ME10304 – Mathematics 1 – Feedback on the 2022/2023 exam.

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General comments.

Last year's ME10304 assessment took the form of a remote two-hour exam using Inspera, as was the imposed requirement. By its nature, that exam was an open-book paper. Despite this, its average mark was about 3% below the typical value for pre-covid papers.

By contrast, this year's assessment was an in-person exam that was invigilated on-campus. I was persuaded to allow students to take two sides of a crib sheet into the exam, something that has happened for an exam of mine only once before and this was for last year's ME10305 Maths 2 paper. The outcome for the present Maths 1 paper was the same as for that ME10305 paper last year, namely that there was a drop in the average mark of well over 10% from pre-covid times.

The overall average was 57.7% and marks ranged from 4% to 98%. In total there were 37 scripts which fell below the 35% level and which will therefore attract a compulsory resit at the end of August. There were a further 19 scripts in the range 35% to 39% which, although formally a failure, may be condoned depending on the performance in all the other units in Year 1. At the other end of the spectrum there were 101 students with marks in the first class range, i.e. 70% or above. Apart from the two extremities of the mark range, the mark distribution was roughly uniform, as may be seen in the following Table.

Table: Mark distribution

$0 \le M \le 9$	3
$10 \le M \le 19$	6
$20 \le M \le 29$	18
$30 \le M \le 39$	29
$40 \le M \le 49$	55
$50 \le M \le 59$	51
$60 \le M \le 69$	47
$70 \le M \le 79$	53
$80 \le M \le 89$	31
$90 \le M \le 100$	17

With a few exceptions, the questions were answered in numerical order; this helps me greatly with the marking – many thanks. That said, you are under no obligation to do that.

While I appreciate that the examination process is stressful and fraught, may I make a plea that the final answers are in an obvious place! Some were found in the middle of workings or on another place on the page, and I fear missing them. This is especially pertinent for those whose natural writing style is, um, not so tidy. It is always better to avoid the use of multiple columns. If inspiration strikes later in the exam and, say, Q5a is answered after, say, Q10, then it is good to make some indication near the Q5b solution that Q5a appears later. Some exercise this welcome courtesy, but some don't.

Finally, please make sure that your candidate number is written both clearly and correctly for it takes quite a while to sort these out. One candidate number used a symbol which looked like a classic lollipop on a stick but with a very small circular lollipop:  and I didn't know if it was a nine or a one!

In the following I concentrate mainly on the negative aspects of the scripts, but I saw a lot of really good work.

Question 1.	**Curve sketching.** Average mark: 3.6/10.
	This question ties with Q10 for the prize of being the worst question, which was a huge surprise to me. Curve-sketching exam questions in previous years have been received much better. Quite a few students spent a lot of time writing huge amounts of text when it just isn't necessary. Some tried to find minima and maxima — again this is not necessary for this question.
(a)	(i) A large number got this one wrong. Instead of xe^{-x^2} they sketched $x^2e^{-x^2}$. (ii) It is necessary to sketch in the envelope for this one, otherwise I don't know if what is being drawn is correct or not. There were some scripts which featured the function, $(\sin x)/(1+x^2)$, but where the "envelope" was $(1+x^2)$. But it is also important to show where the zeros are on the graph, and therefore I would expect to see π, 2π, $-\pi$ and so on on the x-axis. (iii) Many sketched y^2 correctly but didn't follow it up with y. There are two square roots and therefore the sketch should look like an α.
(b)	Many good answers. Some were misled by $y \to 2$ as $x \to \infty$ and answered: $$\left(\frac{x^2-1}{x^2-4}\right)+1,$$ but this function has zeros when $x = \pm\sqrt{5/2}$.

Question 2.	**Complex numbers.** Average mark: 7.0/10.
(a)	Very good. It never feels quite right asking what the argument is for a real number. I would say that zero is the most obvious value to write, but 2π is also correct. I did get a few instances of $\pi/2$ — oops.
(b)	This answer should have been a one-liner. Given that $\pi/6$ radians is the same as $30°$, then the cosine and sine of this angle should be known analytically. I never cease to be amazed why there is a very small subset of the class, about ten in this case, who managed to stretch the working out to about a page.
(c)	We needed the fourth root of a complex number lying in the second quadrant, and this was where most mistakes were made. Some used an argument corresponding to the fourth quadrant, because this is what the calculator gives by default. Ideally I would have liked to see the four roots placed fairly accurately in the complex plane where the $n = 0$ root is about $28°$ away from and above the real axis. Some omitted the sketch of the complex plane.

Question 3.	**Differentiation.** Average mark: 7.1/10.
(a)	All three parts of this question were answered quite well. Although the three increase in complexity, it was the first that had the largest number of errors. If we note that $\ln|3t| = \ln 3 + \ln|t|$, then it is clear that its derivative is not $3/t$. The correct answer is $1/t$, although some thought incorrectly that it was $1/|t|$.
(b)	Also answered well in general. Quite a few people equated $y''(0) = 0$ with the presence of an inflexion point at $t = 0$. Strictly, all this says is that the identity of the critical point is presently inconclusive apart from the fact that it isn't a standard maximum or minimum. For this question one has to go to the third derivative to confirm that it is indeed a rising inflexion point. Many omitted the sketch of the function.

Question 4.	**Partial differentiation.** Average mark: 6.0/10.
(a)	This part was done very well in general. Only a few noticed that the expression for f_y could be obtained from the expression for f_x simply by swapping x and y.
(b)	While very many found z_x and z_y correctly, the rest of the question had frequent errors. There were four critical points, while many thought that there were three or just the one. The main error centred on how to use $z_x = 0$ and $z_y = 0$. Each of these yield two potential conditions, and this naturally gives four possible options to consider. However, many used the two options for $z_x = 0$ simultaneously, not realising that $z_y = 0$ isn't necessarily satisfied. Likewise the other way around. Those who made this mistake obtained three critical points, two of which were incorrect.
	Many used the tabular form that I introduced in the lectures — many thanks. This makes it much easier to see what was going on.

Question 5.	**Integration 1.** Average mark: 5.0/10.
(a)	Generally there was no problem with the partial fractions although some factorised $x^3 + x^2$ using either $x(x+1)^2$ or $x(x^2+1)$. Another common error was to state that the integral of $1/x^2$ is $\ln\|x^2\|$. Yet another was to attempt the partial fractions using only $A/x^2 + B/(x+1)$.
(b)	An integration first by substitution and then by parts. This was usually well-answered. Sometimes the conversion of the limits to the new variable went astray.

Question 6.	**Integration 2.** Average mark: 5.9/10.
(a)	Very good.
(b)	V was usually correct, although some differentiated the πx^6 instead. Some tried to integrate πx^3 instead. The solutions for A were either perfect or completely incorrect. One candidate didn't use the "obvious" $v = x^4$ substitution, but rather attempted to use a sinh substitution which was not only possible and unknown to me but executed perfectly. Seriously impressed with that one.

Question 7.	**Series.** Average mark: 5.3/10.
	Overall, this question was answered better than last year's Series question. Nice work.
(a)	Often very well done. Sometimes the conversion to summation form wasn't quite right. This was because the wrong first value for n, the summation index, was taken.
(b)	An interesting question from the point of view that some people only did the first bit, some only did the second, while some did both. I was hoping for both.
(c)	Very well done.
(d)	Poorly attempted if at all. The most frequent incorrect answer was: $$\lim_{n\to\infty} \frac{(n+2)}{(n+1)}\|x\| = 2\|x\|,$$ from which it was concluded that the radius of convergence is $1/2$. The above limit should be $\|x\|$.

Question 8.	**Vectors 1.** Average mark: 8.8/10.
	The best question on the exam, as is traditional. Generally a walk in the park. Only very occasionally did the cross product go astray. Curiously, part (a), a simple summation, was the next most frequent error.

Question 9.	**Vectors 2.** Average mark: 5.3/10.
	A variety of errors here. Part (a) was usually correct, as was part (b). Parts (c) and especially (d) were not received well. Quite a few students didn't attempt this one, which is why the average is lower than one might expect given my comments. A sizeable minority obtained full marks.
Question 10.	**Miscellaneous.** Average mark: 3.6/10.
	The least favourite question for two reasons: (i) I always design this last one to bring in quite a variety of techniques because that is what we meet in real life engineering, and (ii) many students don't attempt it. Some of those who did attempt the question managed only to do the very first part of Q10a and the whole of Q10c. The correct solution for Q10b is absolutely dependent on Q10a being correct. In those cases where Q10a and Q10b were incorrect but Q10c was correct, the conclusion was drawn that the use of Taylor's series to find the integral is very inaccurate. For those students I feel the need to say that it can be very highly accurate. Indeed, the two solutions here, the numerical and the analytical, differ only in the sixth significant figure. Generally this particular agreement is much better than one would need in practice. So we need to give props to Taylor's series. You may, therefore, have noticed that I have actually used an exam question to pass on some teaching, namely the use of Taylor's series to evaluate integrals. Sorry. However, there were a sizeable group who got this one completely correct — chapeau!