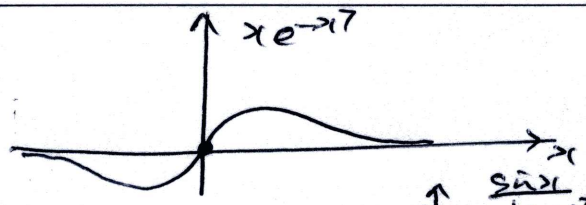
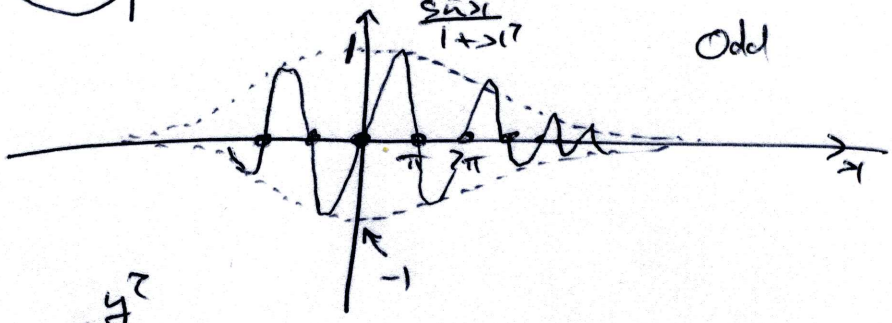
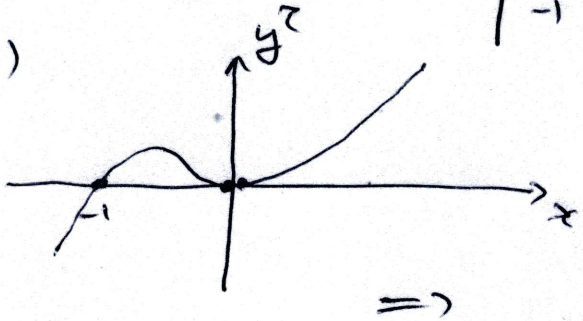
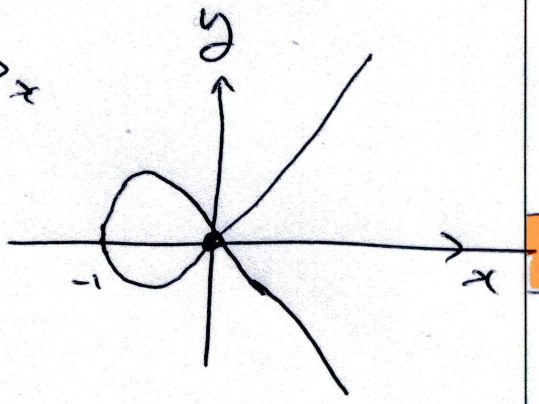
UNIVERSITY OF BATH
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 1	Page 1 of 1	

Part		Mark
(a)	(i) 	2
	(ii) 	2
	(iii)  	3
(b)	$2\left(\frac{x^2-1}{x^2-4}\right)$	3
	Total	10

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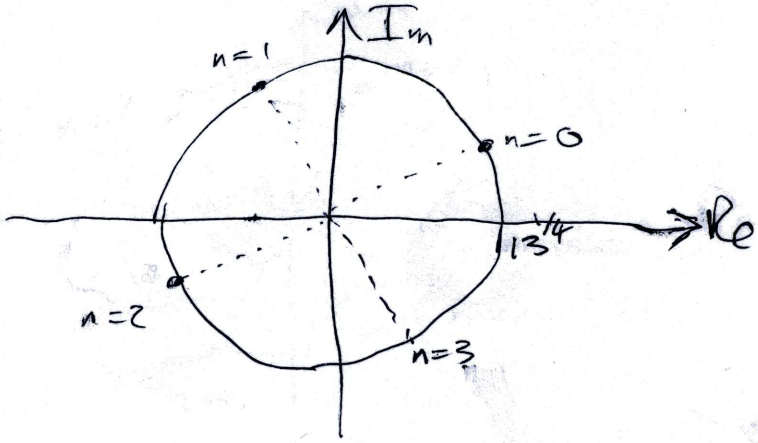
Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 2	Page 1 of 1	

Part		Mark
(a)	$(1+j)(2+j) = (1+3j)$. Hence answer = 1.	3
	Cartesian: 1	
	Complex exp: $1e^{0j}$	
(b)	$\sqrt{3}+j$	2
(c)	If $z = (-5+12j)$ then $z = 13e^{\Theta j}$	3
	where $\Theta = -1.176005 + \pi$	
	$= 1.965587$	
	Hence $z^{1/4} = \left[13e^{(\Theta+2n\pi)j}\right]^{1/4}$ $n=0,1,2,3$	
	$= 13^{1/4} e^{\left(\frac{\Theta}{4}+\frac{n\pi}{2}\right)j}$, $n=0,1,2,3$	
		2
	No. $\Theta = 112.6198°$	
	$\Rightarrow \Theta/4 = 28.1550°$	
	Total	10

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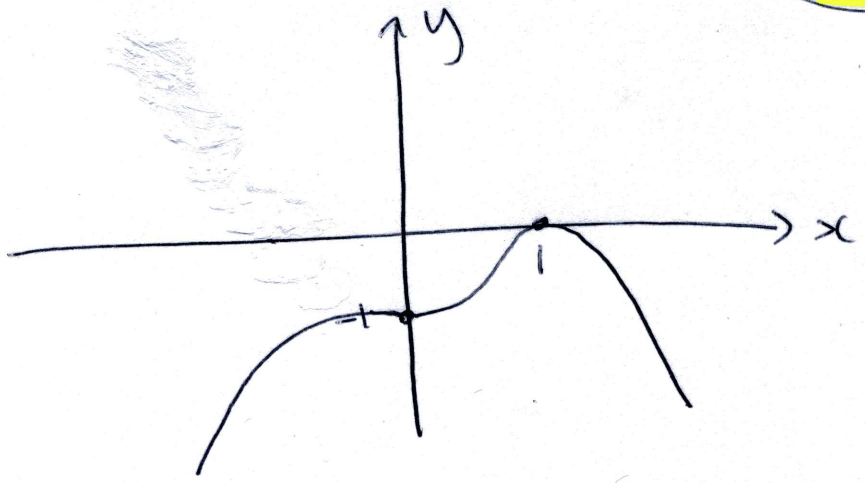
Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 3	Page 1 of 1	

Part		Mark
(a)	(i) $y = \ln\|3t\| = \ln 3 + \ln\|t\| \Rightarrow y' = 1/t$	1
	(ii) $\frac{t^2}{\cos t} \xrightarrow{d/dt} \frac{(\cos t)2t - t^2(-\sin t)}{\cos^2 t}$ $= \frac{2t\cos t + t^2 \sin t}{\cos^2 t}$	2
	(iii) $\sin(te^{-t^2}) = \cos(te^{-t^2}) \times e^{-t^2}(1-2t^2)$	3
(b)	$y = 4t^3 - 3t^4 - 1 \Rightarrow y' = 12t^2 - 12t^3$	
	$y' = 0 \Rightarrow t = 0, 1$ — two critical pts.	
	$y'' = 24t - 36t^2 \Rightarrow \begin{cases} y''(0) = 0 & \text{inconclusive} \\ y''(1) = -12 & \text{maximum} \end{cases}$	
	$y''' = 24 - 72t \Rightarrow y'''(0) = 24$ rising inflexion.	
		4
	Total	10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 4	Page 1 of 1	

Part		Mark
(a)	$f_x = \dfrac{y^2-2xy-x^2}{(x^2+y^2)^2}$ $\quad f_y = \dfrac{x^2-2xy-y^2}{(x^2+y^2)^2}$	3
(b)	$z = xy^2 + x^2y - 3xy$	
	$z_x = y^2+2xy-3y=0 \Rightarrow$ $y[y+2x-3]=0$	
	$z_y = 2xy+x^2-3x=0 \Rightarrow$ $x[2y+x-3]=0$	
	Critical pts. (i) $x=y=0$	
	(ii) $y=0,\ 2y+x-3=0$	
	(iii) $x=0,\ y+2x-3=0$	
	(iv) $[\ ]=0,\ [\ ]=0$	
	$(0,0)$ $(3,0)$ $(0,3)$ $(1,1)$	
	$z_{xx} = 2y \quad z_{yy} = 2x \quad z_{xy} = 2x+2y-3$	
	Table:	

x	y	z_{xx}	z_{yy}	z_{xy}	H	conclusion
0	0	0	0	−3	−9	Saddle
3	0	0	6	3	−9	"
0	3	6	0	3	−9	"
1	1	2	2	1	3	Min $(z_{xx}>0)$

7

Total	10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 5	Page 1 of 1	

Part		Mark
(a)	$\frac{1}{x^3+x^2} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}$ Hence $\int_1^2 \frac{dx}{x^3+x^2} = \int_1^2 \left[\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right] dx$ $= \left[\ln\left\lvert\frac{x+1}{x}\right\rvert - \frac{1}{x}\right]_1^2$ $= \left(\ln\frac{3}{4}\right) + \frac{1}{2}$ or $= 0.212318$	5
(b)	$I = \int_0^1 x^2 \ln x \, dx$ Let $x = e^{-y} \Rightarrow dx = -e^{-y}dy$ $x=0 \Rightarrow y=\infty$ $x=1 \Rightarrow y=0$ Hence $I = \int_\infty^0 e^{-2y}(-y)(-e^{-y})dy = \int_\infty^0 y e^{-3y} dy$ $= -\int_0^\infty y e^{-3y} dy$ $= -\left[(y)\left(\frac{e^{-3y}}{-3}\right) - (1)\left(\frac{e^{-3y}}{9}\right)\right]_0^\infty = -1/9$	5
	Total	10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 6	Page 1 of 1	

Part		Mark
(a)	$V = \int_0^{\pi/2}\int_0^{2} r^3\cos^2\theta \, r\,dr\,d\theta = \int_0^{\pi/2}\left[\frac{1+\cos 2\theta}{2}\right]d\theta \times \int_0^{2} r^3\,dr$ $= \pi/4 \times \frac{2^4}{4} = \pi$	3
(b)	$V = \int_c^1 \pi y^2\,dx = \int_c^1 \pi x^6\,dx = \pi/7$	1
	$S = 2\pi\int_c^1 y\left[1+y'y'\right]^{1/2}dx = 2\pi\int_c^1 x^3\left[1+9x^4\right]^{1/2}dx$ Let $v = 1+9x^4 \Rightarrow dv = 36x^3\,dx$ $x=0 \Rightarrow v=1$ $x=1 \Rightarrow v=10$ $\Rightarrow S = 2\pi\int_1^{10} v^{1/2}\frac{dv}{36} = \frac{\pi}{18}\left[\frac{2}{3}v^{3/2}\right]_1^{10}$ $= \frac{\pi}{27}\left[10^{3/2}-1\right]$ OR 3.56312 (5 DP)	6
		Total 10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 7	Page 1 of 1	

Part		Mark
(a)	$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$	2
(b)	$\lim_{x\to 0}\left[\frac{\sinh x - x}{x^3}\right] = \frac{1}{6}$	2
	l'Hôpital: $\lim_{x\to 0}\left[\frac{\sinh x - x}{x^3}\right] = \lim_{x\to 0}\left[\frac{\cosh x - 1}{3x^2}\right] = \lim_{x\to 0}\left[\frac{\sinh x}{6x}\right] = \lim_{x\to 0}\left[\frac{\cosh x}{6}\right] = 1/6$	2
(c)	$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)(-x)^2}{2} + \frac{(-2)(-3)(-4)(-x)^3}{2\cdot 3} + \cdots = 1 + 2x + 3x^2 + 4x^3 \cdots = \sum_{n=0}^{\infty} (n+1)x^n$	2
(d)	$u_n = (n+1)x^n \Rightarrow u_{n+1} = (n+2)x^{n+1}$ $\Rightarrow \left\lvert\frac{u_{n+1}}{u_n}\right\rvert = \frac{(n+2)}{(n+1)}\lvert x\rvert \longrightarrow \lvert x\rvert$ as $n\to\infty$ $\Rightarrow$ Radius of convergence is 1	2
	Total	10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 8	Page 1 of 1	

Part		Mark
-	(a) $\begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$ (b) 16 (c) 3 (d) 7 (e) $\begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ (f) 40.3676° or 0.704455 radians (g) 90° (h) $\begin{pmatrix} 9 \\ -10 \\ 2 \end{pmatrix}$	-
	Total	10

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Outline Solution to the Resit Examination Question

Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 9	Page 1 of 1	

Part		Mark
(a)	$\underline{r} = \underline{a} + \lambda(\underline{b}-\underline{a}) \Rightarrow r = \begin{pmatrix}-3\\3\\-3\end{pmatrix} + \lambda\begin{pmatrix}8\\-2\\2\end{pmatrix}$ So $0 = \begin{pmatrix}-3\\3\\-3\end{pmatrix}\cdot\begin{pmatrix}8\\-2\\2\end{pmatrix} + \lambda\begin{pmatrix}8\\-2\\2\end{pmatrix}\cdot\begin{pmatrix}8\\-2\\2\end{pmatrix}$ $\Rightarrow 0 = -36 + 72\lambda \Rightarrow \lambda = 1/2$ Hence $\underline{r} = \begin{pmatrix}1\\2\\-2\end{pmatrix} \Rightarrow \lvert\underline{r}\rvert = 3$	2
(b)	$\underline{r} = \underline{a} + \lambda(\underline{b}-\underline{a}) + \sigma(\underline{c}-\underline{a}) = \begin{pmatrix}-3\\3\\-3\end{pmatrix} + \lambda\begin{pmatrix}8\\-2\\2\end{pmatrix} + \sigma\begin{pmatrix}4\\0\\2\end{pmatrix}$	2
(c)	$\underline{p} = \begin{vmatrix} i & j & k \\ 8 & -2 & 2 \\ 4 & 0 & 2 \end{vmatrix} = \begin{pmatrix}-4\\-8\\8\end{pmatrix}$ But $\lvert\underline{p}\rvert = 12 \Rightarrow \hat{p} = \begin{pmatrix}-1/3\\-2/3\\2/3\end{pmatrix}$ (1) From (b) $\underline{r}\cdot\hat{p} = \underline{a}\cdot\hat{p} = -3$ $\Rightarrow$ Distance from the origin is 3.	3
(d)	Eqn (1) gives $-x/3 - 2y/3 + 2z/3 = -3$ or $x + 2y - 2z = 9$. When $z=0$ then $x + 2y = 9$	3
	Total	10

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Examiner: Dr D A S Rees		Date January 2023	
Unit Title: Mathematics 1		Unit Code: ME10304	
Year: 2022/23	Question Number: 10	Page 1 of 1	

Part		Mark
(a)	Eventually get $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $\Rightarrow e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots$ $\Rightarrow x e^{-x^2} = \boxed{x - x^3 + \frac{x^5}{2!} - \frac{x^7}{3!} + \cdots}$	4
(b)	So $\int_0^{0.5} \left[x - x^3 + \frac{x^5}{2!} - \frac{x^7}{3!}\right] dx = \left[\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{12} - \frac{x^8}{48}\right]_0^{0.5}$ $= \boxed{0.110\,596}$ (6 DPs)	3
(c)	Let $y = x^2 \Rightarrow dy = 2x\,dx$. So $\int x e^{-x^2} dx = \frac{1}{2}\int e^{-y} dy = -\frac{1}{2} e^{-y}$ $= -\frac{1}{2} e^{-x^2}$. $\Rightarrow \int_0^{1/2} x e^{-x^2} dx = -\frac{1}{2} e^{-x^2}\Big|_0^{1/2} = \frac{1}{2}\left(1 - e^{-0.25}\right)$ $= \boxed{0.110\,600}$ Numerical error is $\approx \boxed{4 \times 10^{-6}}$.	3
	Total	10