Q8 19/20

(1)

$$x'' + x - y = 0$$
$$1y'' - x + y = \delta(t)$$

$y = y' = 0$
$x = x' = 0$ at $t = 0$

$$\mathcal{L}[f''] = s^2 F - f'(0) - s f(0)$$

Eqs $\longrightarrow$ $s^2 X + X - Y = 0$
$s^2 Y - X + Y = 1$

$$(s^2+1)X - Y = 0$$
$$-X + (s^2+1)Y = 1$$

$$\Rightarrow \begin{pmatrix} s^2+1 & -1 \\ -1 & s^2+1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} X \\ Y \end{pmatrix} = \frac{1}{(s^2+1)^2 - 1}\begin{pmatrix} s^2+1 & 1 \\ 1 & s^2+1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{s^4 + 2s^2}\begin{pmatrix} 1 \\ s^2+1 \end{pmatrix}$$

$$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t) e^{-st} dt$$
$$= e^{-s(0)} dt$$
$$= e^{-0} = 1.$$

$$s^4 + 2s^2 = s^2(s^2+2)$$

$$\frac{A + Bs}{s^2+2}$$

$$\frac{1}{v(v+1)}$$

$$\mathcal{L}[t] = 1/s^2$$
$$\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$$

$$X = \frac{1}{s^2(s^2+2)} = \frac{A}{s^2} + \frac{B}{s^2+2}$$

$$\Rightarrow \boxed{1 = A(s^2+2) + Bs^2}$$

$s^2 = 0 \Rightarrow 1 = 2A \Rightarrow A = 1/2$
$s^2 = -2 \quad 1 = -2B \Rightarrow B = -1/2$

$$\Rightarrow X = \frac{1/2}{s^2} - \frac{1/2}{s^2+2} \Rightarrow \boxed{x = \frac{t}{2} - \frac{1}{2\sqrt{2}} \sin \sqrt{2} t}$$

(1)

Gain $x'' + x - y = 0$

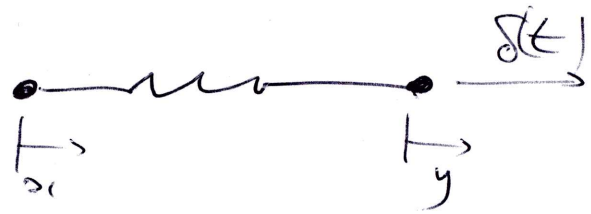
$$\Rightarrow \boxed{y = \frac{1}{2}t + \frac{1}{2\sqrt{2}} \sin\sqrt{2}\,t} \quad \text{— (2)}$$

$$(1) \Rightarrow \dot{x} = \frac{1}{2} - \frac{1}{2}\cos\sqrt{2}\,t$$

$$\dot{y} = \frac{1}{2} + \frac{1}{2}\cos\sqrt{2}\,t.$$

@ $t=0$ $\quad \dot{x} = 0$

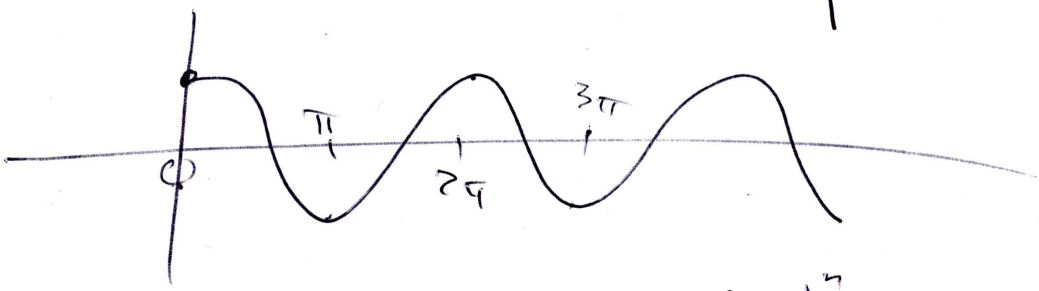
$\dot{y} = 1$



Unit momentum added to right hand mass.

$$\cos n\pi = (-1)^n$$

$$-\frac{\cos n\pi}{n^2} = \frac{(-1)^{n+1}}{n^2}$$



n	$\cos n\pi = (-1)^n$
0	+1
1	−1
2	+1
3	−1
4	+1

(a) $\mathcal{L}[e^{-at}] = \int_0^\infty e^{-at} e^{-st}\, dt = \int_0^\infty e^{-(s+a)t}\, dt = \frac{1}{s+a}$. (3)

$\mathcal{L}[t^2] = 2/s^3$

$\cancel{\frac{1}{s^3+3s^2+3s+1}}$ $\quad \frac{1}{s^3+3s^2+3s+1} = \frac{1}{(s+1)^3}$

s-shift $\mathcal{L}[f(t)] = F(s) \Longrightarrow \mathcal{L}[f(t)e^{-at}] = F(s+a)$

$\mathcal{L}[t^2] = \frac{2}{s^3} \Longrightarrow \mathcal{L}[\frac{1}{2}t^2] = \frac{1}{s^3}$

$\xrightarrow[a=1]{\text{shift}} \mathcal{L}[\frac{1}{2}t^2 e^{-t}] = \frac{1}{(s+1)^3}$.

(c) $\frac{s}{s^2+s-6} = \frac{s}{(s-2)(s+3)} = \frac{A}{s-2} + \frac{B}{s+3}$

$\Longrightarrow \quad s = (s+3)A + (s-2)B$

$s=2 \Rightarrow 2 = 5A \qquad A = 2/5 \quad B = -3/5$

$s=-3 \quad -3 = -5B$

$\Rightarrow \frac{s}{s^2+s-6} = \frac{2/5}{s-2} - \frac{3/5}{s+3}$

$\Rightarrow \mathcal{L}^{-1}\left[\frac{s}{s^2+s-6}\right] = \frac{2}{5}e^{2t} - 3/5\, e^{-3t}$

Using $a = -$ in (a).