**UNIVERSITY OF BATH**
**DEPARTMENT OF MECHANICAL ENGINEERING**

Outline Solution to Examination Question

| | | |
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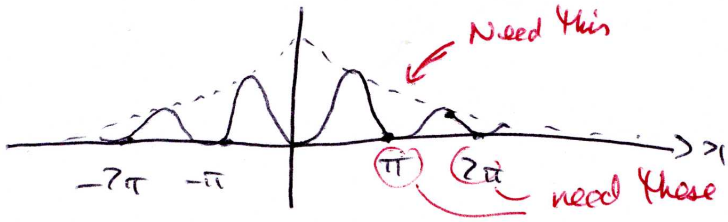
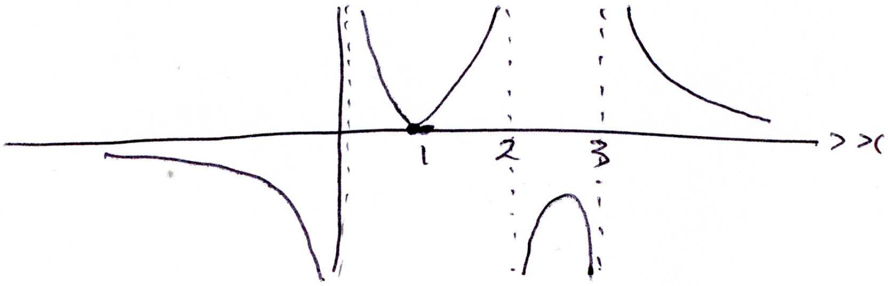
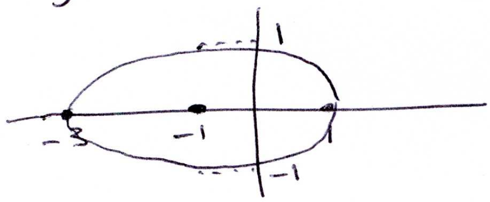
| Examiner: Dr D A S Rees | | Date January 2021 |
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 1 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) |  | 2 |
| (b) | Zeros @ $x=1,1$ Poles at $x=0,2,3$. $y \approx 1/x$ when $\lvert x\rvert \gg 1$<br> | 2 |
| (c) | $x^2+2x+4y^2=3 \Rightarrow (x+1)^2+4y^2=4$<br> | 3 |
| (d) | $-\frac{(x+1)(x-1)}{x^2}$ or $\frac{1-x^2}{x^2}$<br>or $\frac{1}{\lvert x\rvert}-1$ or $2^{\lvert\frac{1}{x}\rvert}-2$ | 3 |
| | Total | 10 |

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| Year: 2020/21 | Question Number: 2 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | $z = \frac{2+3j}{3-j} = \frac{(2+3j)(3+j)}{(3-j)(3+j)} = \frac{3+11j}{10}$. $\boxed{\bar{z} = \frac{3-11j}{10}}$. $\lvert z\rvert = \sqrt{\frac{13}{10}}$ | [3] |
| (b) | $2e^{j\pi/4} = \sqrt{2} + \sqrt{2}j$ | [2] |
| (c) | $z = 117 + 44j = 125\, e^{j(\theta + 2\pi n)}$, $n = 0, 1, 2$ | |
| | & where $\theta = \tan^{-1}\frac{44}{117} = 0.3597$ | [2] |
| | $\Rightarrow \quad z^{1/3} = 5\, e^{j(\theta/3 + 2\pi n/3)}$, $n = 0, 1, 2$ | |
| (d) | $\cos 4\theta + j \sin 4\theta = (c + js)^4$ | |
| | $= c^4 + 4jsc^3 + 6j^2s^2c^2 + 4j^3s^3c + j^4s^4$ | |
| | $= (c^4 - 6s^2c^2 + s^4) + j\, 4sc(c^2 - s^2)$ | [3] |
| | ↓ or $8c^4 - 8c^2 + 1$ | |
| | or $8s^4 - 8s^2 + 1$ | |

| | Total | 10 |
|---|---|---|

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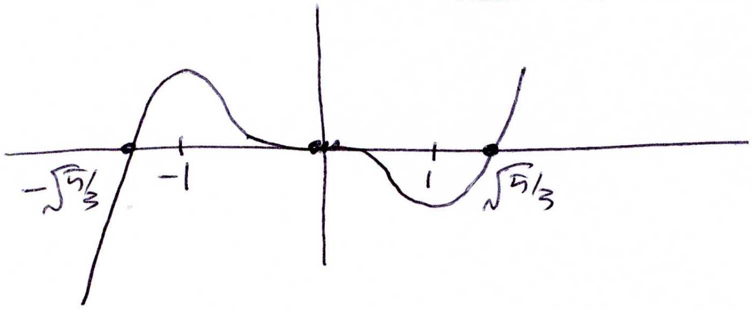
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 3 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | (i) $e^{-2t}(-2)(\cos 2t + \sin 2t)$ | 1 |
| | (ii) $e^{-t}(1-t)\,e^{te^{-t}}$ | 2 |
| | (iii) $\dfrac{t\cos(t^2+1)}{\sqrt{\sin(t^2+1)}}$ | 2 |
| (b) | $y = 3t^5 - 5t^3 \implies y' = 15(t^4 - t^2)$ <br> $\implies$ roots @ $\boxed{t = 0, 1, -1}$ <br> $y'' = 15(4t^3 - 2t)$. $\quad y''(0) = 0 \implies$ inconclusive <br> $y''(1) = 1 \implies$ minimum <br> $y''(-1) = -1 \implies$ maximum. <br> $y''' = 15(12t^2 - 2)$ $\quad y'''(0) = -30$ <br> $\implies$ descending inflexion. <br> need this | 4 |
| |   | 1 |
| | Total | 10 |

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Outline Solution to Examination Question

| | | |
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| Examiner: Dr D A S Rees | | Date January 2021 |
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 4 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | $f_x = \dfrac{4xy^2}{(x^2+y^2)^2}$ $\quad f_y = -\dfrac{4yx^2}{(x^2+y^2)^2}$ | 2 |
| (b) | $z = x^3 - y^2x - 3x^2 + 3y^2$ <br> $\Rightarrow z_x = 3x^2 - y^2 - 6x$ & $z_y = 6y - 2yx = 2y(3-x)$ <br> $z_y = 0 \Rightarrow y = 0$ or $x = 3$; $z_x = 0 \Rightarrow$ $\begin{cases} 3x^2 - 6x = 0 \Rightarrow x = 0, 2 \\ 9 - y^2 = 0 \Rightarrow y = \pm 3 \end{cases}$ | 3 |
| | $z_{xx} = 6x - 6 \quad z_{yy} = 6 - 2x \quad z_{xy} = -2y$ | 1 |

| $x$ | $y$ | $z_{xx}$ | $z_{yy}$ | $z_{xy}$ | $H$ | |
|---|---|---|---|---|---|---|
| 0 | 0 | −6 | 6 | 0 | −36 | Saddle |
| 2 | 0 | 6 | 2 | 0 | 12 | Min ($z_{xx} > 0$) |
| 3 | 3 | 12 | 0 | −6 | −36 | Saddle |
| 3 | −3 | 12 | 0 | +6 | −36 | Saddle |

Using $H = z_{xx}z_{yy} - z_{xy}^2$ — Mark: 4

Total: 10

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| Unit Title: Mathematics 1 | | Unit Code: ME10304 | |
| Year: 2020/21 | Question Number: 5 | Page 1 of 1 | |

| Part | | Mark |
|---|---|---|
| (a) | $I=\int_0^\pi t^3\cos t\,dt = [t^3][\sin t]_0^\pi - [3t^2][-\cos t]_0^\pi + [6t][-\sin t]_0^\pi - [6][\cos t]_0^\pi$ | |
| | $= -3\pi^2+12$ | (2) |
| (b) | $I=\int_0^\infty \frac{2x}{(1+x^4)^{3/2}}dx.$ Let $y=x^2 \Rightarrow dy=2x\,dx.$ Limits are the same. | |
| | $\Rightarrow I=\int_0^\infty \frac{dy}{(1+y^2)^{3/2}}.$ Let $y=\tan\theta \Rightarrow dy=\sec^2\theta\,d\theta$; $y=0\Rightarrow\theta=0$; $y\to\infty\Rightarrow\theta=\pi/2.$ | |
| | $\Rightarrow I=\int_0^{\pi/2}\frac{\sec^2\theta\,d\theta}{\sec^3\theta}=\int_0^{\pi/2}\cos\theta\,d\theta = 1.$ | (6) |
| (c) | Either $f'/f$ form $\Rightarrow I=\ln\lvert t^2+t\rvert+c$ | |
| | or Partial Fractions | |
| | $I=\int\left(\frac{1}{t}+\frac{1}{t+1}\right)dt=\ln\lvert t\rvert+\ln\lvert t+1\rvert+c$ | |
| | $=\ln\lvert t^2+t\rvert+c.$ | (2) |
| | Total | 10 |

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Outline Solution to Examination Question

| Examiner: Dr D A S Rees | | Date January 2021 |
|---|---|---|
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 6 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | $V = \int_0^{\pi/4}\int_0^3 r\cos\theta \, r\,dr\,d\theta = \int_0^{\pi/4}\cos\theta\,d\theta \times \int_0^3 r^2\,dr$ <br> $= [\sin\theta]_0^{\pi/4}\left[r^3/3\right]_0^3 = 9/\sqrt{2}$ | 3 |
| (b) | $V = \pi\int_0^1 x^6\,dx = \pi/7$ | 1 |
| | $A = 2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx$ <br> Let $z = 9x^4 \Rightarrow dz = 36x^3\,dx$ <br> & $x=0 \Rightarrow z=0$ $\quad x=1 \Rightarrow z=9$ <br> Hence $A = 2\pi\int_0^9 \frac{1}{36}(1+z)^{1/2}\,dz$ <br> $= \frac{2\pi}{36}\times\frac{2}{3}(1+z)^{3/2}\Big\vert_0^9$ <br> $= \frac{\pi}{27}\left(10^{3/2}-1\right) = 3.563122$ (6DP) | 5 |
| | Total | 10 |

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Outline Solution to Examination Question

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| Unit Title: Mathematics 1 | | Unit Code: ME10304 | |
| Year: 2020/21 | Question Number: 7 | Page 1 of 1 | |

| Part | | Mark |
|---|---|---|
| (a) | $n$ | $y^{(n)}(x)$ | $y^{(n)}(3)$ <br> 0 | $(1+x)^{-1}$ | $1/4$ <br> 1 | $-(1+x)^{-2}$ | $-1/4^2$ <br> 2 | $+2(1+x)^{-3}$ | $2/4^3$ <br> 3 | $-3\times 2(1+x)^{-4}$ | $-3!/4^4$ <br> 4 | $+4!(1+x)^{-5}$ | $4!/4^5$ | 1 |
| | $\Rightarrow y(x) = \frac{1}{4}\left[1 - \frac{(x-3)}{4} + \frac{(x-3)^2}{4^2} - \frac{(x-3)^3}{4^3} + \dots\right]$ | 1 |
| | $= \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x-3}{4}\right)^n$ | 1 |
| | Radius of convergence is 4 — need $\lvert x-3\rvert < 4$. | 1 |
| | When $x$ is real, need $-1 < x < 7$ for convergence | 1 |
| (b) | $y' = (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \dots$ | 1 |
| | $\Rightarrow y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ (const = 0) | 1 |
| | $= \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$ | 1 |
| (c) | $\lim_{x\to 0}\frac{\sin x - x\cos x}{x^3} \overset{l'H}{=} \lim_{x\to 0}\frac{\cos x - \cos x + x\sin x}{3x^2}$ | |
| | $= \lim_{x\to 0}\frac{\sin x}{3x}$ (cancelling $x$) | |
| | $\overset{l'H}{=} \lim_{x\to 0}\frac{\cos x}{3} = 1/3$ | 2 |
| | Total | 10 |

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Outline Solution to Examination Question

| Examiner: Dr D A S Rees | | Date January 2021 |
|---|---|---|
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 8 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | $\begin{pmatrix}0\\0\\0\end{pmatrix}$ (b) $\underline{a}\cdot\underline{b}=0$ | 1, 1 |
| (c) | $\lvert\underline{a}\rvert = 7$ (d) $\lvert\underline{b}\rvert=\sqrt{34}$ | 1, 1 |
| (e) | $\begin{pmatrix}2/7\\-6/7\\3/7\end{pmatrix}$ (f) $90^\circ$ | 1, 1 |
| (g) | $\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 2 & -6 & 3\\ 3 & 3 & 4\end{vmatrix} = \begin{pmatrix}-33\\1\\24\end{pmatrix}$ | 2 |
| (h) | $\underline{b}\cdot\underline{c} = 34$ and $\underline{b}\cdot\underline{c} = \lvert\underline{b}\rvert\,\lvert\underline{c}\rvert\cos\theta = \sqrt{34}\sqrt{83}\cos\theta$ $\Rightarrow \cos\theta = \dfrac{34}{\sqrt{34\times 83}} = \sqrt{\dfrac{34}{83}}$ $\Rightarrow \theta = 50.206^\circ$ or $0.8763$ rad | 2 |
| | Total | 10 |

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Outline Solution to Examination Question

| Examiner: Dr D A S Rees | | Date January 2021 |
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| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
| Year: 2020/21 | Question Number: 9 | Page 1 of 1 |

| Part | | Mark |
|---|---|---|
| (a) | $\underline{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\-1\end{pmatrix} \equiv \underline{a} + \lambda(\underline{b}-\underline{a})$ | |
| | Closest approach $\Rightarrow \underline{r}\cdot(\underline{b}-\underline{a}) = 0$. | (2) |
| | Hence $\lambda = 0 \Rightarrow \underline{r} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ & $\lvert\underline{r}\rvert = \sqrt{3}$ | |
| (b) | $\underline{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\-1\end{pmatrix} + \sigma\begin{pmatrix}2\\1\\2\end{pmatrix} \equiv \underline{a} + \lambda(\underline{b}-\underline{a}) + \sigma(\underline{c}-\underline{a})$ | (2) |
| (c) | $\underline{p} = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 2 & 1 & 2 \end{vmatrix} = \begin{pmatrix}1\\-4\\1\end{pmatrix}$ | |
| | $\Rightarrow \hat{\underline{p}} = \frac{1}{\sqrt{18}}\begin{pmatrix}1\\-4\\1\end{pmatrix}$ | (3) |
| | So $\underline{r}\cdot\hat{\underline{p}} = \underline{a}\cdot\hat{\underline{p}} = -\frac{2}{\sqrt{18}} = -\frac{\sqrt{2}}{3}$. | |
| | Closest approach is $\sqrt{2}/3$. | |
| (d) | $\underline{r}\cdot\hat{\underline{p}} = -\frac{\sqrt{2}}{3} \longrightarrow \frac{1}{\sqrt{18}}(x-4y+z) = -\frac{\sqrt{2}}{3}$ | |
| | or $x-4y+z = -2$ | (3) |
| | Intersection with the $z=0$ plane $\Rightarrow x-4y=-2$. | |
| | Total | 10 |

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| Examiner: Dr D A S Rees | | Date January 2021 |
| Unit Title: Mathematics 1 | | Unit Code: ME10304 |
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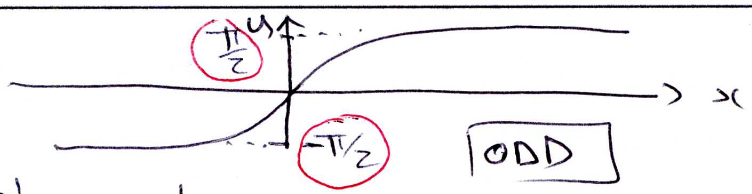
| Part | | Mark |
|---|---|---|
| (a) |   ODD | 2 |
| (b) | $y = \tan^{-1} x \Rightarrow \tan y = x \Rightarrow y' \times \sec^2 y = 1$<br>$\Rightarrow y' = \frac{1}{1+x^2}$ | 2 |
| (c) | $(1+z)^{-1} = 1 - z + z^2 - z^3 + \dots$<br>$\Rightarrow y' = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \dots$ | 2 |
| (d) | $y = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ (using $y(0)=0$)<br>Radius of convergence is 1. | 2 |
| (e) | $\int_0^{0.2} y\,dx = \left[ \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{30} \dots \right]_0^{0.2}$<br>$= 0.019869$<br>(a fourth term is $\approx 4.6 \times 10^{-8}$) | 2 |
| | | Total |