Notes for the ME10304 Mathematics 1 examination (January 2021).

Exam Rubric

The rubric for the exam will state that you should attempt ALL questions. There is no choice, such as 'answer four from six', or 'three from five'. The mathematics units are service units, meaning that the entire content is deemed to be essential for all the other applied units you will be taking at the university.

Each of the ten questions attracts a mark of 10%.

It is important to note the syllabus can change slightly as the years go by. Although the exams of the last five years have the above rubric, the 15/16 paper has a question on Fourier Series — do not attempt it! This topic will be covered in Mathematics 2.

Exam provision

There will be a link to the university's formula book. You will be allowed to use any notes which I have presented to you and likewise the problem sheet solutions. None of the questions will be found in these documents, but there may well be similarities. Obviously any assistance from a live person, a social media group or online software or their equivalents is not allowed — I am hoping to examine your understanding.

Normally you will also be provided with a standard university calculator in the examination room. Given that you unlikely to do anything more than to find an inverse cosine as part of the determination of the angle between two vectors or an inverse tangent when converting complex numbers from Cartesian form to polar form, then any calculator will be fine.

General Comments.

My general aim for the Mathematics units is that you become competent at applying the techniques which have been taught and, in some circumstances, deciding which technique ought to be used if there is a choice. I will be marking what you write. Be aware that "method marks" can be negative as well as positive. A good analysis with an accidental wrong number at the end will attract high marks, whereas a correct answer preceded by demonstrable rubbish will have very few marks if any.

Detailed Comments.

1. Curve sketching. (One question)

   These come in a variety of types including (but not necessarily confined to) products of functions (such as xe^{-x}), functions of functions (such as $e^{\sin x}$ or $\sin(\pi e^{-x})$), envelopes (such as $e^{-x}\sin x$), ratios of polynomials (such as $x(x-2)/(x-1)^2$), and curves for which $y^2 = f(x)$.

   In many cases it is of the utmost importance to find where the zeros and poles are together with their respective multiplicities, and the large-$|x|$ behaviour. The locations of the zeros and poles MUST be shown on the sketch. Occasionally it is good to state that the function is either even or odd (but only if it is, of course!). There is no need at all to find critical points in these questions.

   All of these questions involve the standard functions, i.e. polynomials, exponentials, trigonometric functions and hyperbolic functions, and how they behave when they operate upon one another. Do make sure that you are confident about the shapes of the sinh, cosh and tanh functions — there is a handout on the Maths 1 webpage.

   I may possibly give a sketch and ask you to identify the curve; examples are on problem sheet 1.

2. Complex numbers. (One question)

   You will need to know how to convert complex numbers from $a + bj$ form to $re^{j\theta}$ form and vice versa.

   Make sure that you know the trick (complex conjugate) that's used to divide two complex numbers when in Cartesian form.

   When finding the roots of complex numbers, such as $(a + bj)^{1/k}$ where k is a positive integer, do ensure that your complex exponential form has k different versions:

$$a + bj = re^{j(\theta+2\pi n)}, \qquad n = 0, 1, ..., k - 1,$$

   and then the k^{th} root may be taken to obtain k different answers. I am happy for these to be left in complex exponential form, but θ must be found and given in radians. Any answer which uses degrees will automatically be awarded zero marks — the use of degrees in a complex exponential or within calculus in general is a heinous crime against mathematics, and is the only thing which induces exam-marking rage in me.

3. Differentiation. (Two questions)

   It goes without saying that you should be able to apply the product, quotient and chain rules safely. When finding the derivative of something more complicated, such as $\sin(xe^x)$, which is the function of a product, it might be easier to find the derivative of xe^x before the main analysis is done — this will simplify the presentation quite nicely.

   So-called implicit differentiation may always be treated as an example of the chain rule. If $y = y(x)$, then an example might be:

$$\begin{aligned}(xy^2)' &= (x)'(y^2) + (x)(y^2)' && \text{product rule}\\ &= (1)(y^2) + (x)(2yy') && \text{product rule on } y^2\\ &= y^2 + 2xyy' && \text{tidying up}\end{aligned}$$

   This is why I haven't bothered to use the terminology, as there is nothing new to learn.

   For the identification and classification of critical points I am looking solely for the application of the primary and secondary criteria as outlined in the lectures. Do not evaluate functions or play around with the 'zeros and arrows' method (if you don't understand the latter terminology, then don't worry; I mention it because some have been taught this way in the past). For this exam I am restricting your mode of analysis entirely to differentiation.

   In partial differentiation the bulk of the marks will come from identifying and classifying the critical points of a surface. If the surface is given by $z = z(x, y)$, then you will need to use the formula,

$$H = z_{xx}z_{yy} - (z_{xy})^2.$$

   [Of course, z_{xx}, is shorthand for the second partial derivative of z with respect to x.] The classification of these critical points will then depend on the sign of H and perhaps a secondary criterion.

   You will NOT be tested on the small-increment method of differentiation — this was solely a teaching aid to show where certain formulae came from.

4. Integration. (Two questions)

   I will not always tell you what method ought to be used for any given integral. Therefore the ability to identify the category of the integral is essential: substitution, by parts, using partial fractions, f'/f form. Some integrals may have more than one way of being obtained. But the use of my method of integration by parts may well save you precious minutes and marks in the exam, so I recommend it very strongly to you particularly since it will be needed next semester in at least two topics in Maths 2, and in Modelling Techniques 2 next year.

For volumes under surfaces, do remember that the polar coordinate version requires an extra r. So if we wish to find the volumes under $f(x, y)$ and $g(r, \theta)$, then the respective double integrals need to be

$$\int\int f(x,y)\,dx\,dy, \qquad \int\int g(r,\theta)\,r\,dr\,d\theta.$$

In both cases one may swap the order of integration around if it is advantageous to do so.

5. Series. (One question)

   Definitions of Taylor's series and the binomial series may be found in the formula book, although the lecture notes is a better source.

   You'll definitely have part of a question on d'Alembert's ratio test and l'Hôpital's rule.

6. Vectors. (Two questions)

   You will need to make sure that you can find the vector product securely, particularly if you belong to that part of the class which hasn't met determinants before.

   Angles between two vectors may have the answer quoted in degrees because there is no calculus involved.

   For the geometric applications (lines and planes and points), I have taught the material in as visual a way as possible because you are engineers and I presume that this ability to visualise will be an essential part of your future jobs. Therefore I will generally expect you to do the same in the exam. However, you may wish to use memorised formulae — if you do so (because this was the way it was done in school for those few of you who have covered the topic before) and you have quoted the formula incorrectly then I have no option but to award zero marks for that part of the question.

If you add up the number of questions above, then it comes to nine. There will be a tenth, and this will be drawn from within the above topics. It will involve more than one topic but will have a common theme of some sort. See problem sheet 12 and the last four years' exam papers for a good idea of what this tenth question might be like.

In the 15/16 exam paper one question was on Fourier Series. Ignore it! This topic will be taught in semester 2.

To conclude...

If you have any further doubts about what is required, then check the manner in which the last five years' exam papers have been written. That aspect will not change.

If you still have any further doubts, then do email me. I will endeavour to answer emails within 24 hours, although that won't happen on Christmas Day itself! Indeed, it would be unwise to trust the quality of any email that you might receive from me on Christmas day!

Once the exams are over you will be free to check out the Maths 2 website. However, I will be using the time from now to then to upgrade the typeset notes to a form which is similar to what we have had for Maths 1. Anything that has been updated will have a clear indication that it is ready to read. So at the present moment that is precisely zero.

Best wishes for the exam.