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Some notes on Jean Baptiste Joseph Fourier, 1768–1830

Fourier was born at Auxerre March 21, 1768, the son of a poor tailor. An orphan at eight, he was recommended by a friend to the Bishop of Auxerre, who obtained admission for him in the local military school conducted by the Benedictines of Saint–Maur. He quickly distinguished himself as a student and showed distinct literary ability; at twelve he was writing sermons which were used with great effect in Paris. At the age of thirteen mathematics began to attract him strongly. The prescribed hours of study did not suffice; he arose at night, concealed himself behind a screen, and by the light of candle-ends carefully collected during the day, he pursued his mathematical studies. When he was twenty-one he delivered his first memoir before the Academy of Sciences on the resolution of numerical equations of all degrees.

Educated by monks in a military school, Fourier seems to have considered that only the army or the church could provide a career. With a strong recommendation from Legendre he applied for admission to the artillery. He was refused with the statement, "Fourier, not being of noble birth, cannot enter the artillery, not even if he is a second Newton." He then entered the Benedictine Order, where he remained as a novice from 1787 to 1789. Upon the outbreak of the Revolution he left the convent, although this did not result in any break with the Benedictines, since they immediately appointed him to the principal chair of mathematics at their school in Auxerre. When his colleagues became ill, he took their place, and besides teaching mathematics he also lectured on rhetoric, history, and philosophy.

At Auxerre, Fourier embraced the cause of the Revolution, joined the peoples' party, and served as a publicist, recruiting agent, and member of the Citizens' Committee of Surveillance; in this last function he exercised such moderation that he was himself in danger from the Terror. When, in 1794, the Normal School was instituted at Paris to train a specially selected group of new teachers, Fourier was among the fifteen hundred that were chosen, and, although he began as a student, he was soon made a "master of conference." The school failed after a short time, but Fourier had so impressed the authorities that when the Polytechnic School was founded, he was appointed to its faculty, first as a "superintendent of lectures on fortification" and then as "lecturer on analysis."

Napoleon sometimes attended the sessions at the Polytechnic School, and when he organized the expedition to Egypt in 1798, Fourier was asked to be a part of it, although he was not informed of the role he was expected to play. Fourier was in Egypt for three years, engaged in the most varied activities: organizing factories for the army, constructing machines, leading scientific expeditions, and executing numerous administrative tasks. He acted as the representative of the general-in-chief, receiving complaints from the Egyptian populace, and for one period was virtually governor of half of Egypt. On the death of General Kléber he was called upon to present a eulogy before the French Army. As secretary of the Institute of Cairo he instigated the collection of materials for the famous *Description of Egypt*. In collaboration with Napoleon he wrote the historical introduction to this work, which established his literary reputation and eventually won him membership in the French Academy.

On his return to France in 1802 Fourier was appointed prefect of the Département of Isère and for the next thirteen years lived at Grenoble. He composed the disputes between the different parties and brought order out of the confusion left by the Revolution in his province. As part of the general policy of public improvements, he initiated an extensive road-building project and undertook the reclamation of marsh-lands which had been the source of infection for thirty-seven communes. In recognition of his services he was created Baron of the Empire in 1808.

His many administrative duties as prefect of Isère did not interrupt his work as a mathematician and man of letters. He conducted investigations into the motions of heat in solid bodies with the aim of reducing them to mathematical formulation, and in 1807 submitted his first paper on the subject to the Academy of Sciences. To induce the author to extend and improve his researches the Academy assigned as the problem for its prize competition of 1812, "The mathematical theory of the laws of the propagation of heat and the comparison of the results of this theory with exact experiment." The judges were Laplace, Lagrange, and Legendre, and they awarded the prize to Fourier for his memoir in two parts, *Théorie des mouvements de la chaleur dans les corpes solides.* The first part was published in 1822 as the *Théorie Analytique de la Chaleur.*

Fourier continued to hold his position as prefect through the Revolution of 1814, but Napoleon's return from Elba proved to be his political downfall. As Napoleon was approaching Grenoble, Fourier went to Lyons to

notify the Bourbons that the city would undoubtedly capitulate. They refused to believe him and made him responsible for the safety of the city. Upon his return to Grenoble, which had surrendered, he was taken prisoner and brought before the Emperor. Napoleon confronted him: "You also have declared war against me? ... It only grieves me to see among my enemies an it Egyptian, a man who has eaten along with me the bread of the bivouac, an old friend. How, moreover, could you have forgotten, Monsieur Fourier, that I have made you what you are?" Fourier's loyalty was re-established, although he did not share Napoleon's confidence of victory. The end of the Hundred Days and the Restoration found him deprived of political office, in disgrace, and almost penniless.

A friend and former pupil who was prefect of Paris made it possible for him to become Director of the Bureau of Statistics, which he remained until his death. His political past, however, did not prevent renewed recognition of his scientific abilities. In 1816 he was proposed for membership of the Academy of Sciences, and although Louis XVIII refused his consent at that time, he became a member the following year. He was made permanent secretary of the Division of Mathematical Sciences in 1822, member of the French Academy in 1826, and a year later succeeded Laplace as President of the Council for Improving the Polytechnic School. In 1828 he became a member of the government commission established for the encouragement of literature.

He died May 16, 1830, of aneurism of the heart, which had been aggravated by his habit of wrapping himself in all seasons like "an Egyptian mummy" and living in airless rooms at an excessively high temperature.

From: "Great Books of the Western World", R. M. Hutchins (Editor in Chief) Volume 45 'Lavoisier, Fourier, Faraday', pp163–164, William Benton, *Publisher,* Encyclopaedia Britannica, Inc. (1987).

Power functions comprise only one example of a base set for the expansion of functions: a number of other base sets may be used. In particular a **Fourier series** is an expansion of a periodic function $f(t)$ of period $T = 2\pi/\omega$ in which the base set is the set of sine functions, giving an expanded representation of the form

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \phi_n).$$

Although the idea of expanding a function in the form of such a series had been used by Bernoulli, D'Alembert and Euler (c. 1750) to solve problems associated with the vibration of strings, it was Joseph Fourier (1768–1830) who developed the approach to a stage where it was generally useful. Fourier, a French physicist, was interested in heat-flow problems: given an initial temperature at all points of a region, he was concerned with determining the change in the temperature distribution over time. When Fourier postulated in 1807 that an arbitrary function $f(x)$ could be represented by a trigonometric series of the form

$$\sum_{n=0}^{\infty} (A_n \cos nkx + B_n \sin nkx)$$

the result was considered so startling that it met considerable opposition from the leading mathematicians of the day, notably Laplace, Poisson and, more significantly, Lagrange, who is regarded as one of the greatest mathematicians of all time. They questioned his work because of its lack of rigour, and it was probably this opposition that delayed the publication of Fourier's work, his classic text *Théorie Analytique de la Chaleur* (The Analytical Theory of Heat) not appearing until 1822. This text has since become the source for the modern methods of solving practical problems associated with partial differential equations subject to prescribed boundary conditions. In addition to heat flow, this class of problems includes structural vibrations, wave propagation, and diffusion .... The task of giving Fourier's work a more rigorous mathematical underpinning was undertaken later by Dirichlet (c. 1830) and subsequently Riemann, his successor at the University of Göttingen.

G. James, "Modern Engineering Mathematics" p736 Addison Wesley (1992).