University of Bath
Department of Mechanical Engineering

ME10304 Mathematics 1

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Introduction to ME10304 Mathematics

Dr D A S Rees

1 Who am I?

I am Andrew Rees, more formally written as Dr. D. A. S. Rees. I have been stashed away in 4E in the Department of Mechanical Engineering since 1990 and have been happily whiling away my time doing research and also teaching the ever-increasing numbers of students. In 1991 we had approximately 90 students per year for a three year course, whereas now we have close to 300 on a four year course. In that time the number of academic staff have doubled.

My office is 4E 2.54 and my email address is ensdasr@bath.ac.uk, which reflects the old naming convention we had of ensdasr, which is a shorthand for engineering, staff and my initials. Only a few of us old-stagers still have such an email address. An alias is D.A.S.Rees@bath.ac.uk which I prefer, but the Computer Centre doesn't! In your cases your official email addresses consist of your initials and a random number. I have no idea whether we'll get a fourth convention.....

2 My background.

What authority do I claim to be able to teach you?

I guess that a 1st class degree in Mathematics from Imperial College, London, in 1980 isn't too shabby. I guess too that the fact that Bath University hasn't yet sacked me for being useless is a note in my favour. Somehow I have managed to get away with it for 30 years....

My Ph.D. was in Applied Mathematics from the University of Bristol in 1986 where I concentrated on convective flows and stability in porous materials. Added to this is an ATCL diploma from the Trinity College of Music, London, in 1977 and membership of the National Youth Orchestra of Wales; so I guess that this means that I do have at least one interest outside maths and that I also get to see people who aren't colleagues.

I did have a short spell from 1980 to 1982 working for what was then called British Aerospace. Based in Filton, they wanted me to do some flutter calculations, and to devise feedback laws which would stop aircraft wings falling off when they flew faster than their original design speed.

After my doctorate I stayed at Bristol University with the same supervisor doing essentially the same things but doing them better and quicker. This was followed by two years worth of fixed-term lecturing duties in the Maths Department at Exeter University. Then the job in Bath appeared and I just had to say *yes* to the offer of employment because I had a mortgage to service, and a wife and two children to feed.

3 Outline of ME10304.

Given that we have suffered the unprecedented use of the word, unprecedented, to describe the many consequences of the Covid-19 pandemic, there will be no standard lectures, which is unprecedented. Normally there would be two 50-minute lectures each week with a two-hour problems class, and I generally present 20 lectures out of the 22 slots available. But this year the lectures will be replaced by a set of videos and there will be three hours of in-person problems class spread over five rooms which will take place on Thursdays. The following is the breakdown of the content of the unit:

Topic	Number of lectures
Introduction	0.5
Curve sketching	1.5
Complex numbers	2
Differentiation	4
Integration	5
Series	3
Vectors	4

As you can see, many of these topics are familiar. It is my experience that different people will have covered different things and, given that all of these topics are deemed to be important, I will need to cover them all. So, for each person, some of the material this semester will be revision, but that subset will be different for different people. However, I aim to bring forth some new ideas in various places so that there will be a few nuggets of gold to find among the familar.

I shall make the content available gradually as the semester moves on. This includes (i) my typeset lecture notes, (ii) pre-recorded videos (which will take the place of the usual lectures), (iii) typeset slides (which are closely related to the typeset notes) and which I use in the videos, and (iv) problem sheets. I shall also provide solution sheets which will include detailed workings for most of the problems. Clearly, I will delay the release of the solutions for an appropriate interval after releasing the problem sheets!

4 The detailed syllabus.

The following is how I split the unit content over the 20 lectures last year. I will be following the same ordering of the material.

	Topic	Content
1.	Curve	Polynomials. Modulus. Exponentials. Hyperbolic functions. Logs.
2.	sketching.	Envelopes. Square roots. Ratios of polynomials.
3.	Complex numbers.	Motivation/need for them. Definition. Arithmetic operations. Geometric interpretation. Polar/exponential form.
4.		de Moivre's theorem. Euler's formula and identity. Roots of complex numbers. Further identities.
5.	Differentiation.	Definition and notations. Use of limits. Higher derivatives. Linearity. Product rule. Product of more than two functions.
6.		Chain rule and proof. Nested functions. Advanced cases. Quotient rule.
7.		Critical points. Primary and secondary criteria. Checklist.
8.		Partial differentiation. Clairaut's theorem. Critical points of surfaces.
9.	Integration.	From sums to integrals. Definite and indefinite. Integration by substitution. f'/f form.
10.		Ratios of polynomials and partial fractions. Top heavy ratios. Repeated factors. Irreducible quadratics.
11.		Integration by parts. Derivation of the Rees method – rules of implementation. Miscellaneous cases.
12.		Applications 1. Means and RMS. Recurrence relations. Volumes under surfaces in Cartesians and in polar coordinates.
13.		Applications 2. Length of a line. Volumes and surfaces of revolution. Triple integrals.
14.	Series.	Series and sequences. Binomial theorem and Pascal's triangle. Binomial series.
15.		Taylor's series and Maclaurin. Derivation/justification. Two forms of Taylor's series.
16.		Convergence and d'Alembert's test. Examples of numerical and of power series. Radius of convergence. l'Hôpital's rule. Derivations.
17.	Vectors.	Revision: definition and elementary concepts. Unit vectors. Scalar product. Angle between two vectors.
18.		Vector product. Lexicographical ordering! Area of a triangle using vector products. Coplanarity of three vectors or four points.
19.		Equation of a line. Distance of a line from a point. Closest approach of two lines.
20.		Distance of a point from a plane. Equation of a line - two forms. The direction of the intersection of two planes.

The remaining sections in this document cover a variety of preliminary topics. I am not at all sure that they are emphasized sufficiently in pre-university tuition, but again I would far prefer to play safe by presenting my ideas to you so that you will know what sorts of standard I would expect.

5 Significant figures.

Here's a good question: which is better, 6 decimal places or 6 significant figures?

The answer depends on how large the number is.

The number $\pi/1000 = 0.003142$ is correct to 6 decimal places but only to 4 significant figures, while the number $100\pi = 314.159265$ is also correct to 6 decimal places but is correct to 9 significant figures.

So it is clear that the number of significant figures is the better concept because it is independent of the magnitude of the number being considered.

5.1 The number of significant figures.

From the practical point of view of computing numbers, the big question is, *How many significant figures do we need?* To answer this we need to understand what happens when we do arithmetic with limited precision.

In engineering, we frequently need only 3 or perhaps 4 significant figures for comparison with experimental work, for that is often the greatest precision with which we can measure. There are, of course, counter-examples to this such as the mass of the electron, the speed of light, gravitational acceleration and the density of water, for each of these have been measured to a very much greater degree of accuracy.

If one were to do a Google search for the value of g, the acceleration due to gravity, then one website gives 9.8m/s^2, another gives 9.81m/s^2 while the 'standard value' is generally defined as 9.80665m/s^2. The first of these has 2SFs, the second has 3SFs, while the third is apparently correct to 6SFs. What is the truth of the matter? If we take the third as being correct, then the first two are clearly approximations, so all is well....or is it? Although the third has been designated the standard value, it gives the misleading impression that it is valid everywhere. Although we know that g will decrease as we rise above the Earth's surface and as we descend under the sea, the standard value is not correct everywhere on the earth's surface. In fact it varies between 9.779m/s^2 for Mexico City to 9.819m/s^2 for Oslo; these values represent fairly closely the two extreme cases for the earth's land surface. Therefore it is pointless using all six significant figures in the standard value unless in the official value unless you are at the precise place where it applies. Perhaps we should only use 9.8m/s^2 and simply note that all our calculations will also have only two SFs of accuracy. And even if we know exactly where we are on the globe, g will also vary with the time of day if we are on open water due to the height of the sea.

What are the consequences of this variation? One trivial one is that one would weigh 0.4% less in Mexico than in Oslo. Another is that a pendulum in Mexico has a period which is 0.2% longer than an identical one in Oslo. Admittedly these are small values, but at least we know what the possible error is.

However, despite needing only 3 or 4 figures for experimental work, it is not always good to retain only 3 or 4 figures of accuracy during intermediate theoretical calculations. This is because round-off errors can build up catastrophically. In general, I would recommend using 5 or 6 for all calculations, and if the final result is required to 3 significant figures (SFs), one may then round the final answer to the required accuracy. But even then, one must be aware of how accuracy can get eroded.

Example 1 Calculate $0.301 + 0.478 + 1.42 + 18.4 + 101$ to 3 SFs.

This example illustrates how care must be taken when doing arithmetic. We'll compute the sum using three different 'methods'.

Method (i) Exact arithmetic gives 121.599 which is 122 to 3SFs. This is the way one would do it using a calculator or pencil and paper.

Method (ii) Adding by beginning with the largest number and rounding each computation before adding the next largest and so on.

$$\begin{aligned}
101 + 18.4 = 119.4 &\quad = \quad 119 \text{ (3SF)} \\
119 + 1.42 = 120.42 &\quad = \quad 120 \text{ (3SF)} \\
120 + 0.478 = 120.478 &\quad = \quad 120 \text{ (3SF)} \\
120 + 0.301 = 120.301 &\quad = \quad 120 \text{ (3SF)}
\end{aligned}$$

Method (iii) Adding by beginning with the smallest number and rounding each computation before adding the next smallest and so on.

$$\begin{aligned}
0.301 + 0.478 = 0.779 & \\
0.779 + 1.42 = 2.199 &\quad = \quad 2.20 \text{ (3SF)} \\
2.20 + 18.4 = 20.60 &\quad = \quad 20.6 \text{ (3SF)} \\
20.6 + 101 = 121.6 &\quad = \quad 122 \text{ (3SF)}
\end{aligned}$$

Method (i) works well because the precision we are using (namely, exact for pen and paper, 8 to 10 SFs for a calculator, or either 7 or 15 or more for a computer) uses very many more SFs than the precision of the numbers.

Method (ii) is the poorest. By using the largest number first, we are declaring that almost all the information contained after the decimal point in the rest of the data will not contribute to the final result. This is why it provides the least accurate sum. Note also that if we had included another million numbers under 0.5 in magnitude, then this would have had no effect on the final answer.

Method (iii) yields the safest method when working with a highly restricted number of SFs.

A further danger point is that the original data that we have added together isn't consistent in terms of the number of decimal places, i.e. we have no idea whether they were already rounded to 3SFs before we started to sum them. If whomsoever gave us the data had rounded the data to 3 SFs, then the largest value, 101, could range between 100.500 and 101.500 in real life and this could alter our final answer. We would have more confidence in the answer if either (a) it is stated that each value is exact or (b) all the data were presented to 3DPs. So the moral of the story is that we must enquire about the provenance of the data before we manipulate it.

The reason I have mentioned the above example is that all calculating devices use a fixed number of SFs, and therefore rounding always happens after each addition, subtraction, exponentiation, multiplication, division, square root, etc... It is rare for this to cause any problems, but it is good to be aware that round-off error can very occasionally yield incorrect results.

However, I often see very severe rounding taking place in computations performed in exams. Even if the method used is correct, the final answer can be very poor.

Example 2 Solve the quadratic $x^2 - 2.01x + 1.01 = 0$ giving the answer to 4SFs.

It is straightforward to find the exact answer,

$$\begin{aligned} x &= \frac{2.01 \pm \sqrt{2.01^2 - 4 \times 1.01}}{2} \\ &= \frac{2.01 \pm \sqrt{0.0001}}{2} \\ &= \frac{2.01 \pm 0.01}{2} \\ &= 1.01,\ 1. \end{aligned}$$

Now let us rerun the analysis rounding to 4SFs after each arithmetic operation. First we note that $2.01^2 = 4.0401$ which is equal to 4.040 to 4SFs. Therefore

$$\begin{aligned} x &= \frac{2.01 \pm \sqrt{2.01^2 - 4 \times 1.01}}{2} \\ &= \frac{2.01 \pm \sqrt{4.040 - 4.04}}{2} \\ &= \frac{2.01 \pm 0}{2} \\ &= 1.005,\ 1.005 \end{aligned}$$

In this case, we have an error of 0.5% even with 4SF arithmetic. Although this loss of precision isn't catastrophic, it has changed the qualitative nature of the solution. We had two different roots when using exact arithmetic, and now they are two identical roots. This has arisen because we subtracted two numbers which are almost equal. It is in these situations where the loss of significant figures is felt most.

An alternative view of the above is to analyse the effect of possible inaccuracies in the coefficients, 2.01 and 1.01. These values could have been obtained from an experiment, and, given that they have been quoted to 3SFs or 2DPs, the likely maximum error in their values is ± 0.005. Let us therefore consider the equation

$$x^2 - (2.01 + \epsilon_1)x + (1.01 + \epsilon_2) = 0.$$

Using exact arithmetic, with the final answers rounded to 3DPs, we get the following possible solutions.

$$\epsilon_1 = 0.005, \quad \epsilon_2 = 0.005: \qquad x = 1.015,\ 1.000,$$

$$\epsilon_1 = 0.005, \quad \epsilon_2 = -0.005: \qquad x = 1.108,\ 0.907,$$

$$\epsilon_1 = -0.005, \quad \epsilon_2 = -0.005: \qquad x = 1.005,\ 1.000,$$

$$\epsilon_1 = -0.005, \quad \epsilon_2 = 0.005: \qquad x = 1.0025 \pm 0.1000i.$$

Note that the last case has complex roots!

So for this example we can get an error of up to 10% in real answers for only a 0.5% change in a coefficient. This is a problem which is very sensitive to the degree of accuracy of the basic data — these are dangerous problems! I emphasize that this is an extreme circumstance, but occasionally these circumstances do arise, and it is important to acquire a good intuition of how accurate one's arithmetic truly is.

5.2 Too few significant figures?

This is motivated by the fact that I often see students providing answers with too few significant figures in exams. If the exact answer to a maths problem were to be $1/\sqrt{2}$, then how should we present it? First, I have to come clean and say that I am very happy for the answer to be written as $1/\sqrt{2}$, although some people prefer to have the square root in the numerator: $\sqrt{2}/2$. An alternative notation is $2^{-1/2}$. If this were the final answer, then I would prefer one of these three exact solutions to be written. If someone were to prefer to write it out, then five or six SFs is fine. Hence 0.707107 is generally ok.

Suppose now that we are asked to evaluate $\sin 2\pi$. The great majority of us (assuming that 2π is in radians) would say that the value is zero. The following table shows what happens if 2π is first converted to a number before the sine is computed.

#/SFs	2π	$\sin 2\pi$
2	6.3	0.016814
3	6.28	-0.003185
4	6.283	-0.000185
5	6.2832	0.000015
6	6.28319	0.000005
7	6.283185	-0.000000

So the approximation to 2π using 3 or 4 SFs isn't particularly good given that the maximum value of the sine function is 1 and that the exact value of $\sin 2\pi$ should be zero. It is much better to retain as much accuracy as possible.

5.3 Too many significant figures?

The above went into detail about what could happen if we take too few significant figures. But how many is sufficient?

One may quote π to 36 decimal places,

$$\pi \sim 3.141\,592\,653\,589\,793\,238\,462\,643\,383\,279\,502\,884,$$

as a clever party trick, but the last significant digit shown, which is in the 36th decimal place, represents a very incredibly small quantity. Just how small is it? One way of attempting a visualization is to consider 10^{36} sugar crystals where I will assume that each crystal is a cube of side 0.5mm. It doesn't take long to find out that 10^{36} crystals is equivalent to a cube of side $5,000,000$km, which is roughly 12 times the distance of the moon from Earth. If we compare one sugar crystal with this cube, then that is 10^{-36}. Therefore I conclude that 36 significant figures is a waste of effort!

I tend to use roughly 6 SFs for my calculations and I will only increase that if it turns out to be necessary to do so. So keep your error-checking radar on at all times.

5.4 How accurate is the usual approximation to the value of π?

Well, we know from school days that $\pi \simeq 22/7$, but how good is this? So we need to compare $22/7 \simeq 3.142857142857...$ with π as given above. To do this we may define the relative error as being,

$$\text{R.E.} = \frac{22/7 - \pi}{\pi},$$

and this comes to 0.00040, i.e. four parts in 10,000, or 0.04 percent, which isn't at all bad.

Is there a better one that isn't too complicated? How about $355/113$? This is 3.1415929204 to 10 decimal places. Here's the comparison:

$$\begin{array}{l} 3.1415929204.... \\ 3.14159265358979.... \end{array}$$

The relative error is now about 8.5×10^{-8} which is considerably better.

No doubt you can think of a nice easy way of remembering $355/113$.

Question: How good are the following as approximations to $\sqrt{2}$: $99/70$ and $8119/5741$? I will leave that one to you.

5.5 To conclude....

I have just read that lot through and I reckon that it's somewhat heavy, especially as it is the very first technical material on a Maths unit at university. So perhaps I ought to summarize it in a few recommendations.

1. Check your data. What is its provenance? How accurate is it?
2. Do not do any rounding off until you get to the very final answer.
3. Do not use too few (it's dangerous) or too many (it's pointless) significant figures.
4. Always be on the lookout for potential accuracy issues, e.g. the subtraction of almost equal numbers.

6 How to structure a mathematical argument.

The aim of this document is to cover briefly the frequently-neglected subject of the structuring of a mathematical argument and how to write it down.

When one undertakes a piece of mathematics, be it algebra, trigonometry or calculus or a combination of these, then the mathematics should proceed by sequence of logical steps. It is important to be able to write these steps out in a manner which reflects that logic and which allows someone else to be able to follow that logic. A poor example is the following.

Example 1 (bad) Evaluate the polynomial $(x + 5)^2 - x + 10$ when $x = 2$.

I have seen the following sequence of steps:

$$(2 + 5)^2 = 49 - 2 = 47 + 10 = 57. \tag{1}$$

The answer is correct, but what do the various equals signs mean in this context? Only the last one is used correctly. This way of doing things is a mathematical version of a 'stream of consciousness' writing.

Apart from the obvious simple substitution, $(2+5)^2 - 2 + 10 = 49 - 2 + 10 = 57$, which is fine, one could be very pedantic and write the problem out this way:

When $x = 2$, then $(x+5)^2 = 49$, and hence

$$\begin{aligned} (x+5)^2 - x + 10 &= 49 - 2 + 10 \\ &= 57. \end{aligned} \tag{2}$$

Example 2 (good) Solve the quadratic equation, $x^2 + 2x - 8 = 0$.

We'll do this by completing the square.

$$\begin{aligned} & x^2 + 2x - 8 = 0 && \text{Need to add 9 to both sides...} \\ \Rightarrow\quad & x^2 + 2x + 1 = 9 && \text{...since } x^2 + 2x + 1 \text{ is square} \\ \Rightarrow\quad & (x+1)^2 = 9 && \\ \Rightarrow\quad & x + 1 = \pm 3 && \text{taking square roots} \\ \Rightarrow\quad & x = -1 \pm 3 && \\ & \ = -4,\ 2. && \end{aligned} \tag{3}$$

The 'implies' symbol, $\Rightarrow$, means that the current line follows logically from the preceding line. It must not be used to replace the equals sign. Note that I did not need to use the implies sign on the last line leading to the final solution in Eq. (3), as that was the result of a simple calculation (the same is true for Eq. (2)). Note also the hints on the right, which might aid the reader understand what the logical step was.

So the aim is to write out a mathematical argument the logic of which I (as the marker of your exams) should be able to follow quite easily. This would also mean that you would be able to follow your own work months from now when revising for exams.

One could think of the mathematical argument leading to (3) as being a condensed version of a verbal statement: Given that $x^2 + 2x - 8 = 0$, this implies that it may be rearranged in the form, $x^2 + 2x + 1 = 9$, because the left hand side is now a perfect square and we have a fleeting suspicion that this might be useful to us. Now we take square roots of both sides etc. etc....

7 Notations and functions and the avoidance of ambiguity.

7.1 Inverse functions.

It is a real pain that inverse functions such as the inverse sine are denoted by $\sin^{-1}$. It happens for other functions too, such as $\cos^{-1}$, $\tan^{-1}$ and $\cosh^{-1}$. There is even a general notation: if $y = f(x)$ then $x = f^{-1}(y)$. The pain is that it looks like a reciprocal.

In the olden days, whenever they were, the inverse sine was written as arcsin which isn't at all ambiguous.

If you really wished to write down the reciprocal of $\sin x$, then there are various ways of doing so:

$$\frac{1}{\sin x}, \qquad (\sin x)^{-1} \qquad \text{and} \qquad \text{cosec}\, x.$$

What about the following four functions; are they the same as one another:

$$(\sin x)^{-2}, \qquad (\sin^{-1} x)^2, \qquad \text{arcsin}^2(x) \qquad \text{and} \qquad \sin^{-2} x?$$

The first is the -2 power of $\sin x$. The second is the square of the inverse sine of x, as is the third. The fourth one is ambiguous. Some would say that it is the same as the first one because the power isn't -1, which might be said to be reserved for the inverse function. Others would say that it is the second, although I would hope that almost no-one would fall into that trap. As for me, my view is that, because it is ambiguous, it should never appear; one's mathematical writing *must* never have even the smallest taint of ambiguity.

As for the reciprocal of the inverse sine — I have never seen anyone use this — then the following four will work:

$$(\sin^{-1} x)^{-1}, \qquad \frac{1}{\sin^{-1} x}, \qquad (\text{arcsin}\, x)^{-1}, \qquad \frac{1}{\text{arcsin}\, x}.$$

All of these are correct although the first is an absolute horror.

7.2 Double negatives.

Ah, now these have become very popular in the last ten years or so. By this I mean constructions such as,

$$3 - -2 = 5.$$

In one sense this is ok. However, the first minus sign is formally a subtraction operator while the second indicates the negative of 2, so it isn't really a double negative. Just as we saw that -1 as a superscript could mean two different things and that these usages may be confused, here the difficulty/danger is due to where the two minus signs are when written down. I will illustrate that using the following, which is something that I have often witnessed on exam scripts:

$$3 - -2 \quad = \quad 3 - - 2 \quad = \quad 3 -- 2 \quad = \quad 3 — 2 \quad = \quad 3 - 2 = 1.$$

The above generally appears on successive lines of an exam script as the more complicated parts of an equation are simplified, but the minus signs just keep getting closer and closer! So although this isn't really an ambiguity, my dislike of the double negative is primarily because this often happens. So I would recommend writing, $3 - (-2) = 5$, as a safe way to play the mathematical game.

Being a bit of a purist (and it's also a Great British sport to complain about irrelevant matters!), I would far prefer the following usage: $3 \times (-2) = -6$, the brackets still signifying the negative number, as opposed to 3×-2 — uuurgh.

8 Logarithms.

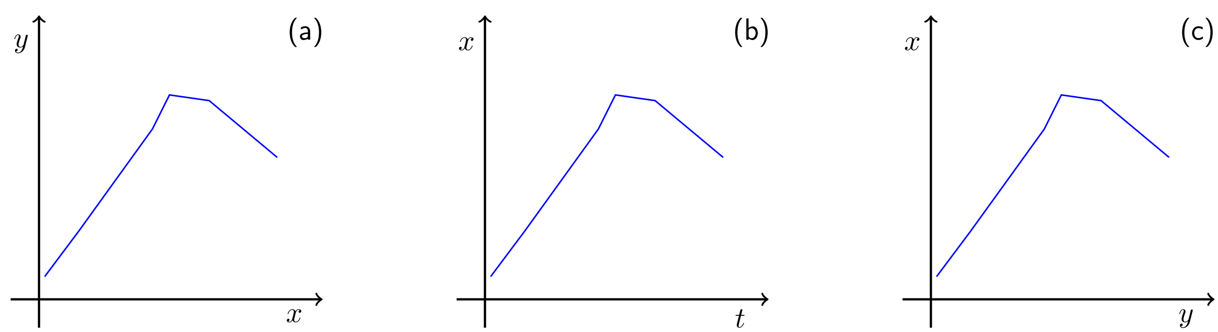
There are three types of logarithm which are used in Engineering. The first is the base-10 logarithm, which is the first one that is taught and which is known as the common logarithm. It is usually written as $y = \log_{10} x$, so if y is increased by 1 (i.e. the addition of 1), then x is multiplied by a factor of 10. Scales such as the Richter Scale behave in the same way: an earthquake which measures 7 on the Richter scale has 10 times the energy of one which measures 6. The amplitude of sound also operates in the same way. I guess that the unit is the *bel* where 10 decibels is equivalent to 1 bel. So a sound of 80 decibels has 100 times the power of one which is rated at 60 decibels.

The natural logarithm, invented by Napier, uses $e = 2.7182818284....$ as its base. This rather strange number arises naturally in very many places and it has many very interesting properties such as the fact that the derivative of the function, e^x, is also e^x. It is generally written as $y = \ln x$, and we simply say that "y is equal to log x" when talking about it. There is no need to say, "lun x", because the base-10 version is hardly ever used in Mechanical Engineering.

The base-2 or binary logarithm is written as $y = \log_2 x$. It too is not used often in Mechanical Engineering, but has great use in Computer Science, Information Theory and Music Theory. Given that many of you will also be musicians it is worth dwelling on this for a moment. Violins in an orchestra generally tune their instruments to a note called A which has a frequency of 440Hz. Perhaps this isn't too well-known, but the highest note that the solo violin plays in Dvŏrák's Violin Concerto is also an A, but its frequency is 3520Hz. The ratio of these frequencies is precisely 8 which is also 2^3. Given that $\log_2 8 = 3$, this tells us that this frequency ratio is precisely 3 octaves. Thus the binary logarithm of a frequency ratio tells us the equivalent number of octaves.

[As an aside for the more geeky musicians, a perfect fifth on a stringed instrument corresponds to a frequency ratio of 1.5, whereas the equivalent on an equally-tempered piano, i.e. seven semitones, corresponds to a frequency ratio of $2^{7/12}$ which is 1.498307. Using $\log_2$, the interval for a violin is 0.584963 of an octave, while for the piano it is $7/12 = 0.583333$ (6SFs). This slight difference between these two intervals is why violinists find it difficult to tune to a piano when both the D and the A are sounded. If the violin A is tuned to the piano A then the two D's do not agree, and vice versa. If the violinist tunes the strings perfectly to the piano, then the simultaneous playing of the D and A strings sounds out of tune. Such matters are unresolvable....]

9 Naming the axes.



These three graphs look the same. If the first one, labelled (a), were to be written in the form, $y = f(x)$, then the other two are $x = f(t)$ and $x = f(y)$, in turn.

The primary reason for introducing these is the very common but very often inaccurate naming of the vertical axis as the y-axis and the horizontal one as the x-axis. Formally this is correct only for the first one. I have noticed that this can even occur in, say, official government graphs of, to choose an example, the rate of inflation against time, where the rate of inflation is then referred to by the TV reporter as the y-axis and the month by the x-axis. Oops....

In the figure labelled, (b), it seems odd to me that one should say that the y-axis corresponds to the value of x while the x-axis corresponds to the value of t. Even worse, should we say that the y-axis gives the value of x while the x-axis gives the value of y in the figure labelled (c)? Again ambiguity is possible.

One simple resolution is to call them the vertical and horizontal axes — no-one will misunderstand you. One could also refer to them as the ordinate and the abscissa, respectively, which are now rather old-fashioned words that are rarely seen. My recommendation is to use 'vertical' and 'horizontal' because again there is no potential for ambiguity.

The final word. Sometimes one needs 3D graphs. They could take various forms: (i) a projection of of the full 3D curve onto say the (x, y)-plane, (ii) a stereoscopic pair where one needs to go slightly cross-eyed to be able to view its 3D nature, or (iii) an anaglyph form which requires red/blue glasses. So such graphs could be formed of lines (e.g. the trajectory of a particle in 3D space) or of surfaces (e.g. the depiction of the height above sea level of an area of land). The three axes could be (x, y, z) or else (x, y, t) or many others. In all these cases it is best to refer to a chosen axis by the name of the coordinate along that axis. Many examples of stereopairs and anaglyphs may be found using Google.