Notes for the ME10304 Mathematics 1 examination (January 2023).

Exam Rubric

The rubric for the exam will state that you should attempt ALL questions. There is no choice, such as 'answer four from six', or 'three from five'. The mathematics units are service units, meaning that the entire content is deemed to be essential for all the other applied units you will be taking at the university.

Each of the ten questions attracts a mark of 10%.

Although the unit's syllabus does change slightly as the years go by it has remained static over the last five years, and therefore all the questions on the exam papers given on the Maths 1 web page may be considered to be typical. That said, do be warned that the 20/21 paper was designed as a open-book paper taken remotely with a three week deadline, and the 21/22 was the same except that it was a two-hour paper. Therefore the questions on the 20/21 paper may be longer and/or more difficult than for an invigilated paper, Do not treat the 20/21 paper as being a typical paper.

Exam provision

You will be provided with the university's formula book. There is a link on the Maths 1 webpage for you to see what the formula book contains.

You will be allowed to prepare your own A4 sheet of crib notes, both sides.

In the exam rooms you will provided with a standard university calculator. The details of the model may also be found on the Maths 1 webpage. Bear in mind that you will be unlikely to need to use it for anything more complicated than finding an inverse cosine or an inverse tangent, but it's still worth having some rehearsal time before the day just in case its operation is unfamiliar.

General Comments.

My general aim for the Mathematics units is that you become competent at applying the techniques which have been taught and, in some circumstances, deciding which technique ought to be used if there is a choice. I will be marking what you write. Be aware that "method marks" can be negative as well as positive. A good analysis with an accidental wrong number at the end will attract high marks, whereas a correct answer preceded by demonstrable rubbish will have very few marks if any. I will want to see some workings-out, especially for more advanced things like a vector cross product.

Please avoid the use of pencils in the exam, especially the hard ones like 2H. My eyes aren't as good as they were 30 years ago!

Detailed Comments.

1. Curve sketching. (One question)

   These come in a variety of types including (but not necessarily confined to) products of functions (such as xe^{-x}), functions of functions (such as $e^{\sin x}$ or $\sin(\pi e^{-x})$), envelopes (such as $e^{-x}\sin x$), ratios of polynomials (such as $x(x-2)/(x-1)^2$), and curves for which $y^2 = f(x)$.

   In many cases it is of the utmost importance to find where the zeros and poles are together with their respective multiplicities, and the large-$|x|$ behaviour. The locations of the zeros and poles MUST be shown on the sketch. Occasionally it is good to state that the function is either even or odd (but only if it is, of course!). There is no need at all to find critical points in these questions.

   All of these questions involve the standard functions, i.e. polynomials, exponentials, trigonometric functions and hyperbolic functions, and how they behave when they operate upon one another. Do make sure that you are confident about the shapes of the sinh, cosh and tanh functions — there is a handout on the Maths 1 webpage.

   I may possibly give a sketch and ask you to identify the curve; examples are on problem sheet 1.

2. <u>Complex numbers.</u> (One question)

   You will need to know how to convert complex numbers from $a + bj$ form to $re^{j\theta}$ form and vice versa.

   Make sure that you know the trick (complex conjugate) that's used to divide two complex numbers when in Cartesian form.

   When finding the roots of complex numbers, such as $(a+bj)^{1/k}$ where k is a positive integer, do ensure that your complex exponential form has k different versions:

$$a + bj = re^{j(\theta+2\pi n)}, \qquad n = 0, 1, ..., k-1,$$

   and then the kth root may be taken to obtain k different answers. I am happy for these to be left in complex exponential form, but θ must be found and given in radians. Any answer which uses degrees will automatically be awarded zero marks — the use of degrees in a complex exponential or within calculus in general is a heinous crime against mathematics, and is the only thing which induces exam-marking rage in me.

3. <u>Differentiation.</u> (Two questions)

   It goes without saying that you should be able to apply the product, quotient and chain rules safely. When finding the derivative of something more complicated, such as $\sin(xe^x)$, which is the function of a product, it might be easier to find the derivative of xe^x before the main analysis is done — this will simplify the presentation quite nicely.

   So-called implicit differentiation may always be treated as an example of the chain rule. If $y = y(x)$, then an example might be:

$$\begin{aligned} (xy^2)' &= (x)'(y^2) + (x)(y^2)' && \text{product rule} \\ &= (1)(y^2) + (x)(2yy') && \text{product rule on } y^2 \\ &= y^2 + 2xyy' && \text{tidying up} \end{aligned}$$

   This is why I haven't bothered to use the terminology, for there is nothing new to learn.

   For the identification and classification of critical points I am looking solely for the application of the primary and secondary criteria as outlined in the lectures. Do not evaluate functions or play around with the 'zeros and arrows' method (if you don't understand the latter terminology, then don't worry; I mention it because some have been taught this way in the past). For this exam I am restricting your mode of analysis entirely to differentiation.

   In partial differentiation the bulk of the marks will come from identifying and classifying the critical points of a surface. If the surface is given by $z = z(x, y)$, then you will need to use the formula,

$$H = z_{xx}z_{yy} - (z_{xy})^2.$$

   [Of course, z_{xx}, is shorthand for the second partial derivative of z with respect to x.] The classification of these critical points will then depend on the sign of H and perhaps a secondary criterion. Here is where the crib sheet will be useful.

   You will NOT be tested on the small-increment method of differentiation — this was solely a teaching aid to show where certain formulae came from.

4. <u>Integration.</u> (Two questions)

   I will not always tell you what method ought to be used for any given integral. Therefore the ability to identify the category of the integral is essential: substitution, by parts, using partial fractions, f'/f form. Some integrals may have more than one way of being obtained, and sometimes a substitution will reduce a integral to one where Integration bu Parts needs to be done afterwards. But the use of my method of integration by parts may well save you precious minutes and marks in the exam, so I recommend it very strongly to you particularly since it will be needed next semester in at least two topics in Maths 2, and also in Modelling Techniques 2 next year.

   For volumes under surfaces, do remember that the polar coordinate version requires an extra r. So if we wish to find the volumes under $f(x, y)$ and $g(r, \theta)$, then the respective double integrals need to be

   $$\int\int f(x, y)\, dx\, dy, \qquad \int\int g(r, \theta)\, r\, dr\, d\theta.$$

   In both cases one may swap the order of integration around if it is advantageous to do so.

5. <u>Series.</u> (One question)

   Definitions of Taylor's series and the binomial series may be found in the formula book.

   You'll definitely have part of a question on d'Alembert's ratio test and l'Hôpital's rule.

6. <u>Vectors.</u> (Two questions)

   You will need to make sure that you can find the vector product securely, particularly if you belong to that part of the class which hasn't met determinants before.

   Angles between two vectors may have the answer quoted in degrees because there is no calculus involved.

   For the geometric applications (lines and planes and points), I have taught the material in as visual a way as possible because you are engineers and I presume that this ability to visualise will be an essential part of your future jobs. Therefore I will generally expect you to do the same in the exam. However, you may wish to use memorised formulae — if you do so (because this was the way it was done in school for those few of you who have covered the topic before) and you have quoted the formula incorrectly then I have no option but to award zero marks for that part of the question.

If you were to add up the number of questions above then it comes to nine. There will be a tenth, and this will be drawn from within the above topics. It will involve more than one topic but will have a common theme of some sort. See problem sheet 12 for some examples, and Q10 examples from the last five years' exam papers for a good idea of what this tenth question might be like.

<u>To conclude...</u>

If you have any further doubts about what is required, then check the manner in which the last five years' exam papers have been written. That aspect will not change.

If you still have any further doubts, then do email me. I will endeavour to answer emails within 24 hours, although that won't happen on Christmas Day itself! Indeed, it would be unwise to trust the quality of any email that you might receive from me on Christmas day!

Over the period between the end of teaching and Christmas itself I will modifying the online notes for Maths 2 and performing any necessary upgrade to other material (e.g. problem sheets). The main course notes were typeset as a new entity two years ago because of covid, but there still remains a few typos. I might even get some extra inspiration over Christmas and add some new example problems in that text. So the message is: don't print these out for the slightly new stuff will be available at the start of the second semester.

Best wishes for the exam.