfrac{e^{t^2} \cdot 2t + 1}{2\sqrt{e^{t^2} + t}}$ and $\frac{1}{2}(t + e^{t^2})^{-1/2} \times 1 + 2te^{t^2}$. One may try to write each of these using the BODMAS/PEMDAS rules to see what they really mean!

- In Q9 the general plane is $x - 4y + z = -2$. Some wrote it as $\underline{i} - 4\underline{j} + \underline{k} = 2$ which is incorrect. This latter equation tells us that a well-defined vector quantity is equal to a scalar quantity. That doesn't happen.

- Finally, never ever quote final answers to two significant figures. The error in doing so can be as large as 5%.