A LONELY IMPULSE OF DELIGHT
DROVE TO THIS TUMULT IN THE CLOUDS;

WILLIAM BUTLER YEATS, *AN IRISH AIRMAN FORESEES HIS DEATH*

L'AVION AVAIT GAGNÉ D'UN SEUL COUP, À LA SECONDE MÊME OÙ IL ÉMERGEAIT, UN CALME QUI SEMBLAIT EXTRAORDINAIRE. PAS UNE HOULE NE L'INCLINAIT. COMME UNE BARQUE QUI PASSE LA DIGUE, IL ENTRAIT DANS LES EAUX RÉSERVÉES. LA TEMPÊTE, AU-DESSUS DE LUI, FORMAIT UN AUTRE MONDE DE TROIS MILLE MÈTRES D'ÉPAISSEUR, PARCOURU DE RAFALES, DE TROMBES D'EAU, D'ÉCLAIRS, MAIS ELLE TOURNAIT VERS LES ASTRES UNE FACE DE CRISTAL ET DE NEIGE.

ANTOINE DE SAINT-EXUPÉRY, *VOL DE NUIT*

THERE WILL BE TIME TO AUDIT
THE ACCOUNTS LATER, THERE WILL BE SUNLIGHT LATER
AND THE EQUATION WILL COME OUT AT LAST.

LOUIS MACNEICE, *AUTUMN JOURNAL*

MICHAEL CARLEY

AIRCRAFT STABILITY AND CONTROL

This is not a textbook

This is not a textbook and should not be read as one. It is a set of notes for a third year unit at the University of Bath, introducing aircraft stability and control to aerospace engineering students. The aim is to develop an understanding of concepts, but only if the notes are read in conjunction with other material, and combined with attendance at lectures. These notes will not be much use on their own. You will have to work hard on ideas which will not be obvious, and were not obvious to the smart people who developed them. You will often feel stupid and confused, and you will wonder why you are doing this. You are doing this because it is worth it: you are taking on a difficult topic which has confused bright people for over a century, but in which it is possible to make a contribution.

Feeling stupid means you are working on something worth the trouble: if you want to feel clever, read the Daily Mail.

Produced using the `tufte-book` class for LaTeX on 8th January 2020.

Contents

1
How aeroplanes fly and how pilots fly them

> Recently several papers have been published dealing with airplane stability and control problems from the pilot's point of view. Although the airplane [sic] designer should not be expected to have any views on a subject so completely in the pilot's domain, he [sic] does have the responsibility of translating the pilot's requirements into concrete airplane proportions.[1]

[1] Otto C. Koppen. Airplane stability and control from a designer's point of view. *Journal of the Aeronautical Sciences*, 7(4):135–140, 1940

The design of aircraft is a problem of translating mechanical and system requirements into a physical form which can be used by a pilot. While the operational parameters can usually be easily stated—speed, payload, range, say—there is still a need to design in such a way that a suitably-trained human can control the aircraft. This requires that the aircraft present itself to the pilot as standard controls, to which the aircraft will respond in a standard and predictable manner. This is true whether we are talking about high performance aircraft or the most basic of microlights: the fundamental task of the designer is to make the aircraft behave "properly" from the point of view of the pilot. In an extreme case, an aircraft may be completely uncontrollable by a human pilot; in the worst case, it may be controllable over almost, but not quite, all of the design flight regime. In any case, our role as designers is to analyze the effect of aircraft configuration on aircraft response to control inputs and to perturbations in flight, such as gusts, and then to understand how to translate the pilot's needs into a flyable configuration.

To start, we consider the problem of longitudinal control which is the question of how to maintain, or change, an aeroplane's incidence, or angle of attack. You should know that lift on a wing or other body is controlled by the incidence, the angle between a reference line on the body and the relative velocity of the flow.[2] The most basic tasks a pilot must perform in an aircraft translate into control about the pitching axis (Figure 1.5 shows the definition of the aircraft axes): steady level flight, change of incidence to change speed and/or height, recovery from stall. Most of the time, most pilots want to maintain steady level flight with a minimum of effort, to leave mental capacity available for other tasks such as navigation, observation, or a cup of tea. For a small part of their time, pilots want to change the state of the aircraft. Then, the pilot wants an

[2] If you believe that wings generate lift because of the "Bernoulli effect" and air speeding up to keep step with itself, you should not be taking this unit.

aircraft to respond predictably to a given input. The first of these requirements is for *stability*; the second is for *control*.

1.1 Equilibrium and stability

The study of stability and control can be viewed as the problem of setting and maintaining equilibrium. In steady level flight or steady climb, for example, the net force and moment on an aircraft are zero and the aeroplane advances in unaccelerated motion.

First, we define equilibrium: a body is in equilibrium when the net force and net moment acting on it are both identically zero. An aircraft which is in equilibrium is said to be in *trim*, or *trimmed*.

Stability relates to the tendency of a system to return to equilibrium if it is disturbed in some way. *Static* stability refers to the instantaneous response of a system when perturbed: a statically stable system will initially move back towards its equilibrium state. A *dynamically* stable system will eventually recover its equilibrium, though not necessarily immediately. Figure 1.1 illustrates the two cases and also that of *neutral* stability, where the system remains in the state to which it has been perturbed.

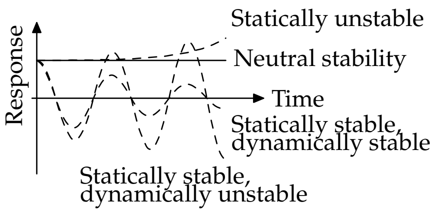


Figure 1.1: Equilibrium and static and dynamic stability

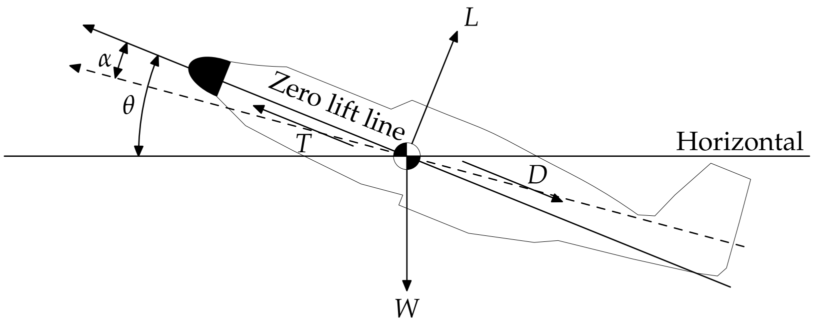


Figure 1.2: Notation for longitudinal stability: the dashed line indicates the flight direction

Figure 1.2 shows the notation for the analysis of aircraft stability. The two angles shown are the incidence α and the inclination θ. The second of these is the angle between a reference line on the aircraft and the horizontal and in practice is of little interest to us in analyzing stability and control, though it is important to a pilot, to whom it is known as "attitude". The incidence, on the other hand, is of great interest and is the angle between the reference line and the direction of flight. As a reference, we take the *zero lift line* (ZLL) which is the angle of attack at which the lift is zero. This choice makes future analysis a little more compact, because then $C_L = a\alpha$, but be careful in consulting other work since the reference system might be different.

Resolving forces and moments from Figure 1.2,

$$T - D - W \sin\theta = 0; \quad (1.1a)$$

$$L - W \cos\theta = 0; \quad (1.1b)$$

$$M_{cg} = 0, \quad (1.1c)$$

where c.g. refers to "centre of gravity" and coordinates are taken in a frame of reference attached to the aircraft. By taking moments about the centre of gravity, we remove the effects of the mass distribution of the aircraft and (1.1c) is a statement about the balance of aerodynamic moments only. If we are to relate scale-test data to full-size aircraft, this is a very useful thing. A pilot brings an aircraft into, or out of, trim by modifying the aerodynamic moments through use of the control surfaces; our analysis lets us deal with the things a pilot changes without worrying about details of the mass distribution.

Having found an equilibrium, we would like to know if it is stable. In aeronautical terms, this can be stated as the requirement that when an aircraft is pitched nose-up (nose-down) by a perturbation, the change in moment must be such as to pitch it nose-down (nose-up). In other words, $\partial M_{\text{cg}}/\partial\alpha < 0$: the change in moment is in the opposite direction to the change in incidence.

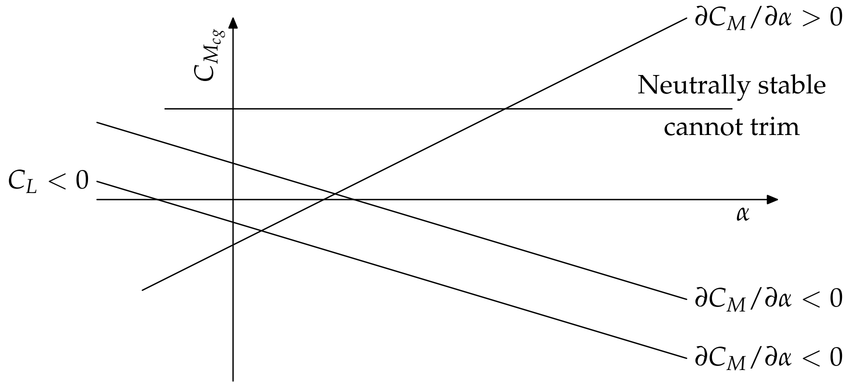


Figure 1.3: Trim and stability behaviour

Figure 1.3 shows some possibilities for equilibrium and stability in terms of moment and lift coefficients $C_{M_{\text{cg}}}$ and C_L.

1.2 *Aerodynamics of wings and controls*

We cannot study equilibrium and stability in the abstract: at some point we must soil our hands with reality and think about actually-existing aircraft and how they behave. In designing any moderately complex system, we usually reduce the elements of the system to a small number of parameters in order to keep the problem tractable. In this case, we do not look at the details of how a wing works, or how the pressure distribution changes when a control is deflected, but only at the overall effect on forces and moments.

As always in aerodynamics we deal in non-dimensional quantities, normalized on air density ρ, velocity V, wing planform area S and, where necessary, wing mean chord $\bar{c}$, or root chord c_0 for tailless aircraft. The 'mean chord' is usually the 'mean aerodynamic chord', or m.a.c. This is a way of representing the wing which gives the same force and moment on the aircraft as the real wing. Figure 1.4 shows some typical planforms and their m.a.c.s. Note that

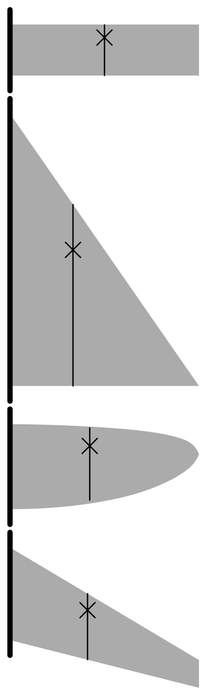


Figure 1.4: Wing planforms (rectangular, delta, bi-elliptical, and swept and tapered) with their mean aerodynamic chords and neutral points

the length and position of the mean chord are both important, since they are used in computing moments as well as forces. Coefficients of lift, drag, and moment are thus given by:

$$C_L = \frac{L}{\rho V^2 S/2}, \; C_D = \frac{D}{\rho V^2 S/2}, \; C_M = \frac{M}{\rho V^2 S\bar{c}/2}. \qquad (1.2)$$

As noted above, $C_L = a\alpha$, where a is the lift curve slope for the wing or other body. As well as stating where our reference angle lies for incidence, this also says that we deal in linear aerodynamics. We do this for two reasons. The first is that, up to stall, aerodynamic behaviour is linear: lift is proportional to incidence. Should an aircraft reach stall, the linearity of the lift curve is not our main concern. Secondly, we design aircraft to be linear, to make them flyable. A pilot wants a linear response to control inputs: a given change in stick force should always give the same change in aircraft response.

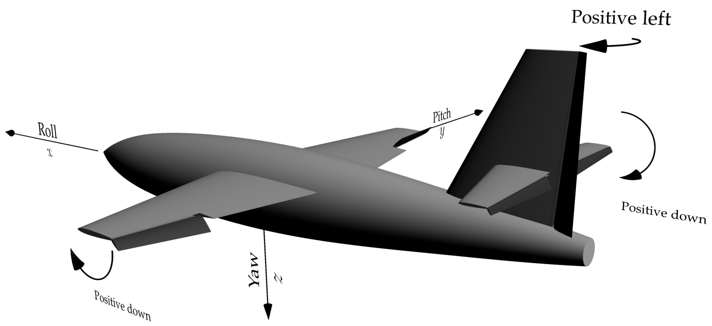


Figure 1.5: Axes and sign conventions for control deflections

Once an aircraft has been built, the aerodynamic properties are fixed, essentially by the choice of wing section and planform, though other effects will need to be considered. This means that the lift curve slope of the wing is constant[3] except in the case of control surfaces—elevator, rudder, and aileron—which can be deflected to change the aerofoil section and thus its properties. You can think of this deflection as a change in section camber, with a corresponding change in lift curve slope.

[3] If flaps or other devices are deployed, the lift curve slope changes, but from one fixed value to another.

Figure 1.5 shows these primary control surfaces on a conventional aircraft, with the corresponding sign conventions. We take a deflection as positive if it generates a positive increment in force. Ailerons work differentially so the deflection is that of both surfaces, with a positive deflection being that which generates a positive rolling moment.

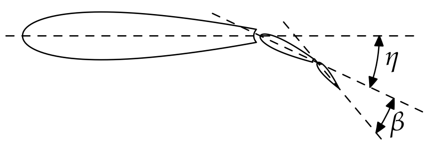


Figure 1.6: Measurement of elevator and tab deflections, η and β

Figure 1.6 shows the sign conventions for deflection of the elevator, the tailplane control surface, and the tab, whose purpose will be explained later. The deflection η is measured from the zero lift line of the tailplane and β from the elevator reference line. As noted above, we deal in linear aerodynamics, so we can write the tailplane lift coefficient

$$C_{L_T} = a_1\alpha_T + a_2\eta + a_3\beta, \tag{1.3}$$
$$a_1 = \frac{\partial C_{L_T}}{\partial \alpha_T}, \quad a_2 = \frac{\partial C_{L_T}}{\partial \eta}, \quad a_3 = \frac{\partial C_{L_T}}{\partial \beta},$$

where α_T is the tailplane incidence, which is not the same as the aircraft incidence. The deflections in (1.3) are the point where a pilot intervenes in the system. The tailplane deflection η is set by the pilot's moving the control, and likewise the tab angle β. On many aircraft, the pilot may also have control of α_T if the aircraft has an all-moving tailplane, such as the X-1, or a trimming tailplane, which is deflected to trim the aircraft, with an elevator for short-term control inputs.

We do not need to know the details of the aerodynamics of control surfaces in order to design a tailplane, but we should know something of how they work. Figure 1.7 shows the change in pressure coefficient over a surface for changes in, respectively, incidence, elevator deflection, and tab angle. You can see that changes in α_T or η give quite large changes in pressure distribution, corresponding to quite large changes in C_{L_T}, making the tailplane useful as a means of adjusting moments on the aircraft. The tab on the other hand seems to have little effect on the pressure distribution or tailplane lift. Why have it?

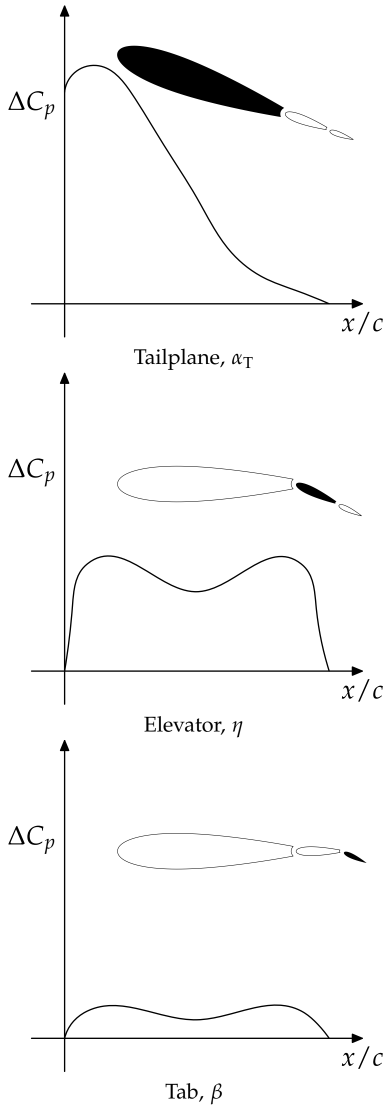


Figure 1.7: Pressure distribution changes with control deflection

Figure 1.8 shows the same data as Figure 1.7 but with the addition of the hinge line, where the elevator joins the tailplane proper. From the shaded regions on the plots, you can see how a deflection modifies the moment about the hinge line. Changing η in order to change C_{L_T} requires quite a large moment; a change in β also gives quite a large change in moment, but with a small change in lift coefficient. This moment is called the hinge moment, and is expressed in non-dimensional form as hinge moment coefficient

$$C_H = \frac{M_H}{\rho V^2 S_\eta c_\eta / 2}, \tag{1.4}$$

given, on linear aerodynamics, by

$$C_H = b_0 + b_1\alpha_{\mathrm{T}} + b_2\eta + b_3\beta. \tag{1.5}$$

The quantities S_η and c_η, the elevator area and chord respectively, are measured behind the hinge line, as shown in Figure 1.9, which also shows an aerodynamic balance, surface ahead of the hinge line which has the effect of reducing the hinge moment for a given deflection. This is one way to alter the stick force required of the pilot, as we will see in Chapter 5.

The hinge moment is fundamental to the control of aircraft, because it corresponds to the force felt by the pilot when they move a control. This means that the hinge moment must be small enough to allow a pilot to move the control surface to any required deflection, but not so small that there is a risk of moving a surface too far and over-accelerating the aeroplane. The control force must also conform to the needs of human physiology and psychology: if control forces are too small, the pilot cannot accurately perceive changes in force even for quite large changes in deflection.[4] The design of a control system for an aircraft is more than simply the sizing of surfaces with respect to some performance criteria; the configuration must also present itself to the pilot in a usable form. Even if the controls are powered, the hinge moment is used to size the actuators which drive the surfaces, so it remains an important consideration in design.

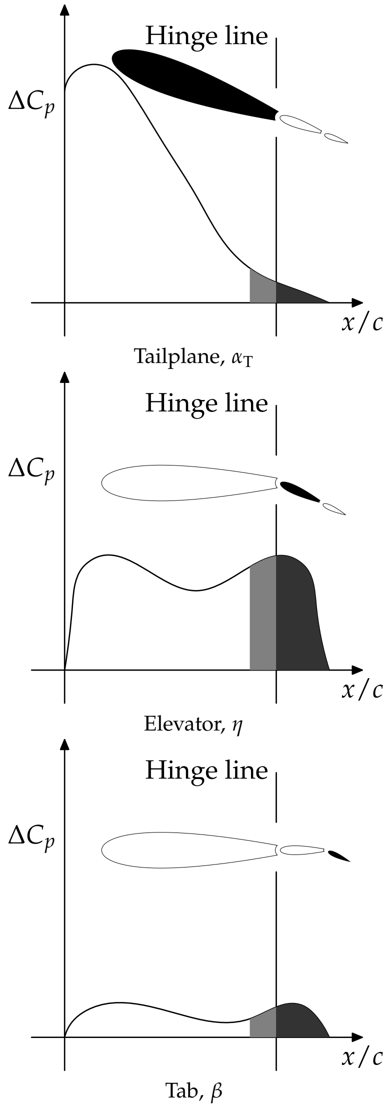


Figure 1.8: Pressure distribution and moment changes with control deflection

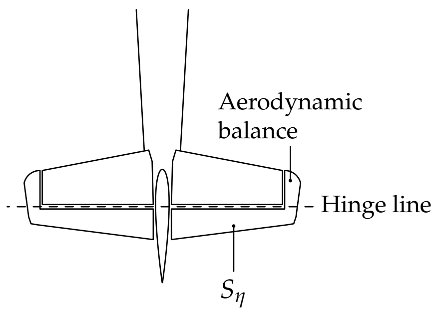


Figure 1.9: Measurement of control geometry

[4] Think Goldilocks.

1.3 Aerodynamic centre

When we come to calculate the moments on an aircraft, it is not enough to know what lift and moment are generated by a wing or control surface; we must also make some choice about where they act. When we compute the moment generated by a lifting body, we do so by viewing it as a force (lift mostly) and a moment placed at some reference point, so that the moment about some other point at a distance x is

$$M(x) = M_0 + Lx.$$

We have some freedom in where we take our reference and we should make best use of it. The obvious reference point on a wing is the centre of pressure, the point about which the aerodynamic moment is zero. Then, $M_0 \equiv 0$ and $M = Lx$, which makes life easy. The problem with this reference point is that the centre of pressure moves with changes in incidence. Given that stability and control are largely concerned with controlling α, the centre of pressure is not very useful as a reference, since it moves as our aircraft pitches.

Instead of the centre of pressure, then, we use an alternative reference point called the *aerodynamic centre* or, for a whole aircraft, the *neutral point*. This is a point about which the moment is independent of incidence, $\mathrm{d}M/\mathrm{d}\alpha \equiv 0$, and

$$M(x) = M_0 + L(x - x_n),$$

where now M_0 is the *zero-lift pitching moment* and subscript n denotes "neutral point". The neutral point of the whole aircraft is one of the most fundamental properties from the point of view of flight.

If we think of the total lift on the aircraft acting at the neutral point, we can sketch some possible relationships between centre-of-gravity position and static stability, Figure 1.10. Remember that if the aircraft pitches nose-up, lift increases, which generates a pitching moment about the centre of gravity. The sign of that moment depends on the relative positions of neutral point and centre of gravity. In Figure 1.10, the first figure shows a stable aircraft because an increase in lift tends to push the nose down: $\partial M_{\mathrm{cg}}/\partial\alpha < 0$; the second is neutrally stable because changes in lift generate no change in moment; the third is unstable because an increase in incidence causes an increase in pitching moment which keeps pushing the nose up, $\partial M_{\mathrm{cg}}/\partial\alpha > 0$.

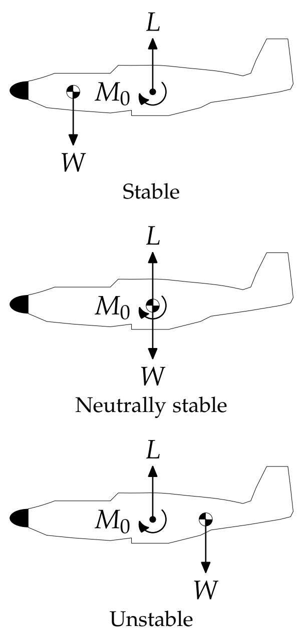


Figure 1.10: Centre of gravity and neutral point positions

1.4 Measures of stability: static and c.g. margins

Being engineers, and comfortable with numbers, we would like to have numerical measures of aircraft stability, if only to bring clarity to the situation. Since, for a stable aircraft $\partial M_{\mathrm{cg}}/\partial\alpha < 0$, we can use $\partial M_{\mathrm{cg}}/\partial\alpha$ as a measure. In non-dimensional terms, we call this measure the static margin,[5]

[5] This is really important. Memorize it.

$$K_n = -\frac{\mathrm{d}C_{M_{\mathrm{cg}}}}{\mathrm{d}C_R}, \tag{1.6}$$

where the resultant force coefficient $C_R = (C_L^2 + C_D^2)^{1/2}$, and the negative sign means that a stable aircraft has a positive margin, which is easier to visualize.

In practice, because $C_D \ll C_L$ and $C_R \approx C_L$, we can use an approximation to K_n, the c.g. margin

$$H_n = -\frac{\mathrm{d}C_{M_{\mathrm{cg}}}}{\mathrm{d}C_L}. \tag{1.7}$$

In the rest of these notes, we will use the approximation $K_n \approx H_n$ without further comment.

1.5 Secondary flight controls

Throughout these notes we consider the so-called 'primary' control surfaces: elevator, rudder, ailerons, and elevons. We should not forget, however, that control is also affected and/or effected by the 'secondary' control systems, such as high-lift devices including flaps and leading edge devices, Figure 1.11. These are not normally used as 'controls', as they are usually moved from one configuration to another and then left in place, but they do have an effect on the aircraft characteristics. In particular, flaps can have a large effect on pitching moment and lift curve slope of the wing, resulting in changes in the stability and handling properties.[6]

While not strictly 'controls', we should also take into account the effect of the engine(s) on handling characteristics. Changes in thrust can have a large effect on aircraft pitching moment and, on multi-engined aircraft, on yaw. Differences in flow over the wing in the wake of a propeller, and engine torque, can also cause quite large rolling moments which must be balanced using the rudder and ailerons or by some other means, such as by making the wing shorter on one side than on the other.[7]

[6] Take a look at the accident report for G-CHNL to see how flap deployment moved the neutral point, making a marginally stable aircraft very unstable.

[7] The Italian Macchi C.202 had exactly this feature.

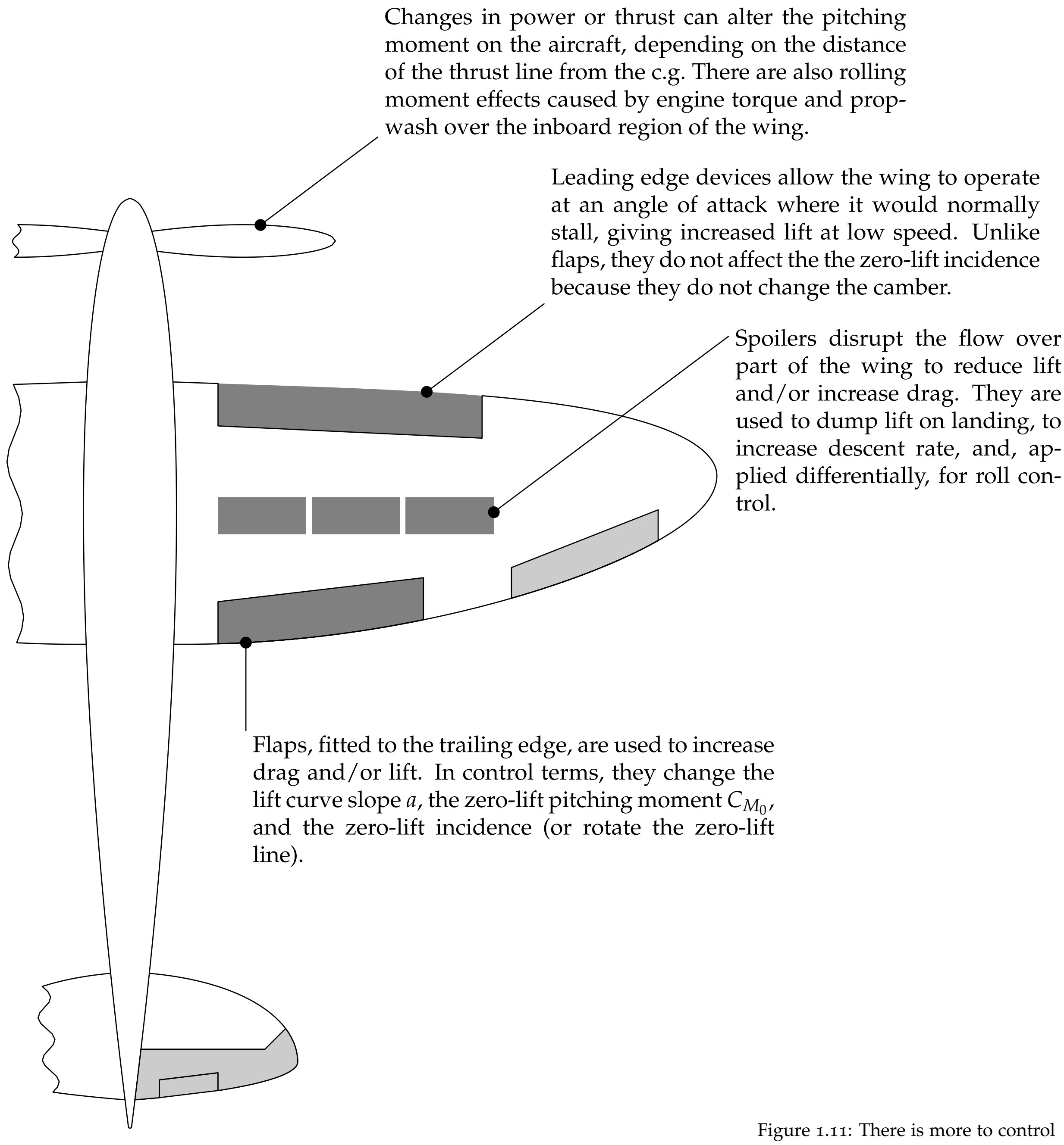


Figure 1.11: There is more to control than the control surfaces.

1.6 What pilots do

The point of aircraft stability and control is that we want to build aeroplanes which are stable and can be controlled. This means that whatever might be the aerodynamic design of the control system, an aircraft must present itself to a pilot in a flyable form: it should respond to a pilot input in the manner a competent pilot expects. The task of a designer is to conceal the design of the control system in such a way that the pilot need know nothing of the connection between the controls and the elevator, but is free to think only of the effect of controls on the aircraft.

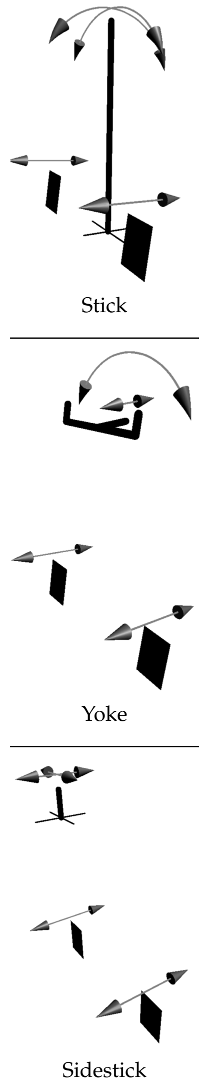


Figure 1.12: Some standard pilot controls: the rudder pedals are almost universal for yaw control, but the pitch and roll controls can vary markedly between aircraft

A fixed-wing aeroplane is controlled through one of a standard set of controls which allow a pilot to move the elevator and ailerons with hand movements, and the rudder through a pair of pedals. Figure 1.12 shows sketches of some typical control arrangements. For now, we only think about the elevator, though the principles are the same for the other controls. Chapter 5 on stick forces discusses the mechanical design of control systems, and talks about how aerodynamic load, i.e. hinge moment, is fed back to a pilot as a stick force which is part of the information available about the state of the aircraft. The movement of the stick reflects in some sense the movement of the control. For example a typical range of elevator deflection is $\pm 30^\circ$; this range of deflection should be achievable using the full range of stick movement. If the pilot can move the elevator beyond its limit, they risk stalling the tailplane which is invariably disastrous; if the pilot cannot move the elevator to its limit, they do not have full control of the aircraft. There is a limit, however, to how precisely a human can control an increment of stick displacement or stick force. Remember that an increment of displacement or force is a control input which accelerates the aircraft. If an aircraft is to be responsive and capable of large accelerations, it should require quite small stick forces to generate a response; if it is to be stable and docile, even large stick forces should not produce excessive accelerations. A numerical statement of the ease of handling of an aircraft is the quality of control a pilot is expected to be able to exercise over stick displacement and force over the range of aircraft accelerations.

Historically, the design of aircraft controls has been concerned with making an aircraft stable, and controllable by a human pilot. With the development of fly-by-wire the mechanical linkage between the pilot control system and the control surfaces was replaced by a computational intermediary, but the design principle remained the same: the aircraft must be flyable by a human being without excessive mental or physical workload.[8]

[8] The meaning of 'excessive' varies with type of aircraft and phase of flight: what would be excessive for a transport aircraft in cruise might be perfectly normal for a fast jet on landing.

The question of how to integrate ergonomics, or human performance, into aircraft design is far too big to be considered in detail in this course, but we will take some account of it, and you should bear it in mind as we proceed. We need to think about what it means to be a competent pilot and how aircraft should be designed to meet the needs and expectations of competent pilots. This means that an aircraft should respond to control inputs in the manner expected by a pilot, and should not stress the pilot psychologically or physiologically. We will look in passing at some crashes. Very often these are attributed to "pilot error"; in those cases, engineers should ask themselves "what led to this error and how could it have been avoided?" In part, if a pilot error has led to a crash, some of the responsibility lies with the designers who made it possible for such an error to lead to catastrophic failure.

2

Longitudinal control and static stability

Longitudinal stability and pitch control of an aircraft are the most basic properties which concern a pilot. Most of the time, a pilot wants to hold an aircraft at a constant incidence, and does so by moving a control surface to the "right" position for moment equilibrium. In order to change the state of flight, the pilot moves the control surface to some other position to impose a finite moment on the aircraft, and force it to rotate. The basic instrument for analysis of the aircraft is thus a moment equation, derived from a free body diagram.

2.1 *The moment equation for aircraft*

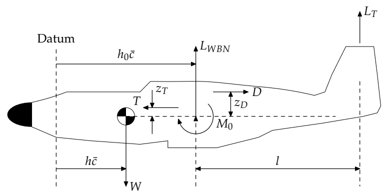


Figure 2.1: Free body diagram for an aeroplane

Figure 2.1 shows the relevant forces and moments acting on an aircraft, with the wing-body-nacelle (WBN) lift placed at the aerodynamic centre $\bar{c}h_0$ and the tailplane (T) contribution placed at the aerodynamic centre of the tailplane. The resulting equation for moment about the centre of gravity is

$$
\begin{aligned}
M_{\text{cg}} &= M_0 - L_{WBN}(h_0 - h)\bar{c} - L_T\left[(h_0 - h)\bar{c} + l\right] - Tz_T + Dz_D \\
&= M_0 - (h_0 - h)\bar{c}(L_{\text{WBN}} + L_T) - L_T l - Tz_T + Dz_D \\
&= M_0 - (h_0 - h)\bar{c}L - L_T l - Tz_T + Dz_D.
\end{aligned}
$$

We can safely assume that lift is much greater than drag and that the combination of drag and thrust is negligible so that the equation can be simplified,

$$M_{\text{cg}} = M_0 - (h_0 - h)\bar{c}L - L_T l.$$

Non-dimensionalizing on $S\bar{c}(\rho V^2/2)$ and noting that the tailplane lift coefficient is based on tailplane area S_T,

$$C_{L_T} = \frac{L_T}{\rho V^2 S_T/2}, \tag{2.1}$$

$$\text{and } C_{M_{\text{cg}}} = C_{M_0} - (h_0 - h)C_L - C_{L_T}\frac{S_T}{S}\frac{l}{\bar{c}}.$$

Defining the *tail volume coefficient*,

$$\overline{V} = \frac{S_T}{S}\frac{l}{\bar{c}} \tag{2.2}$$

results in the fundamental equation of aircraft stability and control:

$$C_{M_{cg}} = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T}. \tag{2.3}$$

This is the most basic equation you need to know[1] since it contains within it all the behaviour of the aircraft which need concern us. Once an aircraft is built and flying, the control problem is how to adjust C_{L_T} for trim, $C_{M_{cg}} \equiv 0$; to design a tailplane, we begin by finding the value of $\overline{V}$ which allows us to achieve stable trim over the required operating range.

[1] You really, really need to know this. Engrave it on your heart with an obsidian dagger. This shall be the whole of the law.

To examine the effect of control deflection, we need to include some detail about the behaviour of the tailplane, and how C_{L_T} is related to pilot input and aircraft operating condition. The big effect we have to include is that of tailplane incidence being affected by downwash, the deflection of the freestream flow caused by lift on the wing.

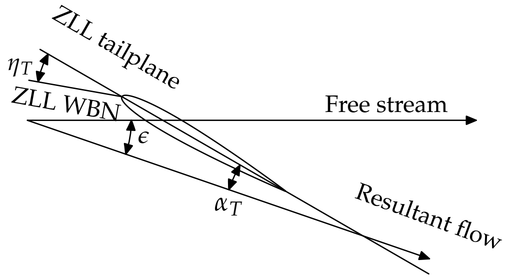


Figure 2.2: Effect of downwash on tailplane incidence

From Figure 2.2, the tailplane incidence is made up of the aircraft incidence α and the angle at which the tailplane is attached to the aircraft η_T, modified by the effect of downwash angle ϵ:

$$\alpha_T = \alpha + \eta_T - \epsilon.$$

For an untwisted wing, ϵ is proportional to the lift on the wing, meaning that in the linear regime, it is also proportional to α,

$$\epsilon = \frac{d\epsilon}{d\alpha}\alpha + \epsilon_0,$$

with ϵ_0 only present for a wing where the zero lift angle of attack varies along the span. Combining these equations,

$$\alpha_T = \alpha + \eta_T - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha\right) = \alpha\left(1 - \frac{d\epsilon}{d\alpha}\right) + (\eta_T - \epsilon_0),$$

and from (1.3),

$$C_{L_T} = a_1\alpha_T + a_2\eta + a_3\beta,$$

so that

$$C_{L_T} = a_1(\alpha + \eta_T - \epsilon) + a_2\eta + a_3\beta,$$

and

$$C_{L_T} = a_1\alpha\left(1 - \frac{d\epsilon}{d\alpha}\right) + a_1(\eta_T - \epsilon_0) + a_2\eta + a_3\beta.$$

We know that

$$C_L = a\alpha,$$

where a is the overall lift curve slope of the aircraft so that

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)C_L + a_1(\eta_T - \epsilon_0) + a_2\eta + a_3\beta. \tag{2.4}$$

In these notes, unless otherwise stated, we assume that $\epsilon_0 = 0$. In practice, we can always do this as long as we modify the value of η_T to take account of zero-lift downwash.

The value of C_{L_T} in (2.3) includes no assumptions about how the lift is generated, so it can be expanded,

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)C_L + a_1(\eta_T - \epsilon_0) + a_2\eta + a_3\beta\right].$$

The most basic thing this equation lets us do is calculate the control input required to trim the aircraft. For example, if we have all the required information about the operating condition of the aircraft, we can calculate the elevator deflection needed for moment equilibrium, $\overline{\eta}$, where the overbar denotes a trim quantity. Also, by using the moment equation as a relation between incidence and pitching moment, we can calculate K_n, our measure of stability.

2.2 *Aircraft stability*

In §1.4, we stated our measure of stability for an aircraft,

$$K_n \approx H_n = -\frac{dC_{M_{cg}}}{dC_L}, \tag{1.7}$$

which, from (2.3),

$$= (h_0 - h) + \overline{V}\frac{dC_{L_T}}{dC_L}, \tag{2.5}$$

into which we can substitute the expression for C_{L_T} from the previous section. This gives us a means of calculating measures of stability under different conditions.

The most basic case is that where the aircraft pitches with the controls locked, known as the *stick-fixed* condition. Then, from (2.4),

$$\frac{dC_{L_T}}{dC_L} = \frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right),$$

$$K_n = h_0 - h + \overline{V}\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right). \tag{2.6}$$

This tells us how stable an aeroplane is for a given position of centre of gravity. For stability, $K_n > 0$, and for any required minimum stability margin (2.6) tells us how far back (aft) we can place the centre of gravity and still meet the requirement.

To summarize this with regard to the aircraft, we call the centre of gravity position where $K_n = 0$ the neutral point h_n, so that the static margin is the non-dimensional distance between the centre of gravity and the neutral point:

$$h_n = h_0 + \overline{V}\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right), \tag{2.7}$$

and $K_n = h_n - h$.

We will see in later chapters how h_n can be measured on an aircraft using flight-test data, so that we do not need to rely on estimates of aerodynamic parameters to find the safe loading conditions for an aeroplane. Given that we can always measure or estimate the centre of gravity position on an aircraft, this is sufficient information to meet a minimum stability requirement.

As well as the stick-fixed case, we consider the stick-free, where the control is free to move until it reaches moment equilibrium,

$$\begin{aligned} C_H &= b_0 + b_1\alpha_\mathrm{T} + b_2\eta + b_3\beta = 0, \\ \text{and } \eta &= -\frac{b_0 + b_1\alpha_\mathrm{T} + b_3\beta}{b_2}, \end{aligned}$$

yielding

$$\begin{aligned} C_{L_T} = &\left(a_1 - \frac{a_2 b_1}{b_2}\right)\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + \left(a_1 - \frac{a_2 b_1}{b_2}\right)(\eta_\mathrm{T} - \epsilon_0) \\ &+ \left(a_3 - \frac{a_2 b_3}{b_2}\right)\beta - \frac{a_2 b_0}{b_2}. \end{aligned}$$

For concision, we introduce some auxiliary variables,

$$\overline{a_1} = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right), \overline{a_3} = a_3\left(1 - \frac{a_2 b_3}{a_3 b_2}\right), \tag{2.8}$$

so that, in the stick-free case,

$$C_{L_T} = \frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}.$$

Inserting C_{L_T} into the pitching moment equation gives

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right],$$

which allows us to find the tab angle to trim with zero stick force, $\overline{\beta}$.

The reason we have a trim tab is that it gives the pilot a means of zeroing the stick force. The tab has very little effect on the tailplane lift, but has quite a large effect on the elevator hinge moment. By moving the tab to deflection $\overline{\beta}$, the pilot can remove their hands from the controls in order to perform other tasks, reducing their physical and mental workload. Also, by zeroing the stick force for the required flight condition, the pilot can use small control inputs, which gives them much finer control over the aircraft than if they had to use some large force to hold the stick in its trim position.[2]

[2] You can try this yourself: hold your hand steady palm upwards. Now do the same thing with a heavy book on your palm. Can you keep your hand stationary? Now do the same, but try moving your hand a small distance under full control.

The measure of stability is the same as it ever was, so we can calculate a static margin using

$$\frac{\mathrm{d}C_{L_T}}{\mathrm{d}C_L} = \frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right),$$

which gives the *static margin stick-free*,

$$K_n' = h_0 - h + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right), \tag{2.9}$$

with a corresponding *neutral point stick free*

$$h_n' = h_0 + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right). \tag{2.10}$$

Note that stick-free values are denoted by a prime symbol.

2.3 *Aircraft control*

The two control conditions which we have seen, stick-fixed and stick-free, give different trim behaviour to the aircraft. In the stick-fixed case, the pilot is actively holding the stick to generate the elevator deflection needed for trim; in the stick-free case, at least notionally, they can release the stick and the aircraft will continue to fly at the same incidence, because the elevator is already in aerodynamic equilibrium.

Variable	Significance
h	Forward centre of gravity limit
C_L	In flight, trim speed
$\overline{V}$	Tailplane size
η	Stick-fixed, elevator angle to trim
β	Stick-free, tab angle to trim
η_{T}	Tailplane setting (possibly to trim)

Table 2.1: Significance of solution for trim equation variables for $C_M = 0$ and all other variables held fixed

Using the trim equation for design, we can size a tailplane for a given operating condition or, given the final aircraft geometry, we can set operating limits or calculate behaviour in flight. There are six variables which we can change in the trim equation, h, C_L, $\overline{V}$, η, β, and tailplane setting η_{T}. On aircraft with an all-moving tailplane we can also change η_{T} in flight. If the aircraft is in trim, $C_M \equiv 0$, so fixing five of the variables gives us a solution for the sixth. They can be interpreted using Table 2.1. In each case, a solution for the variable can be found and has a particular significance in flight. For example, an aircraft flying at a given weight and centre of gravity position will have a trim speed determined by the tab angle β. Likewise, for a given speed and weight, take-off speed and MTOW for example, the forward limit for centre of gravity can be found by solving for h with maximum elevator deflection.

In terms of flying the aeroplane, a useful quantity to consider is $\partial C_L/\partial\overline{\eta} = -\overline{V}a_2/K_n$. It relates the change in lift coefficient to the change in elevator deflection. Elevator deflection is perceived by the pilot as stick deflection, so this corresponds to how far a pilot needs to move the stick to change the lift coefficient, or speed, by a given amount. Also, since pilots typically fly on attitude, i.e. by using a visual reference to establish the aircraft incidence, and C_L corresponds directly to incidence, this is also a measure of how changes in K_n alter the perceived handling qualities of the aircraft.[3]

[3] Now relate this to changes in aircraft speed.

2.4 Tailless aircraft

The analysis up to now has been developed for conventional aircraft, which have a tail. Not all aeroplanes do, and in the interests of inclusivity, we should welcome them into the group, Figure 2.3.

Figure 2.4 shows the control surfaces on a tailless aircraft. The rudder operates as on a conventional layout, but elevators and ailerons are combined into "elevons" which operate differentially for roll control, and together in pitch.


Vulcan: delta wing


Saab Gripen: close-coupled canard

Pegasus Quantum 15-912: flex-wing

Figure 2.3: Some tailless aircraft planforms

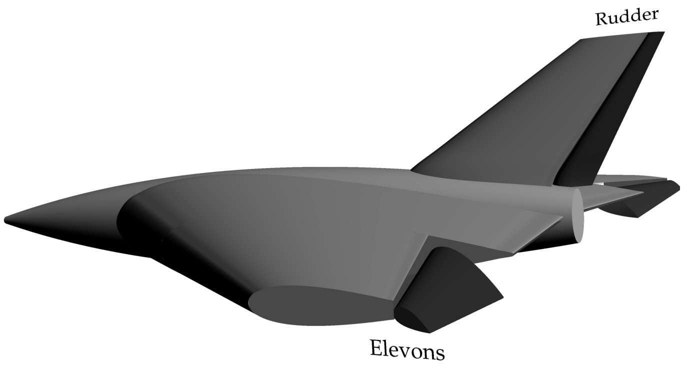


Figure 2.4: Control surfaces for tailless aircraft: elevons operate together for pitch control and differentially for roll

The variables relevant to analyzing a tailless aircraft are shown in Figure 2.5. Clearly, there is no tailplane contribution to include in calculating the pitching moment, but there is a complication because elevon deflection generates a change in lift coefficient and a change in pitching moment. The coupling of these two effects can make tailless aeroplanes quite challenging to control, especially on landing.

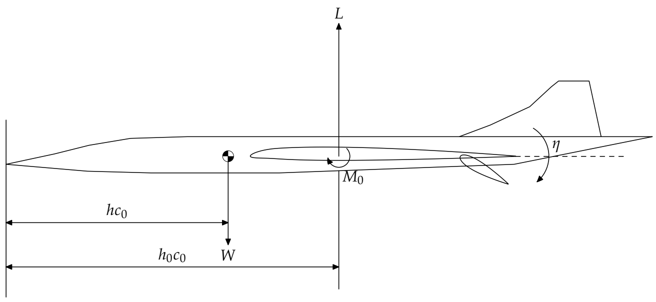


Figure 2.5: Representation of tailless aircraft

The lift coefficient for a tailless aircraft looks a bit like the corresponding expression for a tailplane, because there is a control deflection to include,

$$C_L = a_1\alpha + a_2\eta,$$

but there is no tab, because tailless aircraft usually have none.

As before, we can analyze a free body diagram to find a pitching moment equation,

$$M_{\text{cg}} = M_0 + \frac{\partial M_0}{\partial \eta}\eta - (h_0 - h)c_0 L,$$

which we then non-dimensionalize to give

$$C_M = C_{M_0} + \frac{\partial C_{M_0}}{\partial \eta}\eta - (h_0 - h)C_L.$$

Our definition of static margin is the same as before so

$$K_n = -\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = h_0 - h,$$

with no need to consider the stick-free case because such aeroplanes usually have powered controls.

3
How to design a tailplane

All things considered, an aeroplane needs a wing: most aircraft cannot fly without one. After that, it usually needs a tailplane: most aircraft cannot fly for very long without one, though see Figure 3.1. Engines come later: push come to shove, you can always glide. The functions of a tailplane are to stabilize the aircraft and allow the pilot to control it, with a minimum weight and drag penalty.


Figure 3.1: Wings are obligatory; fins are optional

The requirements for stability and control can be stated in various ways, but we assume that we are given some combination of a minimum value of K_n, and a forward centre of gravity limit, or centre of gravity range, the distance between the forward and aft limits. The desire for minimum weight is an obvious one for any part of an aircraft, and can be restated as wanting the smallest tailplane area we can get away with. The minimum drag requirement comes from the problem of *trim drag*. As you should know, generating lift inevitably means generating drag. Given that a conventional tailplane mostly generates negative lift, in order to generate a nose-up pitching moment, there is a double drag penalty: the drag generated by the tailplane itself, and the extra drag on the wing which has to produce a lift greater than the aircraft weight to compensate for the down force on the tail. Minimizing the tailplane area minimizes the drag and weight penalty.

3.1 Basic tailplane sizing

The most basic design requirement is to size a tailplane for a given minimum static stability margin K_n, and centre of gravity range $\Delta h = h_{\text{aft}} - h_{\text{fwd}}$. We can write two equations

$$K_n = h_0 - h_{\text{aft}} + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right), \tag{3.1a}$$

$$\begin{aligned} C_{M_{\text{cg}}} &= C_{M_0} - (h_0 - h_{\text{fwd}})C_L \\ &\quad - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_{\text{T}} - \epsilon_0) + a_2\overline{\eta} + a_3\beta\right] = 0. \end{aligned} \tag{3.1b}$$

The first of these equations should be obvious: the aft limit on centre of gravity fixes the minimum stability margin. The second is a trim requirement based on the pitching moment which can be generated by the tailplane. The maximum moment is required when the centre of gravity is at its forward limit. There is also a limit on the elevator deflection to trim. The first limit is that the elevator may not be deflected beyond some point fixed by aerodynamic considerations such as stall. This limit is usually about 30°. A second limit is set by the need for the pilot to have a reasonable range of options at any point. For example, if the aircraft is trimmed with a forward centre of gravity in the flight condition set by (3.1b), the pilot might want to manouevre by changing η. If η is already at its aerodynamic limit, the pilot has no options available. Another way to state this constraint is to impose a limit on C_{L_T} to keep it well within its linear operating range, avoiding the risk of tailplane stall.

In any case, there will be a flight condition given in terms of C_L and η. The three equations can then be combined to find $\overline{V}$, h_{fwd}, and h_{aft}. Given basic information about the aircraft geometry, l, $\bar{c}$, and S, the tailplane area S_T can be determined from $\overline{V}$. Some calculations of this type are given as tutorial questions at the end of the notes.

3.2 *Scissors plots*

When the tailplane and centre of gravity range must conform to multiple, possibly conflicting, requirements, the standard design method is the *scissors plot*, which is a graphical method for determining a tailplane area and centre of gravity range for an aircraft. The approach is to rearrange the various constraint equations to give $\overline{V}$ as a function of h, plot them, and read from the plot the value of $\overline{V}$ which gives the required range of h. Typical constraints are the stability limit on K_n, the limiting cases for trim at low speed[1] such as landing approach and climb after take-off, and other important operating conditions such as take-off rotation and the need to have sufficient load on the nose wheel to be able to steer the aircraft on the ground.

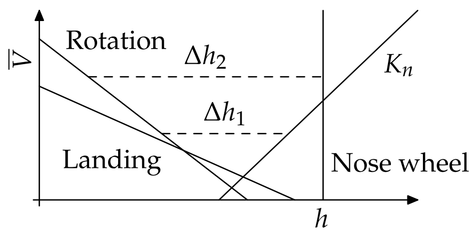


Figure 3.2: A simple scissors plot

[1] Why are these important?

Figure 3.2 shows a simple fictional scissors plot, with two centre-of-gravity ranges indicated. The first, Δh_1, is quite a narrow range and the limiting cases are the take-off rotation and stability constraints. If the designer wants a larger range, Δh_2, the aft constraint is no longer aerodynamic but the requirement to keep sufficient load on the nose wheel. This knowledge of which constraint is driving the tailplane size can be used in subsequent design iterations for the whole aircraft.

3.3 *Designing a useful tailplane*

The scissors plot gives us an estimate of tailplane area for a given set of aerodynamic parameters, a, a_1, etc., but does not tell us if the tailplane is "good" in some sense. The aerodynamic parameters themselves will usually be estimated using published methods, such as ESDU, or by copying a tailplane which is known to work well. In order to be useful, however, the tailplane must be usable by a pilot, which imposes constraints on the hinge moment coefficients, and it must continue to function when things are going wrong.

In aeronautical terms, "going wrong" usually translates as "stall". Stall occurs when the wing reaches its maximum lift coefficient because its incidence is too great. Stall is a simple problem to deal with, and is one of the first things student pilots learn about: the pilot moves the stick forward to reduce incidence which also has the effect of increasing the aircraft speed. Implicit in this procedure, however, is the assumption that the tailplane has not stalled. If the tailplane stalls, the pilot has no control over the angle of attack of the aircraft and cannot recover. The tailplane must be designed to stall later than the wing. In practice, this means that it has slightly more sweep and consequently a lower value of lift curve slope, and may have a constrained choice of η_{T}.

In discussing the scissors plot, we talked about how to select a suitable value of $\overline{V}$. The value we obtain is one suitable for the stability and control requirements we have set for the design but it is not enough information for us to size the tailplane proper. If we assume the wing geometry is fixed, we have no choice of S and $\bar{c}$, so we have to achieve a particular value of $S_T l$. Obviously, if we make l large enough, we can make S_T as small as we like, but we cannot choose l arbitrarily. On any aircraft which must fit a particular footprint, there is a maximum length for the aeroplane, which places one limit on l. There are further constraints which arise from structural considerations and wanting to avoid "dead space" in a fuselage, between the rear bulkhead and the empennage. The major exception to these limits is glider or sailplane design, where it is structurally feasible to have the tailplane on a long boom or long slender fuselage.

4
Flight testing and aircraft handling

Having designed an aircraft to have given stability characteristics, we must test the production model to find what the real behaviour is. In the early stages of design, we use approximate analyses and correlations and semi-empirical methods (for example, ESDU sheets) to estimate the aerodynamic parameters such as lift curve slopes, largely because early in design we have not fixed the exact shape and size of the aircraft or of its subsystems. When we have a detailed geometry, we can use computational methods to refine our estimates. When the first few aircraft are produced, or after modifications to a design, we test them to see what the real behaviour of the real aircraft is.

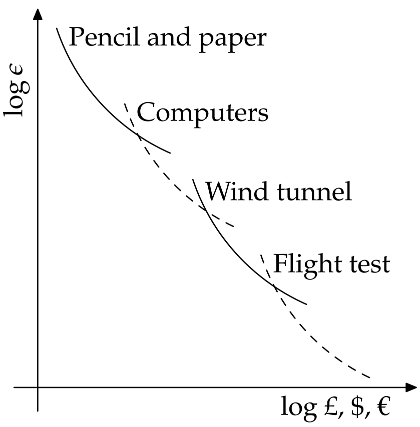


Figure 4.1: Accuracy versus cost for different methods of estimating aircraft properties

Figure 4.1 gives an indication of how ϵ, the error or uncertainty in estimated aircraft properties, varies with the cost of different methods. The simplest methods using pencil and paper are cheap but have a relatively large uncertainty, which is considered acceptable because the methods introduce uncertainties no greater than the uncertainty in the input data. In other words, the precision of the method matches its accuracy. Computational methods give estimates with less uncertainty but take longer and cost more. Wind tunnel testing gives data based on physical testing, but in idealized conditions with uncertainties introduced by rig and interference effects and model scaling: it is also very costly. Finally, flight testing gives the least uncertainty but is the most expensive way to gather data.

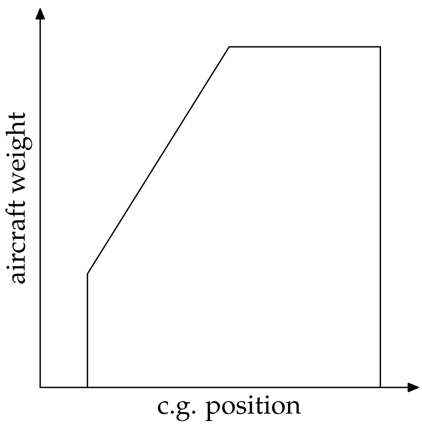


Figure 4.2: A typical weight and balance envelope for a small aircraft

This information is used in setting the limits to be observed in service—the 'flight envelope' of Figure 4.2. Before flight, the aircraft weight and centre of gravity are plotted on the diagram and must lie within the limits indicated.[1] If they do not, then the weight must be reduced or the centre of gravity must be moved by adding ballast. This guarantees that the aircraft will fly within the limits set at the design stage. The rear centre-of-gravity limit, the vertical line on Figure 4.2, is fixed by the minimum stability requirement; the forward limit is set by the maximum moment which the tailplane can generate in order to maintain pitch equilibrium in all phases of flight.

[1] Federal Aviation Administration Flight Standards Service. *Aircraft weight and balance handbook*. US Department of Transportation, 2007

4.1 Measuring stick-fixed stability

Flight testing depends on being able to use quantities we can measure to estimate the things we want to know. For a given aircraft, we know the shape of the aeroplane, because we can measure it. For a given flight, we know the centre of gravity position and the aircraft weight, because we can calculate them or measure them on the ground. For a given flight condition, we know the aircraft speed and the control deflections. This gives us h, C_L, η, and β, and we know that $C_M = 0$ in trim. How far can we get with this information?

As always, we start from the fundamental moment equation,

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T},$$

and the tailplane lift coefficient,

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta,$$

giving

$$\begin{aligned} C_M = 0 = C_{M_0} - (h_0 - h)C_L - \\ \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta\right]. \end{aligned}$$

As in §2.2, we differentiate to find the static margin,

$$K_n \approx H_n = -\frac{\partial C_M}{\partial C_L} = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right),$$

and since we can also calculate the elevator angle to trim,

$$\begin{aligned} \overline{\eta} = \frac{1}{\overline{V}a_2}\Bigg\{ & C_{M_0} - (h_0 - h)C_L \\ & - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_3\beta\right]\Bigg\}, \end{aligned}$$

we find that $\overline{\eta}$ and K_n are related,

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = -\frac{1}{\overline{V}a_2}\left[(h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\right] = -\frac{K_n}{\overline{V}a_2}. \tag{4.1}$$

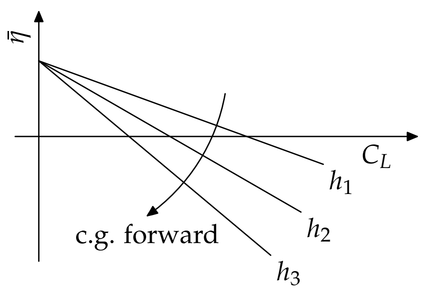


Figure 4.3: Elevator angle to trim at various lift coefficients

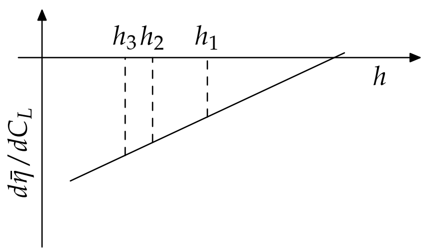


Figure 4.4: Measurement of neutral point location

Figure 4.3 shows $\overline{\eta}$ plotted against C_L, while Figure 4.4 shows the relationship between $\mathrm{d}\overline{\eta}/\mathrm{d}C_L$ and h. It is worth noting that the elevator angle to trim at zero lift is independent of centre-of-gravity position, as the moment equation makes clear.[2]

[2] Can you think of a physical reason why this should be so?

Given this information, one way of finding the aircraft neutral point stick-fixed is: fly the aircraft straight and level at various speeds, recording the elevator angle to trim. This is repeated for various different centre of gravity positions, yielding a plot like Figure 4.3. To find the neutral point, plot the gradients of the lines of Figure 4.3, as in Figure 4.4. Extrapolating to $\mathrm{d}\overline{\eta}/\mathrm{d}C_L$ gives the centre of gravity position where $K_n = 0$, the neutral point h_n.

4.2 *What does this mean for the pilot?*

Equation 4.1 expresses a relationship between stability and control, from the pilot's point of view. A more stable aircraft (one with large K_n) requires larger changes in control deflection for given changes in incidence (α or C_L) or speed. The limits on elevator deflection and operating condition for the aircraft thus impose a limit on the maximum static margin which will allow the aircraft to be flown by a pilot. Likewise, a very small value of K_n makes it difficult for the pilot to control the aircraft, because small changes in control deflection can give quite large changes in incidence, or speed.

We can also directly examine the relationship between speed and elevator deflection. The elevator angle to trim is a function of speed via the lift coefficient,

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}V} = \frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L}\frac{\mathrm{d}C_L}{\mathrm{d}V}.$$

We know that

$$C_L = \frac{L}{\rho V^2 S/2},$$

giving

$$\frac{\mathrm{d}C_L}{\mathrm{d}V} = -\frac{2L}{\rho V^3 S/2} = -\frac{2C_L}{V},$$

and

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}V} = -\frac{2C_L}{V}\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = \frac{4}{\rho}\frac{W}{S}\frac{1}{V^3}\frac{K_n}{\overline{V}a_2},$$

which is sketched in Figure 4.5.[3]

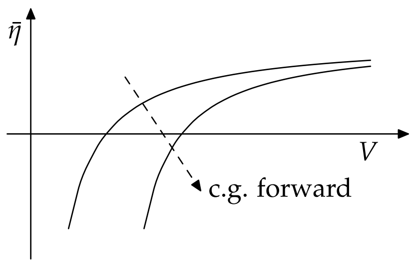


Figure 4.5: What the pilot experiences

[3] What happens to the curves in Figure 4.5 as the wing loading changes?

From Figure 4.5, it is clear that the aircraft is uncontrollable below some minimum flight speed—it is not possible to move the elevator far enough to trim. This happens because at low speed, the control surfaces cannot generate enough force to balance the moment about the centre of gravity. Likewise, above a certain speed, small changes in $\overline{\eta}$ lead to large changes in trim speed and the aircraft is also very hard to control. The useful range of speeds for an aircraft lies between these two limits, although the limits in question will be a function of the aircraft type and of the skill assumed of the pilot.

These effects become apparent to the pilot as "sloppy" handling at low speed, which is taken as a sign of incipient stall. One way to interpret this poor handling is as the result of insufficient ρV^2 for the wing and control surfaces to generate enough moment or force for the controls to be responsive. One of the basic properties of any aircraft is thus the minimum control speed, which is largely determined by the size of the control surfaces with respect to the available dynamic pressure.

4.3 Measuring stick-free stability

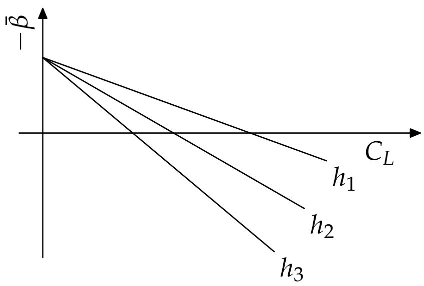


Figure 4.6: Tab angle to trim at varying lift coefficients

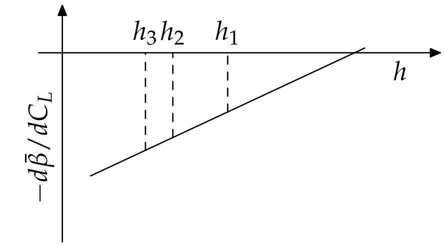


Figure 4.7: Measurement of stick free neutral point location

To find the neutral point stick-free, we can use the same approach as in the stick-fixed case, but using the tab to trim, rather than the elevator. Once again,

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V} C_{L_T} = 0,$$

and

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta,$$

so that

$$\eta = -\frac{b_0 + b_1\alpha_\mathrm{T} + b_3\beta}{b_2},$$
$$C_{L_T} = \frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2},$$

and

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right].$$

This gives the tab angle to trim for the flight condition,

$$\overline{\beta} = \frac{1}{\overline{V}\overline{a_3}}\Bigg\{C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) - \frac{a_2 b_0}{b_2}\right]\Bigg\},$$

which can be differentiated,

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{1}{\overline{V}\overline{a_3}}\left[h_0 - h + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\right].$$

and related to K_n',

$$K_n' = (h_0 - h) + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right),$$

with

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{K_n'}{\overline{V}\overline{a_3}}.$$

So to find the neutral point stick free, we vary the aircraft speed at fixed centre of gravity, trimming with the tab, giving us Figure 4.6. We then plot the gradients from that figure against C_L, Figure 4.7, to find h_n'.

5
Piloting: stick forces

The physicist, philosopher and motorcycle mechanic Matthew Crawford has written extensively on how we experience the world and draw non-verbal information from the sensory data which physical reality feeds back to us, for example through our sense of touch. After talking about the way an ice-hockey player uses the information transmitted by his stick, Crawford looks at the more generic problem of using a probe, for example to feel inside a component to check its surface finish.

> Consider the experience of using a probe to explore an unseen space, or the way a blind person feels his [sic] way by tapping with a stick. At first you feel the varying pressure of the probe against your palm and fingers, and you have to *interpret* this pressure, mapping it in some as yet uncertain way onto a spatial representation that you are developing of the object. But as you learn to use the probe, your awareness of this pressure at the handle end is transformed into something quite different. What you have eventually is a direct, unmediated sense of the probe's tip touching the objects you are exploring [page 47].[1]

[1] Matthew B. Crawford. *The world beyond your head: How to flourish in an age of distraction*. Viking, 2015

Part of the experience of controlling a machine, such as a car or an aeroplane, is using the information which the machine transmits to the driver or pilot. This is fundamental to our experience of control, and is something which we internalize early in learning how to drive or fly. Part of the problem of design in aircraft control is balancing the sometimes conflicting requirements for aerodynamic control of the aircraft, information transmission to the pilot, and allowing the pilot to move control surfaces without excessive physical effort.

Crawford's thoughts on the equivalent problem in car design are worth reading.

> A car that interposes layers of electronic mediation between the driver and the road demands an effort of interpretation by the driver, because each of those layers is based on a representation that has no inherent, necessary relationship to the states being represented. Some committee of engineers had to make a whole series of decisions about how the pedal pressure felt by a driver in a car with brake-by-wire, for example, should map onto the braking force delivered and, crucially, the readiness of the system to keep delivering it. Should the pedal effort change with sustained or heavy braking, to convey

> the fact that those little DC motors doing the work are getting hot? Brake rotors get hot under heavy use and, in doing so, become less effective. This fact gets conveyed to the driver *in a necessary and lawlike way* with the familiar "brake fade" in conventional hydraulic brakes [page 82].[2]

[2] Matthew B. Crawford. *The world beyond your head: How to flourish in an age of distraction*. Viking, 2015

The majority of aircraft, even large ones, have a direct mechanical linkage between the pilot controls and the corresponding control surface. As well as being mechanically simpler than "electronic mediation", with the resulting advantages of reliability and ease of maintenance, direct linkages give the pilot physical feedback from the control surface, which is incorporated into the pilot's picture of how the aircraft is behaving.[3]

[3] To get an idea of how important the pilot's construction of the state of the aircraft is, read some accounts of the AF447 crash.

5.1 *Aerodynamics, stick force, and piloting*

The question which then arises is how to design a control system which gives the aerodynamic forces and moments required to allow a pilot to predictably and reliably control an aircraft, without the risk of accidental overloading. Allowable control forces are laid out in the regulations governing aircraft certification and operation. Table 5.1 gives the important numbers.

	Pitch			Roll			Rudder	
	Stick	Wheel		Stick	Wheel		(Push)	
Temporary	267	111	222	334	133	222	667	N
Prolonged	44.5	44.5	—	22	22	—	89	N
application		One hand	Two hands		One hand	Two hands		

Table 5.1: Allowable control forces, from EASA CS 23.143, CS 25.143.

Good design practice is to make sure that the maximum rudder force is greater than the maximum elevator force which is in turn greater than the maximum aileron force. A further criterion is to aim for the controls to be 'harmonized', meaning that aileron, elevator, and rudder forces required for a given control response have the ratio 1:2:4. For example, the rudder force for a 10°/s yaw should be twice the elevator force for a 10°/s pitch.

The pilot input to the system, from a designer's point of view, is the stick force to trim $\overline{P_e}$, which is the force required on the pilot control to balance the hinge moment at the control surface,

$$\overline{P_e} = m_e \frac{\rho V^2}{2} S_\eta c_\eta \overline{C_H},$$

where m_e is the gearing ratio between the stick and control deflections.

The stick force to trim must lie within reasonable limits over the operating range of the aircraft: too high and the pilot will not be able to move the elevator over the full range of deflections needed; too low and a small stick deflection will generate a large acceleration on the aircraft with a risk of overloading the structure. The first piece of information we need is the hinge moment to trim, which depends on the flight condition and on the tab setting.

We already know that

$$C_H = b_0 + b_1\alpha_\mathrm{T} + b_2\eta + b_3\beta,$$

which gives η as a function of C_H,

$$\eta = \frac{C_H - b_0 - b_1\alpha_\mathrm{T} - b_3\beta}{b_2}.$$

Tailplane lift coefficient is

$$C_{L_T} = a_1\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta,$$

and so

$$C_{L_T} = \overline{a_1}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(C_H - b_0),$$

a general form of the stick-free expression with $C_H \neq 0$.

The pitching moment equation is then

$$\begin{aligned}C_M &= C_{M_0} - (h_0 - h)C_L \\ &\quad - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(C_H - b_0)\right],\end{aligned}$$

which can be re-arranged to find $\overline{\beta}$,

$$\begin{aligned}\overline{V}\overline{a_3}\overline{\beta} &= C_{M_0} - (h_0 - h)C_L \\ &\quad - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) - \frac{a_2 b_0}{b_2}\right],\end{aligned} \tag{5.1}$$

or hinge moment to trim,

$$\begin{aligned}\overline{V}\frac{a_2\overline{C_H}}{b_2} &= C_{M_0} - (h_0 - h)C_L \\ &\quad - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right].\end{aligned} \tag{5.2}$$

Now, if we subtract (5.2) from (5.1),

$$\overline{V}\left(\overline{a_3}\overline{\beta} - \frac{a_2}{b_2}\overline{C_H}\right) = \overline{V}\overline{a_3}\beta,$$

yielding

$$\overline{C_H} = \frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta)$$

and

$$\overline{P_e} = m_e\frac{\rho V^2}{2}S_\eta c_\eta\frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta),$$

so that $\overline{C_H}$ and the stick force to trim depend linearly on the difference between the current tab setting and the tab angle to trim for the flight condition.

In theory we could find the stick-free neutral point from measurements of stick force, via

$$\overline{C_H} = \frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta),$$

which gives

$$\frac{\partial \overline{C_H}}{\partial C_L} = \frac{b_2}{a_2}\overline{a_3}\frac{\partial \overline{\beta}}{\partial C_L}.$$

In §4.3, we found that

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{K_n'}{\overline{V}}\frac{1}{\overline{a_3}},$$

so that

$$\frac{\mathrm{d}\overline{C_H}}{\mathrm{d}C_L} = -\frac{b_2 K_n'}{\overline{V} a_2}.$$

In principle, by measuring the stick force or hinge moment at different flight conditions, we can work out the stick free neutral point. In practice, however, we cannot measure the stick force accurately enough for a reliable estimate, because of errors introduced by such things as friction in the system. We can, however, use a measurement of hinge moment, taken at the control proper, to find $\mathrm{d}\overline{C_H}/\mathrm{d}C_L$ and perform the required analysis. Note once again the effect of static margin—a stability measure—on the control characteristics of the aircraft where the relationship between hinge moment and incidence is a function of K_n'.

5.2 *Modification of stick forces*

Having designed an aircraft with suitably-sized control surfaces, it can happen that the stick forces do not lie in a range usable by a pilot. In this case, there are a number of means of modifying the stick forces to make the aircraft controllable. A couple of these require no aerodynamic redesign. The first approach is to change the gearing ratio between the stick and the control deflections, but this is limited because it can affect the range of control movement available. A more flexible approach is to add power assistance to reduce or increase the pilot input, or to eliminate it, though this then requires a feedback system to give the pilot force information.

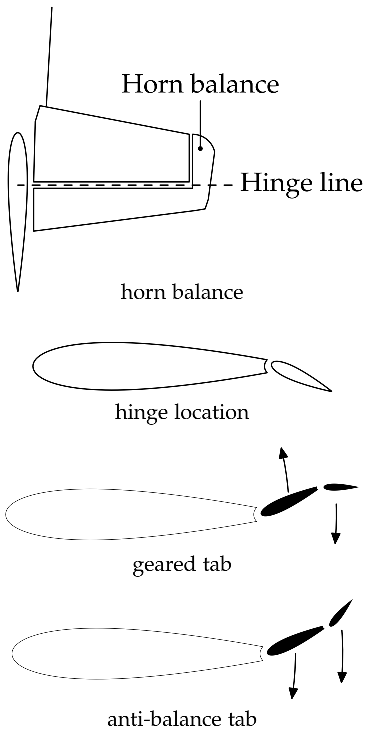


Figure 5.1: Aerodynamic assistance

Figure 5.1 shows some aerodynamic methods for modifying stick force. The first two modify the moment required for a given control deflection by adding surface ahead of the hinge line (aerodynamic balancing) or by moving the hinge line.

The aim is to change the hinge moment required for a given increment in elevator deflection, $\mathrm{d}P_e/\mathrm{d}\eta$. In this case,

$$\overline{P_e} = m_e \frac{\rho V^2}{2} S_\eta c_\eta C_H,$$

but since

$$C_H = b_0 + b_1 \alpha_\mathrm{T} + b_2 \eta + b_3 \beta,$$

$$\frac{\mathrm{d}\overline{P_e}}{\mathrm{d}\eta} = m_e \frac{\rho V^2}{2} S_\eta c_\eta b_2.$$

To reduce the stick force, we want to reduce b_2, but $b_2 \mathrm{d}P_e/\mathrm{d}\eta$, must be negative for correct feel of the controls. Reducing b_2 is useful at high speed (because of the effect of V^2) but at low speed, the pilot might not have enough feel for the controls and other methods of reducing the stick force may be needed.

The third and fourth methods for modifying stick force in Figure 5.1 are to gear the tab to the movement of the elevator. In one case, as the elevator moves, the tab deflection changes in such a way that it reduces the hinge moment on the elevator; in the other, the tab movement increases the hinge moment. By a suitable choice of gearing, it is possible to modify the stick forces to make the aircraft fully controllable by the pilot within a normal range of force.

6
Manoeuvre

The history of all hitherto existing stability has been the history of equilibrium. Most of the time, this is what a pilot wants, but it is clearly important to be able to change state of flight, or manouevre. The most basic manouevre is a steady 'pullout' where a descending aircraft makes a transition to horizontal flight, at a constant speed. We can approximate the dynamics of the process in terms of flight on a circular path in the vertical plane, which will allow us to analyze the behaviour of the aircraft and the relationship between control input and aircraft acceleration. This is also a good approximation to the state of an aircraft in a banked turn.

Aircraft acceleration is important for two reasons. First, the maximum acceleration which can be imposed on an aircraft is a statement of agility: how rapidly can the aeroplane change from one flight condition to another? Second, there is a maximum acceptable acceleration which arises from structural considerations or, for aerobatic aircraft which can sustain high loads, the peak acceleration which a pilot can tolerate.

6.1 Steady pullout

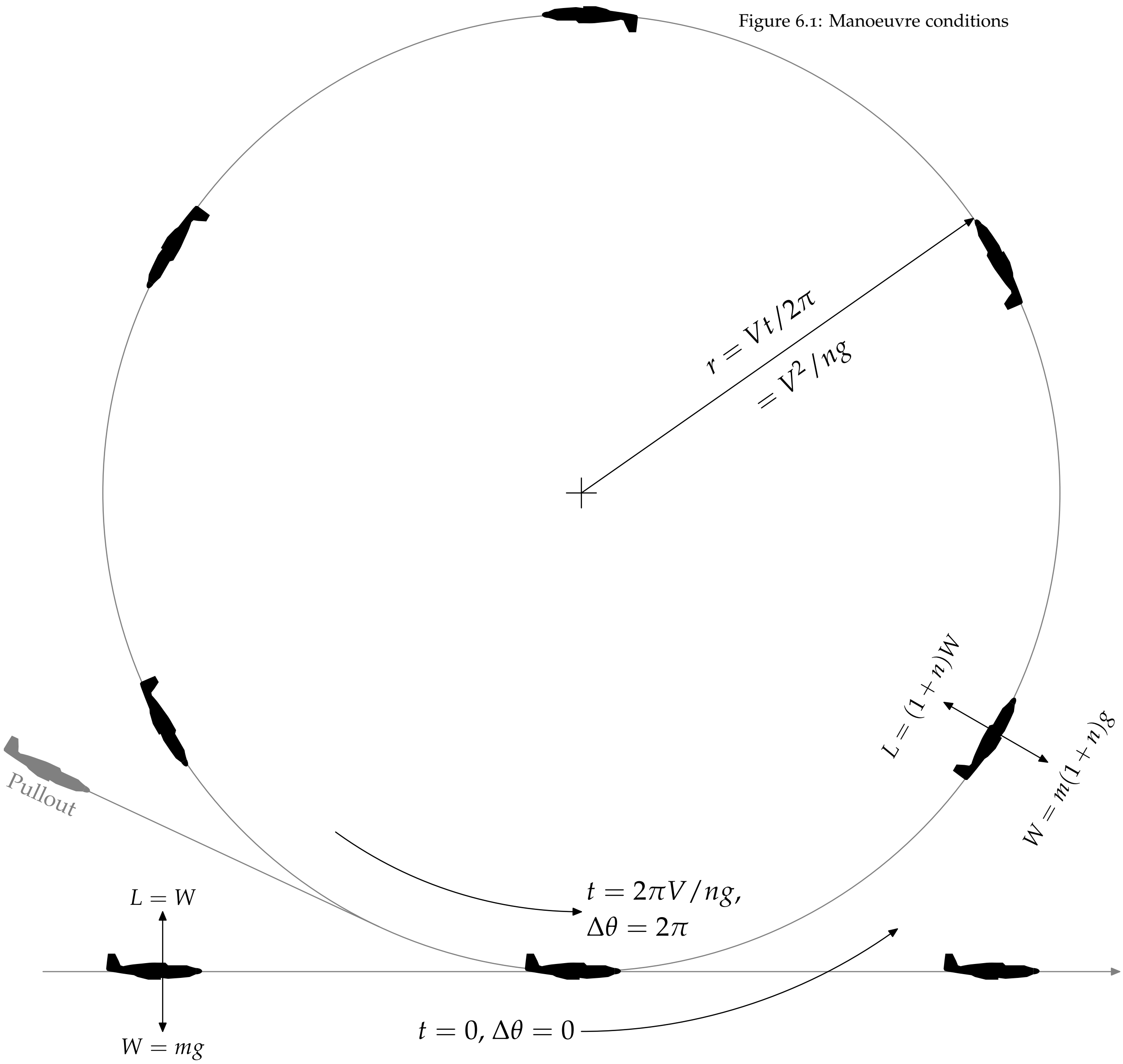


Figure 6.1: Manoeuvre conditions

Our basic model for a manouevre is shown in Figure 6.1, which illustrates an aircraft flying around a circle in the vertical plane. Also shown in grey is the more common case of a descending aircraft making a transition to level flight, which can be modelled as motion along part of the vertical circle. We take the aircraft as moving at constant speed V on a circle of radius r. Clearly, this is accelerated motion and there is an additional force on the aircraft to keep it moving on a circular path. We call the increment in force nmg, where g is acceleration due to gravity, and n is then the number of "gees pulled". In steady level flight, $n \equiv 0$, and $L = W = mg$.

In the manouevre, there is a change in lift, $L = W = m(1+n)g$

and the new lift coefficient is $C_L + \Delta C_L$,

$$\Delta C_L = nC_L = \frac{nW}{\rho V^2 S/2}$$

where C_L is the lift coefficient in the straight and level case.

We want to relate the dynamics of the aircraft to the acceleration n, which we can do by looking at the details of the manouevre, Figure 6.1. In travelling round the circle, the aircraft pitches through an angle of 2π. We know the aircraft speed, so we can find the time required to traverse the loop,

$$t = \frac{2\pi r}{V}.$$

The acceleration on the aircraft is related to its speed and the radius of the circle,

$$r = \frac{V^2}{ng},$$

so

$$t = \frac{2\pi V}{ng},$$

and the aircraft pitch rate is

$$q = \frac{2\pi}{t} = \frac{ng}{V}.$$

The aircraft is rotating about its centre of gravity at angular velocity q, which means that there is a change of incidence at the tailplane,

$$\Delta\alpha_{\mathrm{T}} = \frac{ql_T}{V},$$

where l_T is the tail arm *measured from the centre of gravity*.[1] Inserting the pitch rate,

$$\Delta\alpha_{\mathrm{T}} = \frac{ngl_T}{V^2},$$

and non-dimensionalizing to remove the explicit dependence on V,

$$\Delta\alpha_{\mathrm{T}} = \frac{nC_L}{2\mu_1},$$
$$\mu_1 = W/\rho g S l_T.$$

The term μ_1 is called the *longitudinal relative density*. Note that it uses the true density at altitude, and not the sea-level value.[2]

The pitch-induced change in tailplane incidence generates a corresponding change in tailplane lift coefficient, $a_1\Delta\alpha_{\mathrm{T}}$, and the total lift coefficient is then

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta + a_1\frac{nC_L}{2\mu_1}.$$

[1] This is important: moving the centre of gravity affects the stick force required for a given acceleration, which is why aerobatic piston engine aircraft are taildraggers.

[2] Why is this so?

We now have the elements we need to complete the pitching moment equation,

$$C_M = C_{M_0} - (h_0 - h)(1+n)C_L \\ - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)(1+n)C_L + a_1(\eta_T - \epsilon_0) + a_2\eta + a_3\beta + a_1\frac{nC_L}{2\mu_1}\right].$$

In steady level flight,

$$C_M = 0 = C_{M_0} - (h_0 - h)C_L \\ - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)C_L + a_1(\eta_T - \epsilon_0) + a_2\overline{\eta} + a_3\beta\right]. \quad (6.1)$$

For a trimmed steady manoeuvre, the pitching moment is zero,[3] and we can write the elevator angle to trim as $\overline{\eta} + \Delta\overline{\eta}$, so that

[3] Do you believe this?

$$0 = C_{M_0} - (h_0 - h)(1+n)C_L \\ - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)(1+n)C_L \\ + a_1(\eta_T - \epsilon_0) + a_2(\overline{\eta} + \Delta\overline{\eta}) + a_3\beta + a_1\frac{nC_L}{2\mu_1}\right]. \quad (6.2)$$

In practice, what we want to know is the elevator deflection per g, or the control input required for a given acceleration, which we can find by subtracting (6.1) from (6.2),

$$0 = -(h_0 - h)nC_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)nC_L + a_1\frac{nC_L}{2\mu_1} + a_2\Delta\overline{\eta}\right],$$

which we rearrange to find

$$\frac{\Delta\overline{\eta}}{n} = -\frac{C_L}{\overline{V}a_2}\left\{(h_0 - h) + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{a_1}{2\mu_1}\right]\right\}. \quad (6.3)$$

This must always be negative, otherwise the aircraft would pitch nose-down when the pilot pulls back.

6.2 *Stick fixed manoeuvre stability*

Much of the information contained in the expression for $\Delta\overline{\eta}/n$ is based on calculations which may have no independent verification until flight testing has taken place. In the same way as we stated static margins in terms of distance between centre of gravity and a neutral point, we can express manoeuvre characteristics in terms of a manoeuvre point, whose position can be estimated on a real aircraft using measured data.

When $\Delta\overline{\eta}/n = 0$ the centre of gravity is at the *stick fixed manoeuvre point*, so

$$h_m = h_0 + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{a_1}{2\mu_1}\right].$$

Returning to the definition of stick-fixed neutral point, we can see that h_m is $a_1\overline{V}/2\mu_1$ aft of h_n.

The *stick fixed manoeuvre margin*, H_m, is then defined in the obvious way,

$$H_m = h_m - h.$$

From §6.1,

$$\frac{\Delta\overline{\eta}}{n} = -\frac{C_L}{\overline{V}a_2}\left\{(h_0 - h) + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right]\right\},$$

and,

$$h = h_0 + \frac{\overline{V}a_2}{C_L}\frac{\Delta\overline{\eta}}{n} + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right],$$

so that there is a relationship between the stick-fixed manoeuvre margin and the elevator angle to trim for a given acceleration,

$$H_m = -\frac{\overline{V}a_2}{C_L}\frac{\Delta\overline{\eta}}{n},$$

in the same way that the static margin stick-fixed is related to the elevator angle to trim in steady level flight, (4.1).

6.3 *Stick free manoeuvre stability*

Given the stick-fixed manoeuvre characteristics of an aircraft, we can determine how it will perform aerodynamically in a manoeuvre, but we must also consider the pilot input required for a given acceleration.

From §5.1, we know the hinge moment required for trim in steady level flight,

$$\begin{aligned} C_M = 0 = C_{M_0} &- (h_0 - h)C_L \\ &- \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(\overline{C_H} - b_0)\right]. \end{aligned}$$

To assess the pilot input needed for manoeuvre, we require the change in hinge moment $\Delta\overline{C_H}$ for a given acceleration, which we can find from the usual moment equation, with the manoeuvre lift coefficient $(1+n)C_L$,

$$\begin{aligned} C_M = 0 = C_{M_0} &- (h_0 - h)(1+n)C_L - \\ \overline{V}&\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L\right. \\ &\left.+ \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta + \overline{a_1}\frac{nC_L}{2\mu_1} + \frac{a_2}{b_2}(\overline{C_H} + \Delta\overline{C_H} - b_0)\right]. \end{aligned}$$

Subtracting these expressions

$$\frac{\overline{V}a_2}{b_2 C_L}\frac{\Delta\overline{C_H}}{n} = -(h_0 - h) - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right].$$

Adopting the usual notation, we define the *stick free manoeuvre point*, h'_m, the centre-of-gravity position where $\Delta\overline{C_H}/n = 0$,

$$h'_m = h_0 + \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right].$$

The corresponding *stick free manoeuvre margin*, H'_m, is

$$H'_m = h'_m - h,$$
$$H'_m = -\frac{\overline{V}a_2}{b_2 C_L}\frac{\Delta\overline{C_H}}{n}.$$

The stick force per g is calculated from the hinge moment,

$$\frac{\Delta P_e}{n} = m_e \frac{\rho V^2}{2} S_\eta c_\eta \frac{\Delta\overline{C_H}}{n}.$$

The stick force per g is a fundamental piloting property of the aircraft and must lie within reasonable limits to avoid the risk of a pilot accidentally overloading the aeroplane. For aerobatic aircraft, on the other hand, it can be relatively low, but with a requirement for greater skill on the part of the pilot.

6.4 Tailless aircraft

Tailless aircraft have similar dynamics to conventional aeroplanes, but we need to introduce some extra notation to allow for the different configuration. We already know that for a tailless aircraft,

$$C_M = 0 = C_{M_0} + \frac{\partial C_{M_0}}{\partial\eta}\overline{\eta} - (h_0 - h)C_L,$$

and when the aircraft is in a steady pullout with acceleration g and pitch rate q,

$$C_M = 0 = C_{M_0} + \frac{\partial C_{M_0}}{\partial\eta}(\overline{\eta} + \Delta\overline{\eta}) - (h_0 - h)(1+n)C_L + \frac{\partial C_M}{\partial q}q.$$

Subtracting one equation from the other gives the change in elevator angle for the manoeuvre,

$$\Delta\overline{\eta} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h)nC_L - \frac{\partial C_M}{\partial q}q\right],$$

where $\partial C_M/\partial q$ is an *aerodynamic derivative*.[4] When we look at the dynamic behaviour of aircraft, we will come across more aerodynamic derivatives which are used to express the relationship between the motion of the aircraft and the resulting forces and moments. For consistency, these derivatives are expressed in a standard non-dimensional form. In this case $\partial C_M/\partial q$ is non-dimensionalized as

[4] Does a conventional aircraft have a corresponding aerodynamic derivative? What is it?

$$m_q = \frac{1}{\rho V S c_0^2}\frac{\partial M}{\partial q}.$$

Then,

$$\frac{\partial C_M}{\partial q} = \frac{1}{\rho V^2 S c_0/2}\frac{\partial M}{\partial q} = \frac{2c_0}{V}m_q$$

and we know that

$$q = \frac{ng}{V},$$

so that

$$\Delta\overline{\eta} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h)nC_L - m_q n C_L \frac{\rho g c_0 S}{W}\right].$$

The longitudinal relative density, μ_1, for a tailless aircraft is

$$\mu_1 = \frac{W}{\rho g S c_0},$$

so that

$$\frac{\Delta\overline{\eta}}{n} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h) - \frac{m_q}{\mu_1}\right] C_L.$$

This gives us a means of defining a manoeuvre point, the centre-of-gravity position where $\Delta\overline{\eta}/n = 0$,

$$h_m = h_0 - \frac{m_q}{\mu_1}.$$

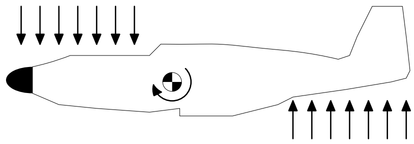


Figure 6.2: Aerodynamic forces during pitching motion

Consulting Figure 6.2 should convince you that $m_q < 0$, because the aerodynamic forces introduced by a positive pitch rate generate a negative moment: the aerodynamic loads oppose the rotation, and the pitch damping increases the stability of the aircraft, which is also clear from the definition of h_m.

The manoeuvre margin for a tailless aircraft, H_m, is defined in the same way as before

$$H_m = h_m - h,$$

and

$$H_m = (h_0 - h) - \frac{m_q}{\mu_1} = K_n - \frac{m_q}{\mu_1}.$$

Then, as for the elevator deflection on a conventional aircraft, the elevon angle per g is proportional to H_m,

$$\frac{\Delta\overline{\eta}}{n} = \frac{H_m C_L}{\partial C_{M_0}/\partial\eta}.$$

6.5 *Static and manoeuvre margins*

We have shown that the static margins, stick-fixed and stick-free, for conventional aircraft are

$$K_n = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right),$$
$$K_n' = (h_0 - h) + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right).$$

Also, the manoeuvre margins for conventional aircraft are

$$H_m = (h_0 - h) + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{a_1}{2\mu_1}\right],$$
$$H_m' = (h_0 - h) + \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right],$$

and

$$H_m = K_n + \frac{\overline{V}a_1}{2\mu_1},$$
$$H'_m = K'_n + \frac{\overline{V}\overline{a_1}}{2\mu_1}.$$

Manoeuvre points are aft of the corresponding neutral points, because of the stabilizing effect of pitch damping.

We have shown that the static margin for tailless aircraft is

$$K_n = h_0 - h$$

and that the manoeuvre margin is

$$H_m = (h_0 - h) - \frac{m_q}{\mu_1}.$$

So,

$$H_m = K_n - \frac{m_q}{\mu_1}.$$

6.6 *Piloting qualities: changing the stick force*

It can happen that an aircraft is aerodynamically acceptable in manoeuvre, but does not have good handling properties from the point of view of the pilot. This leads us to modify the primary control input, the stick force, by changing the relationship between the input force and the aircraft response.[5]

[5] USAF Test Pilot School. *Flying qualities textbook, USAF-TPS-CUR-86-02*, volume II. United States Air Force, Edwards Air Force Base, 1986

Adding a spring into the circuit, Figure 6.3, generates a moment on the stick which corresponds to a control force. The tension T on the spring is approximately constant over the range of stick travel, so that the force P is also approximately constant. This means that a spring in the system imposes a constant increment, or decrement, on the control force perceived by the pilot.

A bob-weight on the other hand introduces a moment which varies with aircraft acceleration. From Figure 6.3, the stick force generated by the weight varies as $W(1+n)$ and the force required on the stick changes as the aircraft accelerates, a modification of the stick force per g. Increasing the stick force per g can be interpreted as shifting h'_m aft.

In practice, springs and weights are used in combination to give the required handling properties. For example, if a weight is introduced in order to change the stick force per g, a spring may be used to balance the moment from the weight at zero acceleration.

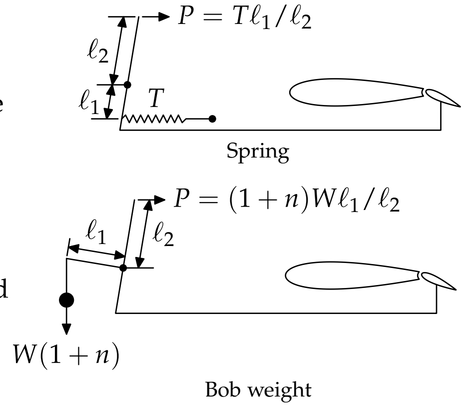


Figure 6.3: Modification of stick force and stick force per g using spring and bob-weight respectively

7
Aircraft configurations and control

There are many different tail configurations in use, in response to the many requirements which must be satisfied by any aircraft design. It is not sufficient for a tail to generate the moments required to allow a pilot to control the aircraft, and to maintain some measure of stability: it must be integrated onto a particular aircraft which has a particular role. This leads to various configurations. The most common tail unit, or empennage, is a combination of a vertical fin, with a rudder, and a horizontal tailplane, with an elevator and tab. On some aircraft, there is no tab and the whole tailplane is moved for trim. Even this basic layout has variations, however, depending on the vertical positioning of the horizontal tailplane. Further variations include combining the horizontal and vertical surfaces into a 'V' or 'Y' shape. These choices are often made for reasons which are not directly related to stability and control, but to other requirements of the overall design.

Figure 7.1 shows a few otherwise conventional aircraft with tails of varying outlandishness. Engineering being engineering, the reasons for choosing such configurations do not always arise from stability considerations alone, but are often attempts to provide the required handling and stability properties while meeting other requirements. For example, the tee tail, shown here on a Grob 109, may be chosen on transport aircraft such as the A400M or C17 in order to leave room for cargo handling equipment and to avoid aerodynamic interference during airdrops, or on rear-engined aircraft such as the Boeing 717 to keep the tailplane clear of the engine exhausts.[1] It does, however, also act as an "end plate" on the fin which can help improve its aerodynamic efficiency, and a tee tail can allow a greater lever arm for the tailplane while keeping the fuselage proper the same length. Tee tail aircraft, however, are particularly prone to a phenomenon called "deep stall" which led to a number of fatal crashes in aircraft under test before the problem was recognized. This occurs when the outboard region of the wing stalls, shifting the tip vortices inboard. This leads to increased downwash on the tailplane, which lies roughly in the plane of the tip vortices. If the aircraft is stable in pitch beyond the stall, there may be insufficient control authority for the pilot to recover by pushing the nose down.[2]

[1] We draw a discreet veil over three-engined aircraft.

[2] Malcolm J. Abzug and E. Eugene Larrabee. *Airplane stability and control: A history of the technologies that made aviation possible.* Cambridge University Press, Cambridge, 2002

Also shown in Figure 7.1 are a vee and an inverted-vee tail. These have the advantage of having fewer surfaces to build, and reduced interference drag, but with the disadvantage of greater control complexity since the elevators are used for both pitch and yaw control. Vee tails are also prone to stall in sideslip which has been proposed as a possible cause of the high accident rate of the Beech 35. It appears that the inverted-vee tail of the Predator was not chosen for pure stability reasons but to protect the propeller during landings, by acting as a bumper.[2]

Finally, at the bottom of Figure 7.1 are a pair of quite unusual designs, the triple vertical tail of the OV-1 light attack aircraft and the vertical tail of the C-2 carrier supply aircraft, which has four vertical surfaces, and three rudders.

Grob 109

Handley–Page Jetstream

Sukhoi 27

De Havilland Vampire

Fouga Magister

General Dynamics Predator

Grumman OV-1 Mohawk

C2 Greyhound

Figure 7.1: Tailplane configurations on aircraft of otherwise conventional layout

7.1 Canard aircraft

The first real aeroplane, the Wright Flyer, was a canard but the now conventional arrangement with a tailplane was soon found to be better for most purposes. There are a number of canard aircraft in operation, however, so they clearly have their uses. Probably the leading designer of canard aircraft is the legendary Burt Rutan, founder of Scaled Composites. Two of his designs, the home-built VariEze and the Beechcraft Starship, are shown in Figure 7.2, with the Piaggio Avanti, a three-surface aircraft.

The principal advantages of a canard configuration lie in the design of highly manoeuvrable aircraft, where their disadvantages are outweighed by the possibilities of post-stall control and supermanoeuvrability, which is why a canard layout is often seen on modern fast jets.

In more conventional flight regimes, the main reason for choosing a canard is that the aircraft becomes very difficult to stall. If the angle of attack increases sufficiently, the canard stalls first, and the lift on the wing pushes the nose back down, giving an inherent stability which recovers from the incipient stall. If the pitch rate is too high, however, the aircraft can rotate past the canard stall angle to the point where the wing enters dynamic stall and the canard does not have sufficient control authority to recover. This can be mitigated using a three-surface layout with both a tailplane and a canard as on the Piaggio Avanti of Figure 7.2.[3]

Rutan VariEze

Beechcraft Starship

Piaggio Avanti

Figure 7.2: Some canard aircraft

[3] Malcolm J. Abzug and E. Eugene Larrabee. *Airplane stability and control: A history of the technologies that made aviation possible.* Cambridge University Press, Cambridge, 2002

7.2 Weight-shift and microlight aircraft

A large number of aircraft operated by recreational pilots fall into the microlight category, which broadly speaking means a maximum takeoff weight of 300kg for a single-seat and 450kg for a two-seat aircraft.[4] These aircraft include powered parachutes, as well as flex-wing and conventional aeroplanes.

[4] There is more to the definition than weight.

The dynamics of such small aircraft can be quite different from the behaviour expected of larger designs, because of the importance of added mass effects: this is especially true of Human Powered Aircraft (HPA). When a body moves in air, its apparent mass and moment of inertia include a contribution from the loads imposed by aerodynamic effects. For a relatively dense, or heavy, aircraft these effects represent only a small percentage of the overall mass and can be neglected. For aircraft which are already quite small, these effects may have a large influence on the dynamics of the aeroplane. Given that such aircraft have lightweight, i.e. flexible, structures, deformation effects must also be accounted for in the dynamics, resulting in behaviour which is not what might be intuitively expected.

Deformation effects are especially important on flex-wing aircraft where the elastic properties of the wing ("sail") must be managed in order to maintain safe handling and acceptable performance, in particular to ensure that the wing holds a reasonable profile over the whole speed and incidence range.[5] The delta wing configuration used for flex-wings is inherently stable in all three axes, but there is one particular instability called the "tumble mode" which invariably leads to loss of the aircraft and is almost always fatal.[6] In this case, the aircraft rotates rapidly, at up to 400°/s, about its pitch axis, usually because of aircraft modification or an attempt to fly beyond the aircraft or pilot's capability. The acceleration leads to structural failure and destruction of the airframe.

[5] G. B. Gratton. The weightshift-controlled microlight aeroplane. *Proceedings of the IMechE*, 215 Part G: 147–154, 2001

[6] G. Gratton and S. Newman. The 'tumble' departure mode in weightshift-controlled microlight aircraft. *Proceedings of the IMechE*, 217 Part G:149–166, 2003

8
High-speed flight: compressibility effects

So far in these notes, the assumption, tacit or otherwise, has been that the aerodynamics are linear and the constants are constant, in effect the assumption of low speed flight. In the transonic flow regime, compressibility effects can lead to large changes in the stability and control characteristics of aircraft with possibly catastrophic results. In the 1940s, when aircraft began to enter the transonic regime, the study of these effects began, largely in order to investigate anomalous handling, as seen, in almost correct form, in The Sound Barrier.[1]

[1] Malcolm J. Abzug and E. Eugene Larrabee. *Airplane stability and control: A history of the technologies that made aviation possible*. Cambridge University Press, Cambridge, 2002

8.1 High speed effects

Figure 8.1 summarizes the principal effects which modify the control of aircraft at high speed. First, there is the motion of the neutral point h_0 from wing quarter chord to half chord as Mach number M increases. This raises two problems. The first is the change in h_0 with speed as the aircraft passes through $M = 1$. This leads to control problems for a pilot as the pitching moment changes with no corresponding control input. The second problem is that the change in h_0 leads to a large increase in K_n. Remembering the relation between elevator angle to trim and static margin (page 28),

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = -\frac{1}{\overline{V}a_2}\left[(h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\right] = -\frac{K_n}{\overline{V}a_2}, \tag{4.1}$$

we can see that an increase in K_n increases the change in elevator angle needed to trim for a given change in C_L or, equivalently, speed. A large increase in K_n can thus make the aircraft uncontrollable because there is insufficient elevator travel to change speed or to manoeuvre, as can be seen by considering changes in $\Delta\overline{\eta}/n$, (6.3).

The next two plots in Figure 8.1 show more bad things: the zero-lift pitching moment coefficient changes with Mach number, as does the wing zero-lift incidence. The combination of these effects means that purely by virtue of its changing speed, the aircraft wants to pitch as it approaches and exceeds a Mach number of unity.[2]

[2] If the aircraft geometric incidence is held constant and α_0 changes, what are the implications for stall?

The second row of Figure 8.1 shows how changes in lift curve slopes, a and a_1, and elevator effectiveness a_2 manifest themselves. Considering only the upper curve in each case, that for a rigid aircraft, we can see large increases in lift curve slope around $M = 1$, followed by a drop-off with increasing Mach number. This can cause various problems for stability and control, as you can see by looking at the expressions for stability and manoeuvre margins, and also causes difficulties for aircraft design: an aeroplane designed for good qualities with the high speed values of a and a_1 may well not have good handling qualities at low speed, unless special measures are taken. Then, it is clear from the plot of a_2 that above a certain speed, the elevator simply stops working, and cannot be used for pitch control. It was some time before this effect was recognized, and led to the use of all-moving tailplanes for longitudinal control. Note also the effect of aircraft deformation, which is a function of altitude and the corresponding change in density, and leads to further changes in the variation of the aerodynamic coefficients.

The final row of Figure 8.1 shows, first, a large change in downwash which will have an effect on the tailplane behaviour, and then the so-called "Mach tuck" phenomenon. As an aircraft reaches high speed, the net pitching moment can decrease before increasing and then returning to its equilibrium value. This leads to a control problem for the pilot, who may well input a wrong stick force.[3] The solution to this problem is shown in the final plot of the figure: a

[3] This is the grain of truth in a scene from the David Lean film The Sound Barrier, which is well worth watching as an account of test flying.

Mach sensor is used to generate an additional stick force which depends on speed so that the stick force gradient remains "correct" through the speed change.

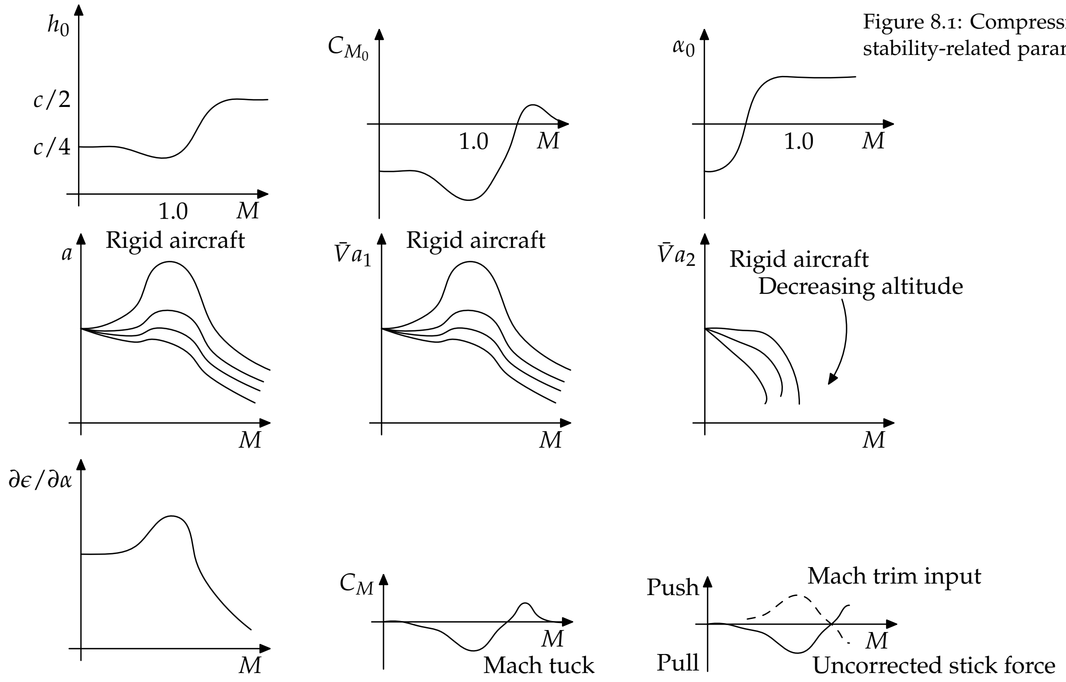


Figure 8.1: Compressibility effects on stability-related parameters

9
Dynamic behaviour of aircraft

So far, we have only looked at the static stability of aircraft: how they initially respond to a perturbation. To understand the flying qualities of an aeroplane, we need to consider dynamic stability, how the aircraft responds over time. Start with a simple example in pitch. The motion of a rotating body is governed by

$$M = I\ddot{\theta},$$
$$\text{or, for an aeroplane, } B\ddot{\alpha} - M_{\text{cg}} = 0,$$

where B is the moment of inertia about the pitch axis. Assuming the controls are locked, M_{cg} is related to α via the static margin stick-fixed,

$$\begin{aligned} M_{\text{cg}}(\alpha) &= \frac{\rho V^2 S\bar{c}}{2} C_{M_{\text{cg}}}, \\ &= -\frac{\rho V^2 S\bar{c}}{2} K_n a\alpha + M_{\text{cg}}(0), \end{aligned}$$

so that

$$B\ddot{\alpha} + \frac{\rho V^2 S\bar{c}}{2} K_n a\alpha = 0, \tag{9.1}$$

ignoring the zero-incidence pitching moment. This is the equation of motion of a simple harmonic oscillator with natural frequency ω,

$$\begin{aligned} \alpha &= \mathrm{e}^{\mathrm{j}\omega t}, \\ \omega^2 &= \frac{\rho V^2 S\bar{c}}{2B} K_n a. \end{aligned}$$

If the static margin is negative, (9.1) becomes

$$\ddot{\alpha} - \frac{\rho V^2 S\bar{c}}{2B} |K_n| a\alpha = 0, \tag{9.2}$$

and the response to a perturbation is no longer oscillatory, but grows exponentially,

$$\begin{aligned} \alpha &= \mathrm{e}^{\lambda t}, \\ \lambda^2 &= \frac{\rho V^2 S\bar{c}}{2B} |K_n| a. \end{aligned}$$

This oscillation is a simple model for what happens when an aircraft encounters a gust, or the pilot changes elevator deflection.[1]

[1] How would this change if you assumed a stick-free condition?

We will look at this in more detail in the next chapter when we consider the dynamics of the whole aircraft.

We can also look at another form of motion, where the aircraft flies at constant incidence at varying speed. In this case, the aircraft can be approximated as a mass acted on by a force perpendicular to the flight path.

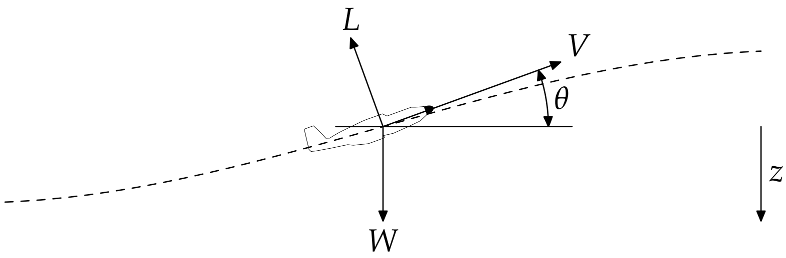


Figure 9.1: Motion of a body under lift and gravity

Figure 9.1 shows the notation.[2] We assume that the aircraft flies at constant incidence with thrust balancing drag, so that the lift $L = \rho V^2 S C_L/2$ and varies only with speed V. The inclination of the flight path is θ so the net force normal to the aircraft is given by

$$L - W\cos\theta = \frac{W}{g}\frac{V^2}{R}, \tag{9.3}$$

where R is the radius of curvature of the path. We also know that energy is conserved so that $V^2/2 - gz$ is constant, with z taken positive downwards. We can choose an origin for z such that the total energy is zero and then $V^2 = 2gz$: the aircraft trades kinetic and potential energy (height) so we can write speed in terms of height and *v.v.*

If we take V_1 as the speed the aircraft would have in steady level flight at the prescribed C_L, we can rewrite (9.3),

$$\frac{z}{z_1} - \cos\theta = \frac{2z}{R}, \tag{9.4}$$

and since

$$\frac{1}{R} = \frac{\mathrm{d}\theta}{\mathrm{d}s} \quad \text{and} \quad \sin\theta = -\frac{\mathrm{d}z}{\mathrm{d}s},$$

where s is the arc length along the flight path and R is the radius of curvature of the trajectory, (9.4) can be rewritten

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(z^{1/2}\cos\theta\right) = \frac{z^{1/2}}{2z_1}, \tag{9.5}$$

and integrating gives a solution for $\cos\theta$ and R,

$$\cos\theta = \frac{1}{3}\frac{z}{z_1} + C\left(\frac{z_1}{z}\right)^{1/2}, \tag{9.6a}$$

$$\frac{z_1}{R} = \frac{1}{3} - \frac{C}{2}\left(\frac{z_1}{z}\right)^{3/2}, \tag{9.6b}$$

where $Cz_1^{1/2}$ is the constant of integration. We cannot solve directly for the flight path, called the *phugoid*, but we can say something about its behaviour as a function of C.

[2] The analysis presented here is based on Milne-Thomson, L. M., *Theoretical aerodynamics*, MacMillan, fourth edition, 1966, pp 376–378.

First, if the aircraft flies at constant height $z = z_1$, it is obvious that $C = 2/3$. If the aircraft is to fly in a loop, at some point $\cos\theta = -1$, which can only happen if $C < 0$, because z and z_1 are never negative. When $C = 0$, $R = 3z_1$ and the flight path is a sequence of semi-circles. When $0 < C < 2/3$, the flight path is an oscillation in z. Some possibilities are shown in Figure 9.2, found by numerically solving the equations of motion,

$$\dot{V} = -\sin\theta,\ \dot{\theta} = \frac{V^2 - \cos\theta}{V}. \tag{9.7}$$

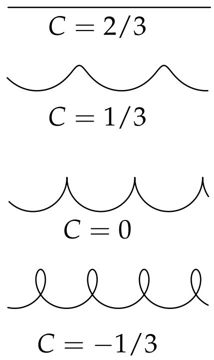


Figure 9.2: Phugoid flight paths

These two examples illustrate the essential properties of longitudinal dynamics of an aeroplane. To properly understand the handling qualities of an aircraft, we need to develop a systematic analysis of its dynamics, incorporating rotation and translation in three axes and the coupled effects of aerodynamic interactions.

9.1 *Analysis of aircraft dynamics*

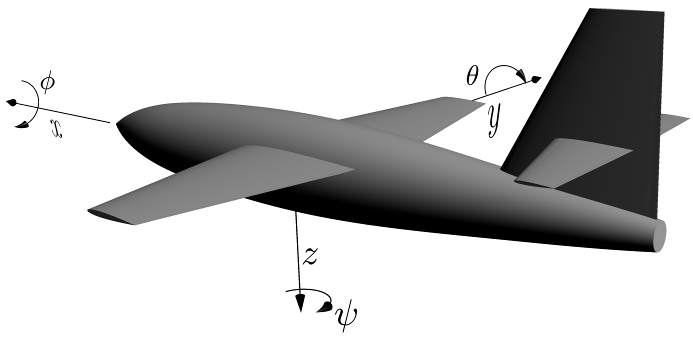


Figure 9.3: Notation for analysis of dynamic stability

Axis	Perturbation force	Mean velocity	Perturbation velocity	Rotation angle	Angular velocity	Moment of inertia	Moment
x	X	U	u	ϕ	p	A	L
y	Y	V	v	θ	q	B	M
z	Z	W	w	ψ	r	C	N

The first step is, as always, to define our notation. Figure 9.3 shows the system of axes. The axes are attached to the aircraft, rather than to an inertial frame, and have their origin at the centre of gravity. The table of quantities gives the notation for the displacements, rotations, forces, moments and moments of inertia. It is literally as easy as A-B-C. In practice, to examine problems of stability we will linearize the system and write quantities as the sum of a mean value for steady level flight, and a small perturbation.

The logic of our analysis is the same as for any dynamic problem: identify the forces and moments which act on a free body, insert these in the appropriate dynamic equations, and calculate the motion of the body. The difficulties arise from the aircraft's having six degrees of freedom and from coupling between motion in those degrees of freedom. We need to develop a systematic way of modelling this coupling, ideally using quantities which can be measured in flight as well as calculated on the ground.[3]

[3] The analysis which follows is taken from Milne-Thomson, L. M., *Theoretical aerodynamics*, MacMillan and Company, 1966.

As always in this course, we assume linearity, so we can work to first order in perturbation quantities. For example,

$$M = M_0 + \frac{\partial M}{\partial u}u + \frac{\partial M}{\partial v}v + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial p}p + \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial r}r.$$

The moment in this case is written as a sum of inputs from the velocities and angular velocities on all three axes. The constants, which are effectively the first terms in a Taylor series, are called "aerodynamic derivatives" or "stability derivatives". One of them, $\partial M/\partial q$ has already arisen in §6.4, with regard to pitch damping during manoeuvres. If we know these quantities, and the state of the aircraft, we can compute its motion using Newtonian dynamics. This is feasible using computational methods, but does not give us insight into the qualitative behaviour of an aeroplane, so we will have to do some maths.

We know that there are two sets of forces on the aircraft, aerodynamic $\mathbf{F}$, and gravitational $\mathbf{g}$,

$$\mathbf{F} = X\hat{\mathbf{\imath}} + Y\hat{\mathbf{\jmath}} + Z\hat{\mathbf{k}}, \tag{9.8}$$

$$m\mathbf{g} = mg_1\hat{\mathbf{\imath}} + mg_2\hat{\mathbf{\jmath}} + mg_3\hat{\mathbf{k}}, \tag{9.9}$$

where the components of $\mathbf{g}$ are needed because the reference frame is fixed to the aircraft and rotates about three axes. We also need to know the motion of the aircraft,

$$\begin{aligned} \mathbf{v} &= u\hat{\mathbf{\imath}} + v\hat{\mathbf{\jmath}} + w\hat{\mathbf{k}}, \quad \text{velocity,} \\ \mathbf{\Omega} &= p\hat{\mathbf{\imath}} + q\hat{\mathbf{\jmath}} + r\hat{\mathbf{k}}, \quad \text{angular velocity,} \\ \mathbf{h} &= h_1\hat{\mathbf{\imath}} + h_2\hat{\mathbf{\jmath}} + h_3\hat{\mathbf{k}}, \quad \text{angular momentum.} \end{aligned}$$

The equations of motion are then

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v}) = m\dot{\mathbf{v}} + \boxed{\mathbf{\Omega} \times (m\mathbf{v})} = \mathbf{F} + m\mathbf{g}, \tag{9.10a}$$

$$\frac{\mathrm{d}\mathbf{h}}{\mathrm{d}t} = \dot{\mathbf{h}} + \boxed{\mathbf{\Omega} \times \mathbf{h}} = \mathbf{L}, \tag{9.10b}$$

where the boxed terms are required because the frame of reference is rotating. The applied moment $\mathbf{L}$ is

$$\mathbf{L} = L\hat{\mathbf{\imath}} + M\hat{\mathbf{\jmath}} + N\hat{\mathbf{k}}.$$

We now approximate these equations to examine how the aircraft responds when it is perturbed from steady level flight.

In steady flight, we write

$$\mathbf{v} = \mathbf{V},\ \mathbf{\Omega} = 0,\ \mathbf{F} + m\mathbf{g} = 0,$$

and add small perturbations so that

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_1 + \mathbf{u}, \\ \mathbf{V}_1 &= U\hat{\mathbf{\imath}}, \\ \mathbf{u} &= u\hat{\mathbf{\imath}} + v\hat{\mathbf{\jmath}} + w\hat{\mathbf{k}}, \\ \boldsymbol{\omega} &= p\hat{\mathbf{\imath}} + q\hat{\mathbf{\jmath}} + r\hat{\mathbf{k}}. \end{aligned}$$

For a small rotation χ,

$$\chi = \phi\hat{\mathbf{i}} + \theta\hat{\mathbf{j}} + \psi\hat{\mathbf{k}},$$
$$\boldsymbol{\omega} = \dot{\phi}\hat{\mathbf{i}} + \dot{\theta}\hat{\mathbf{j}} + \dot{\psi}\hat{\mathbf{k}},$$

and likewise, the perturbation forces are

$$\mathbf{F} + \delta\mathbf{F},\ m(\mathbf{g} + \delta\mathbf{g}),$$

so it can be shown that

$$\delta\mathbf{g} + \chi \times \mathbf{g} = 0.$$

Inserting these assumptions into (9.10) gives the equations of motion for small perturbations,

$$m\dot{\mathbf{u}} + m(\dot{\chi} \times \mathbf{V}_1 + \chi \times \mathbf{g}) = \delta\mathbf{F}, \tag{9.11a}$$
$$\dot{\mathbf{h}} = \delta\mathbf{L}. \tag{9.11b}$$

Some reasonable assumptions will now help us to make the system tractable. First, we can assume that forces and moments depend on velocities but not on accelerations, except for M, which has a dependence on $\dot{w}$, so

$$\delta\mathbf{F} = \delta X\hat{\mathbf{i}} + \delta Y\hat{\mathbf{j}} + \delta Z\hat{\mathbf{k}},$$
$$\delta\mathbf{L} = \delta L\hat{\mathbf{i}} + \delta M\hat{\mathbf{j}} + \delta N\hat{\mathbf{k}},$$

and, for example,

$$\delta X = \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial v}v + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial p}p + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial r}r,$$
$$\delta M = \frac{\partial M}{\partial u}u + \frac{\partial M}{\partial v}v + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial \dot{w}}\dot{w} + \frac{\partial M}{\partial p}p + \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial r}r.$$

We also assume that the aircraft is laterally symmetric so that symmetric perturbations cause symmetric responses: a pitch disturbance cannot cause yaw or roll. Furthermore, the symmetric response to an asymmetric input is symmetric: a given roll rate has the same response in pitch whether the roll rate is negative or positive. Taking these assumptions together,

$$\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial w} = \frac{\partial Y}{\partial q} = \frac{\partial L}{\partial u} = \frac{\partial L}{\partial w} = \frac{\partial L}{\partial q} = \frac{\partial N}{\partial u} = \frac{\partial N}{\partial w} = \frac{\partial N}{\partial q} \equiv 0,$$
$$\frac{\partial X}{\partial p} = \frac{\partial X}{\partial r} = \frac{\partial X}{\partial v} = \frac{\partial Z}{\partial p} = \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial v} = \frac{\partial M}{\partial p} = \frac{\partial M}{\partial r} = \frac{\partial M}{\partial v} \equiv 0.$$

Eliminating zero terms,

$$\delta\mathbf{F} = \left(\frac{\partial X}{\partial u}u + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial q}\dot{\theta}\right)\hat{\mathbf{i}} + \left(\frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial p}\dot{\phi} + \frac{\partial Y}{\partial r}\dot{\psi}\right)\hat{\mathbf{j}}$$
$$+ \left(\frac{\partial Z}{\partial u}u + \frac{\partial Z}{\partial w}w + \frac{\partial Z}{\partial q}\dot{\theta}\right)\hat{\mathbf{k}},$$
$$\delta\mathbf{L} = \left(\frac{\partial L}{\partial p}\dot{\phi} + \frac{\partial L}{\partial r}\dot{\psi} + \frac{\partial L}{\partial v}v\right)\hat{\mathbf{i}} + \left(\frac{\partial M}{\partial q}\dot{\theta} + \frac{\partial M}{\partial u}u + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial \dot{w}}\dot{w}\right)\hat{\mathbf{j}}$$
$$+ \left(\frac{\partial N}{\partial p}\dot{\phi} + \frac{\partial N}{\partial r}\dot{\psi} + \frac{\partial N}{\partial v}v\right)\hat{\mathbf{k}}.$$

With the further assumption that there is no inertial coupling between yaw and roll, we find that the only moments of inertia we need consider are A, B and C.

Perturbing from horizontal flight and expanding the products in (9.11) gives

$$m\dot{u} = \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial q}q - mg\theta, \tag{9.12a}$$

$$m(\dot{w} - Uq) = \frac{\partial Z}{\partial u}u + \frac{\partial Z}{\partial w}w + \frac{\partial Z}{\partial q}, \tag{9.12b}$$

$$B\dot{q} = \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial u}u + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial \dot{w}}\dot{w}. \tag{9.12c}$$

and

$$m(\dot{v} + Ur) = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial p}p + \frac{\partial Y}{\partial r}r + mg\phi, \tag{9.13a}$$

$$A\dot{p} = \frac{\partial L}{\partial p}p + \frac{\partial L}{\partial r}r + \frac{\partial L}{\partial v}v, \tag{9.13b}$$

$$C\dot{r} = \frac{\partial N}{\partial p}p + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial v}v. \tag{9.13c}$$

The first of these systems of equations covers symmetric motion, e.g. pitch oscillations, while the second covers lateral motion, such as yaw and roll. An important point to note is that these equations are *uncoupled* so that longitudinal motion does not affect lateral and *vice versa*.

10
How aircraft wobble: normal modes

Given the equations of motion for an aircraft, we would like to extract some solutions which characterize the dynamic behaviour. In any dynamic system, these solutions are the normal modes and arise from an eigenvalue analysis of the equations of motion. Since longitudinal and lateral motion are uncoupled, we can treat them separately as two three-degree-of-freedom systems, which is rather simpler than dealing with the full six-degree-of-freedom problem.[1]

[1] The following analysis, with different notation, is given in GRAHAM, W., 'Asymptotic analysis of the classical aircraft stability equations', *Aeronautical Journal*, February 1999, pp 95–103.

10.1 Longitudinal symmetric motion

Normal modes for the multi-degree of freedom system are found as a natural frequency and a set of amplitudes for the motion. We begin in the usual manner by inserting assumed forms for the solution of (9.12),

$$u = u_0 e^{\lambda t},\ v = v_0 e^{\lambda t},\ \theta = \theta_0 e^{\lambda t},$$

so that, for example,

$$m u_0 \lambda e^{\lambda t} = \frac{\partial X}{\partial u} u_0 e^{\lambda t} + \frac{\partial X}{\partial w} w_0 e^{\lambda t} + \frac{\partial X}{\partial q}\theta_0 \lambda e^{\lambda t} - mg\theta_0 e^{\lambda t}.$$

Non-dimensionalizing,

$$(\Lambda - x_u)u' - x_w w' - \left(\frac{x_q \Lambda}{\mu_c} - \frac{C_L}{2}\right)\theta_0 = 0, \quad (10.1a)$$

$$-z_u u' + (\Lambda - z_w)w' - \Lambda\left(1 + \frac{z_q}{\mu_c}\right)\theta_0 = 0, \quad (10.1b)$$

$$-\left(\frac{m_{\dot{w}}}{\Lambda}\mu_c + m_w\right)w' + \frac{\Lambda(b\Lambda - m_q)}{\mu_c}\theta_0 = 0. \quad (10.1c)$$

where the non-dimensional parameters are given on the data sheet and primes denote velocities scaled on U, $u' = u_0/U$, $w' = w_0/U$.

The first solution we consider is a low frequency oscillation. We state without proof that there is a solution with Λ and u'/θ of order one and w'/θ_0 of order $1/\mu_c$. This means that the vertical motion is negligible or, equivalently, the incidence is almost constant.[2] We can rewrite (10.1) in matrix form, with the negligible terms in each

[2] What does this imply about the aircraft attitude or inclination?

equation removed:

$$\begin{bmatrix} \Lambda - x_u & 0 & C_L/2 \\ -z_u & 0 & -\Lambda \\ 0 & -m_w & \Lambda(b\Lambda - m_q)/\mu_c \end{bmatrix} \begin{bmatrix} u' \\ w' \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

This equation can only have a non-trivial solution if the determinant of the matrix is zero,

$$\Lambda^2 - x_u\Lambda + \frac{-z_u}{2}C_L = 0,$$

which gives

$$\Lambda = -\frac{(-x_u)}{2} \pm j\Omega_{\text{ph}} \left[1 - \left(\frac{-x_u}{2\Omega_{\text{ph}}}\right)^2\right]^{1/2}.$$

This is a solution for oscillatory motion (note the imaginary part in Λ) with

$$\Omega_{\text{ph}} = \left[\frac{-z_u C_L}{2}\right]^{1/2}, \quad \text{natural frequency,} \tag{10.2a}$$

$$c_{\text{ph}} = \frac{-x_u}{2\Omega_{\text{ph}}}, \quad \text{damping.} \tag{10.2b}$$

This is the *phugoid* mode, a lightly damped long period oscillation, which we examined in simplified form in Chapter 9. The incidence is almost constant and the aircraft varies altitude at constant energy, trading potential for kinetic energy and back again, Figure 10.1.

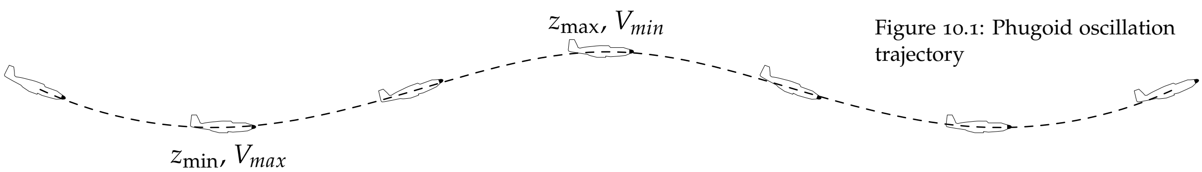


Figure 10.1: Phugoid oscillation trajectory

An important point to note is that the damping is proportional to $(-x_u)$, the rate of change of horizontal force with horizontal speed, which depends largely on drag.[3] Since drag acts in the opposite direction to velocity, $x_u < 0$ and the damping is positive, stabilizing the motion.

[3] What else might it depend on?

The second solution for longitudinal oscillation is for the case where Λ is of order $\mu_c^{1/2}$, u_0/θ_0 is of order $\mu_c^{-1/2}$ and w_0/θ_0 is of order one. In this case, the approximation to (10.1) is

$$\begin{bmatrix} \Lambda - x_u & -x_w & -\left(\dfrac{x_q\Lambda}{\mu_c} - \dfrac{C_L}{2}\right) \\ 0 & \Lambda - z_w & -\Lambda \\ 0 & -\left(\dfrac{m_{\dot{w}}\Lambda}{\mu_c} + m_w\right) & \dfrac{\Lambda(b\Lambda - m_q)}{\mu_c} \end{bmatrix} \begin{bmatrix} u' \\ w' \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Again, we find the natural frequency by requiring that the determinant of the matrix be zero,

$$\Lambda(\Lambda - x_u)\left[\Lambda^2 - \left(z_w + \frac{m_q + m_{\dot{w}}}{b}\right)\Lambda + \frac{z_w m_q - m_w \mu_c}{b}\right] = 0,$$

which, on solving the quadratic, gives a result for the non-dimensional natural frequency and damping:

$$\Omega_{\text{spo}} = \left[\frac{\mu_c(-m_w) + m_q z_w}{b}\right]^{1/2}, \quad \text{natural frequency}, \tag{10.3a}$$

$$c_{\text{spo}} = -\frac{1}{2\Omega_{\text{spo}}}\left(z_w + \frac{m_q + m_{\dot{w}}}{b}\right), \quad \text{damping}. \tag{10.3b}$$

This is the *short period oscillation*, which you saw at the start of Chapter 9. It is a heavily damped mode with period typically of a few seconds. The aircraft pitches rapidly about its centre of gravity which continues to fly at almost constant speed in a straight line. The periodic time is typically a few seconds, but must not be less than about 1.25s, otherwise there is a risk of Pilot Induced Oscillation (PIO).[4]

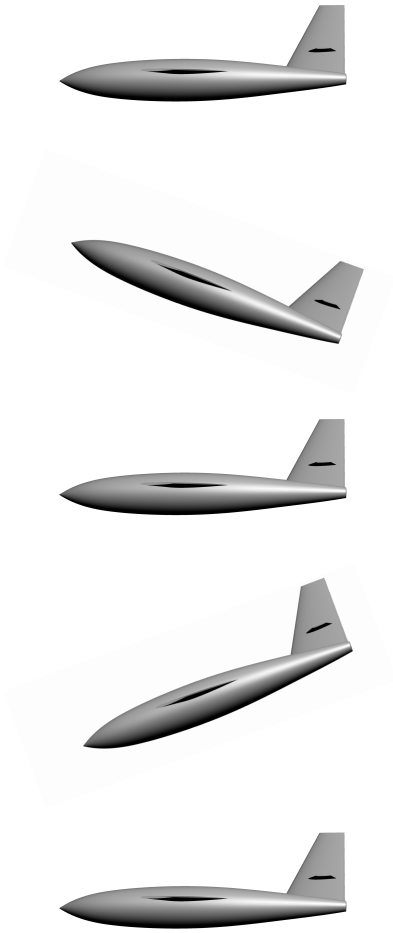


Figure 10.2: Short period oscillation

[4] If you are in the humour, you might like to try modelling Pilot Induced Oscillation.

The frequency is proportional to $K_n^{1/2}$, and increases with dynamic pressure, $\rho V^2/2$. Therefore the aircraft will have the highest frequency SPO, and hence the shortest time period, at high speed with the centre of gravity in the furthest forward position.[5] The SPO is always stable for a statically stable aircraft.

[5] Using this information, could you relate the static margin stick-fixed to the aerodynamic derivatives?

10.2 *Lateral motion*

In the case of lateral motion, we insert the assumed form for the solution

$$v = v_0 e^{\lambda t},\ \phi = \phi_0 e^{\lambda t},\ r = r_0 e^{\lambda t},$$

into (9.13), and non-dimensionalize, using wingspan s as our reference length,

$$(\Lambda - y_v)v' - \left(\frac{y_p\Lambda}{\mu_s} + \frac{C_L}{2}\right)\phi_0 + \left(1 - \frac{y_r}{\mu_s}\right)r' = 0, \tag{10.4a}$$

$$-l_v v' + (a\Lambda - l_p)\frac{\Lambda}{\mu_s}\phi_0 - \frac{l_r}{\mu_s}r' = 0, \tag{10.4b}$$

$$-n_v v' - \frac{n_p\Lambda}{\mu_s}\phi_0 + \frac{c\Lambda - n_r}{\mu_s}r' = 0. \tag{10.4c}$$

Again, the non-dimensional quantities are given on the data sheet.

The first lateral mode we consider is *Dutch roll* which has oscillations of roughly equal magnitude in pitch, yaw and roll. In this case, (10.4) reduces to

$$\begin{bmatrix} \Lambda & 0 & 1 \\ -l_v & a\Lambda^2/\mu_s & 0 \\ -n_v & 0 & c\Lambda/\mu_s \end{bmatrix} \begin{bmatrix} v' \\ \phi_0 \\ r' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

As before the determinant of the matrix must be zero for a non-trivial solution,

$$\Lambda^2(c\Lambda^2 + \mu_s n_v) = 0,$$

and the frequency of the oscillation is, on the approximations we are using,

$$\Omega_{\text{dr}} = \left(\frac{\mu_s n_v}{c}\right)^{1/2}. \tag{10.5}$$

In Dutch roll, yawing oscillation (analogous to the longitudinal SPO) causes alternating sideslip. This in turn causes a rolling oscillation via $L_v v$. The periodic time is typically a few seconds, but as for the SPO it should not have a period of less than 1.25s to avoid PIO.

Dutch roll is not permitted to be divergent. Divergent Dutch roll can be 'fixed' by a yaw damper on the rudder which damps the yawing oscillation, and hence the roll response as well.

There are two further solutions to the dynamic equations which have small values of Λ. These are dominated by yaw and roll with weak sideslip and the corresponding approximation to (10.4) is

$$\begin{bmatrix} 0 & C_L/2 & 1 \\ -l_v & (a\Lambda - l_p)\Lambda/\mu_s & -l_r/\mu_s \\ -n_v & -n_p\Lambda/\mu_s & (c\Lambda - n_r)/\mu_s \end{bmatrix} \begin{bmatrix} v' \\ \phi_0 \\ r' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The requirement for a non-trivial solution is then that

$$an_v\Lambda^2 + [l_v(n_p - cC_L/2) - l_p n_v]\Lambda + (l_v n_r - l_r n_v)C_L/2 = 0.$$

The two roots of this equation can be approximated as:

$$\Lambda_{\text{rs}} = -\frac{(-l_p)n_v + (-l_v)[cC_L/2 + (-n_p)}{an_v}, \tag{10.6}$$

and

$$\Lambda_{\text{sm}} = -\frac{C_L}{2}\frac{l_v n_r - l_r n_v}{(-l_p)n_v + (-l_v)[cC_L/2 + (-n_p)]}. \tag{10.7}$$

Note that both of these roots are real and so they do not describe oscillations. The first, Λ_{rs}, describes *rolling subsidence*, a pure rolling motion which is generally heavily damped, and is usually stable. The damping is primarily from the wings, where the incidence along the wing is changed by the roll-rate. This is experienced by the pilot as a lag in roll response. Roll control is not like pitch and

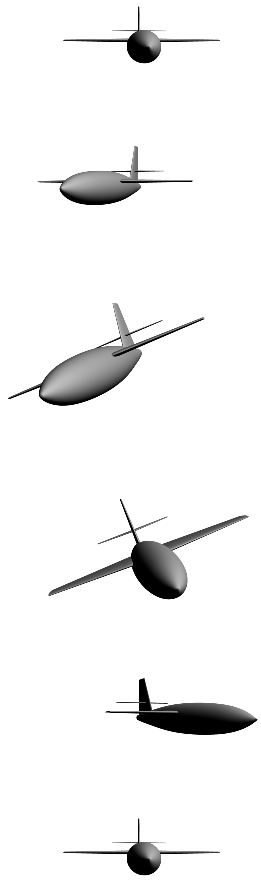


Figure 10.3: Motion of an aircraft undergoing Dutch roll

yaw control because the control input sets roll *rate* rather than roll angle. A lag in response means that the required roll rate is not reached immediately, and the pilot must change the control input slightly early to stop the aircraft rolling at the required roll angle.

This roll-rate results in a rolling moment $L_p p$. Therefore, if L_p is negative the rolling subsidence mode is stable. This is usually the case. However, if L_p becomes positive, usually due to non-linearities in the lift curve slopes at high roll rates, auto-rotational rolling can occur. This is what happens when an aircraft spins.

The second root Λ_{sm}, which is much smaller than Λ_{rs}, corresponds to the *spiral mode* of the aircraft. This is a combined yaw and roll motion which is allowed to be unstable (i.e. negatively damped) as long as it does not double amplitude in less than twenty seconds, so that it can be controlled out. The spiral mode normally happens so slowly that it can only be perceived visually or using instruments, but not by the pilot's inner ear, so that it can be fatal in reduced visibility when no visual reference is available for aircraft attitude.

The dynamics of the spiral mode are that if the aircraft rolls slightly, it will start to sideslip, and the fin then tries to turn the aircraft into the relative wind due to a yawing moment $N_v v$. However, the rolling moment due to sideslip $L_v v$ tries to roll the wings back level. Depending on which of the effects prevails, the aircraft will be spirally unstable or stable, as can be seen from the numerator of (10.7).

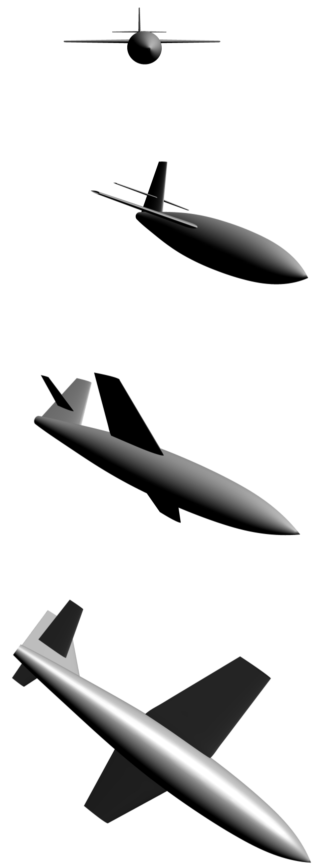


Figure 10.4: Spiral mode

10.3 Dihedral effect and weathercock stability

The aerodynamic derivatives L_v and N_v establish whether an aircraft is stable or unstable in rolling subsidence and Dutch roll. L_v and N_v are known as the 'dihedral effect' and 'weathercock stability' respectively. The effect of the two aerodynamic derivatives on the lateral stability of the aircraft is shown in Figure 10.5.

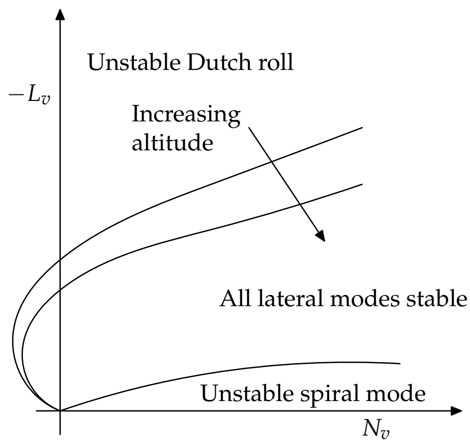


Figure 10.5: Stability of lateral modes

L_v is known as the dihedral effect since the majority of the rolling moment caused by sideslip comes from dihedral (on an aircraft with unswept wings), as shown in Figure 10.6. Positive dihedral combined with positive sideslip results in a negative rolling moment (and hence negative L_v).

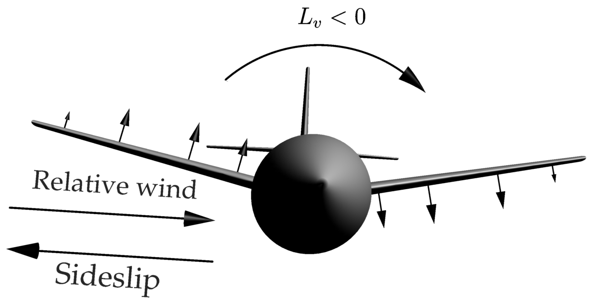


Figure 10.6: Dihedral effect

Wing sweep has a large, negative, effect on L_v because of reduced or increased effective sweep for positive sideslip. This is shown in Figure 10.7.

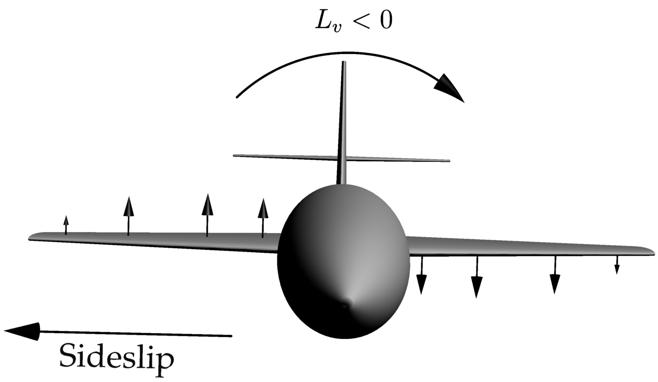


Figure 10.7: Wing sweep effects on L_v: reduced effective sweep in the direction of sideslip generates a higher lift

Wing–fuselage interference effects give contributions to L_v because sideslip changes the effective incidence near the wing root. These contributions are negative for high mounted wings and positive for low mounted wings, as shown in Figure 10.8.

A reasonable value of L_v may be achieved by using anhedral with swept and high mounted wings (e.g. Harrier). Ground clear-

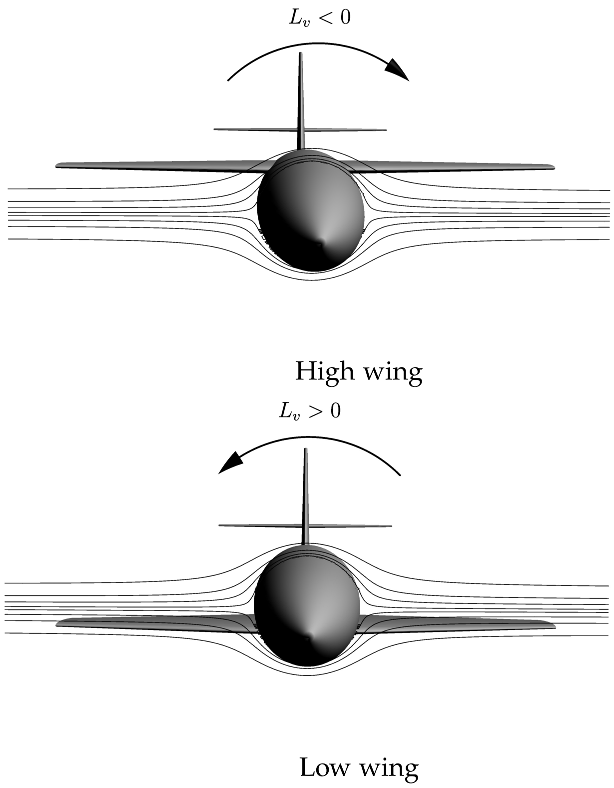


Figure 10.8: Wing-fuselage interference effects on L_v

ance issues may limit anhedral on low wing aircraft, resulting in an unstable Dutch roll mode.

The aerodynamic derivative N_v is known as weathercock stability since it is, effectively, the tendency of an aircraft to turn into the wind. It is produced mainly by the side force of the fin in sideslip, and should always be negative. However, as shown in Figure 10.5, if N_v is too large the aircraft may be spirally unstable.

11
Flying aeroplanes

These notes are mainly intended to introduce the ideas you need in order to design the mechanical elements of aeroplanes. As in many engineering problems, the requirements are a translation into numerical form of a set of human needs, in this case the requirement that a complex machine be controllable by a human being in order to carry out some set of functions. This human element of the design question is what makes the difference between adequate aircraft and great ones. The field is normally called "handling properties" or "flying qualities" and is the area where mechanical design, aerodynamics, physiology, psychology, and ergonomics intersect.

The idea that there is such a thing as flying qualities and that these qualities can be specified numerically is not an obvious one, and it is worth reading a history of how these qualities were first recognized and defined and then stated as ranges of numerical values.[1] In short, over a period of about twenty five years after the First World War, test pilots and research engineers working together developed an understanding of what it means to fly an aeroplane in terms which allow for the discussion of the qualities of the aircraft, so that it becomes possible to properly design to make the aircraft useable by a human being. By 1949, one textbook was dealing with "the comparatively new art of designing the airplane [sic] for adequate flying qualities": the existence of flying qualities had been recognized and engineers were being taught to design for them, rather than hoping the aircraft's first pilot survived long enough to report on the aircraft's handling.

[1] Walter G. Vincenti. Establishment of design requirements: Flying-quality specifications for American aircraft, 1918–1943. In *What engineers know and how they know it: Analytical studies from aeronautical history*. Johns Hopkins, Baltimore, 1990

In Vincenti's words:

> Flying qualities comprise those qualities or characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the task of controlling the vehicle. Flying qualities are thus a property of the aircraft, though their identification depends on the perceptions of the pilot.

Vincenti gives examples of aircraft from the mid-thirties which were considered quite adequate at the time, but would not now be thought flyable such as the Martin M-130 flying boat which was commercially successful but whose range was limited by the length of time the pilots could withstand the control forces, and the P-35, in service with the US Army Air Corps, but which Charles

Lindbergh found too sensitive to be easily flown.

The problem was stated by an officer at the US Navy's Bureau of Aeronautics quoted by Vincenti:

> At present we simply specify that the airplane [sic] shall be perfect in all respects and leave it up to the contractor to guess what we really want in terms of degree of stability, controllability, maneuverability [sic], control forces, etc. He [sic] does the best he [sic] can and then starts building new tails, ailerons, etc. until we say we are satisfied.

The development of flying handling qualities definitions and specifications resulted in the numerical requirements which are stated in regulations and specifications, for example, the stick force requirements of Table 5.1. For an aircraft to be flyable, there are also requirements on stick force gradient (modified by weights, page 45), linearity of response, and dynamic response, so that an aircraft shifts predictably from one state of flight to another and behaves predictably in any phase of operation.

11.1 The Cooper–Harper scale

The piloting qualities of an aircraft relate to the human experience of an aircraft's response to inputs. In order to design an aircraft, we need some way of specifying which properties of the aircraft form part of the flying qualities and of specifying an acceptable range for those properties so that the handling of the aircraft can be assessed at the design stage. In these notes, we have looked at such numerical parameters as stick deflection and stick force per g, and at how we might design for particular values. These are examples of the general approach developed by Robert Gilruth of NACA. In the words of two historians of aircraft stability,[2]

> Gilruth's seminal achievement was to rationalize flying qualities by separating airplanes into satisfactory and unsatisfactory categories for some characteristic, such as lateral control power, by pilot opinion. He then identified some numerical parameter that could make the separation. That is, for parameter values above some number, all aircraft were satisfactory, and vice versa. The final step was to develop simplified methods to evaluate this criterion parameter, methods that could be applied in preliminary design.

[2] Malcolm J. Abzug and E. Eugene Larrabee. *Airplane stability and control: A history of the technologies that made aviation possible*. Cambridge University Press, Cambridge, 2002

Thus by 1943 there were numerical criteria relating the design of an aeroplane to the qualities required by pilots, and methods for applying these criteria at the design stage.

The other side of flying qualities specification is the requirement to measure, in some sense, the handling of the aircraft from the point of view of a pilot. The standard method for this is the Cooper–Harper scale, developed at NASA in the 1960s.[3]

[3] George E. Cooper and Robert P. Harper, Jr. The use of pilot rating in the evaluation of airplane handling qualities. Technical Report TN D-5153, NASA, 1969

Aircraft characteristic	Pilot demands	Rating
Excellent Highly desirable	Pilot compensation not a factor for desired performance	1
Good Negligible deficiencies	Pilot compensation not a factor for desired performance	2
Fair—some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5
Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	6
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation; controllability not in question	7
Major deficiencies	Considerable pilot compensation is required for control	8
Major deficiencies	Intense pilot compensation is required to retain control	9
Major deficiencies	Control will be lost during some portion of required operation	10

Figure 11.1: The Cooper–Harper scale for handling qualities

The scale, Figure 11.1, assigns numerical scores to the aircraft in particular phases of flight so that a test pilot can report the adequacy of the aircraft for a given task in terms of the workload which it imposes on a pilot. The scale can then be used in specifications and regulations, and can also be mapped to contours of frequency and damping of the aircraft modes to link numerical data, which can be estimated at the design stage, to pilot perception. An early example of this is the "bullseye" or "thumbprint" plot of iso-opinion contours, shown in Figure 11.2, which has been replotted from the published original.[4]

[4] Malcolm J. Abzug and E. Eugene Larrabee. *Airplane stability and control: A history of the technologies that made aviation possible.* Cambridge University Press, Cambridge, 2002

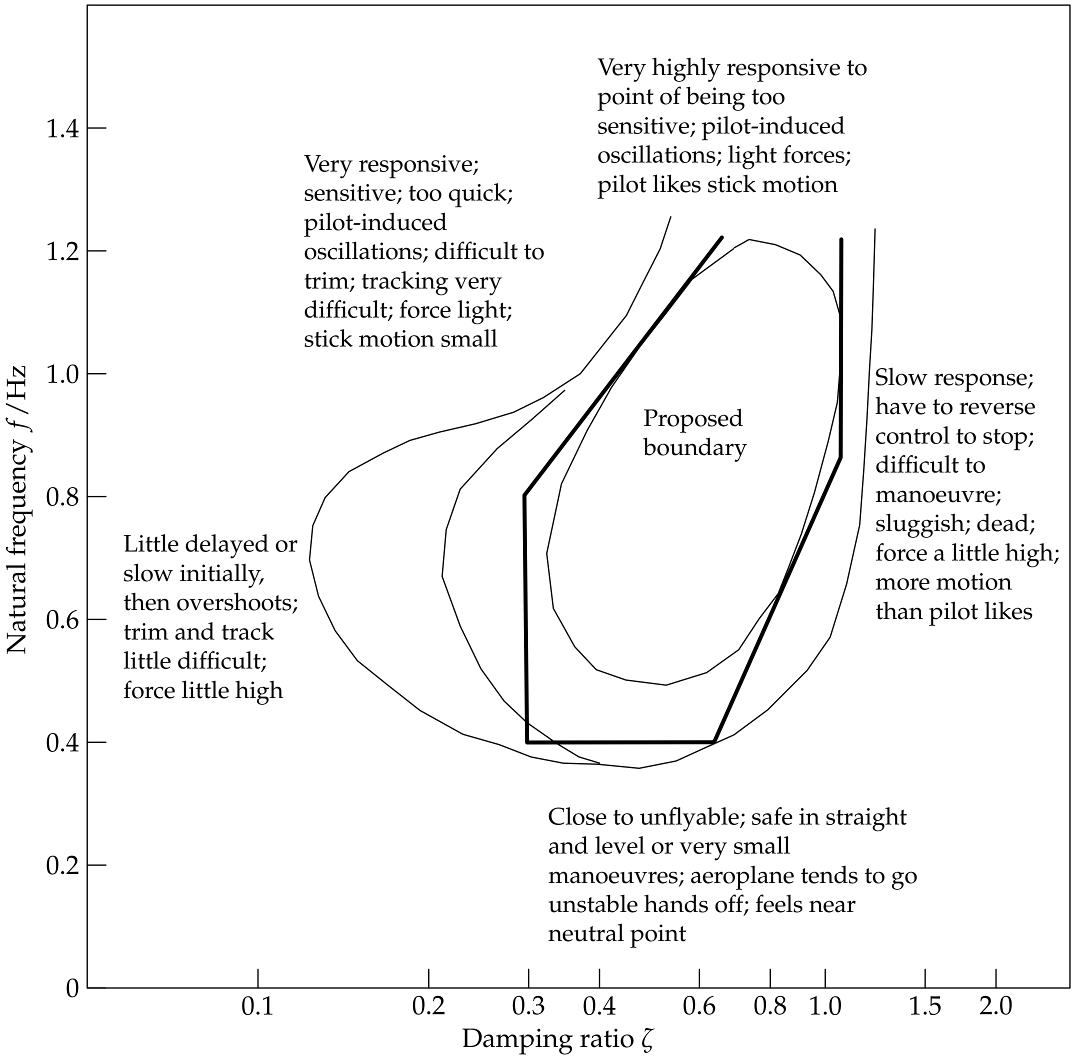


Figure 11.2: Iso-opinion contours for the short-period oscillation from tests on a variable stability F-94F, replotted from data in Abzug and Larabbee

The curves on the plot are contours of "constant opinion" (of flying qualities) as rated by pilots on a variable stability F-94F. By varying the stability of the aircraft, the contours could be plotted against the properties (natural frequency and damping ratio) of the short period oscillation. This gives designers an indication of a range of properties, within the heavy boundary, which give good handling for a particular type of aircraft, allowing them to account for flying properties at the design stage, and to see how changing those properties will affect the pilot's perception of the aircraft's qualities.

Modern approaches are more sophisticated and incorporate models of human psychology and physiology but still work on the principle of linking predicted dynamic properties of an aircraft to a human assessment of the more intangible qualities of the design. In design, much of the assessment of flying qualities is now carried out by having pilots "fly" simulations of the aircraft so that the handling properties can be tuned before moving into production.

12
Questions

These questions are intended to give you practice in applying the methods and theory of the text, and to encourage you to think about how what you are learning fits into the general context of aircraft design, and engineering more generally. You should work through the questions in this section in order, but you can take the questions in §12.1 in any order you please, depending on when you get around to watching the films recommended.

1. For the two situations shown in Figure Q1 calculate the values of L_W and L_T required to give both a total lift equal to the aircraft weight and give zero net moment about the aircraft c.g.

   [$L_W =$ 99.3kN, $L_T = 0.7$kN, $L_W =$ 95.3kN, $L_T = 4.7$kN]

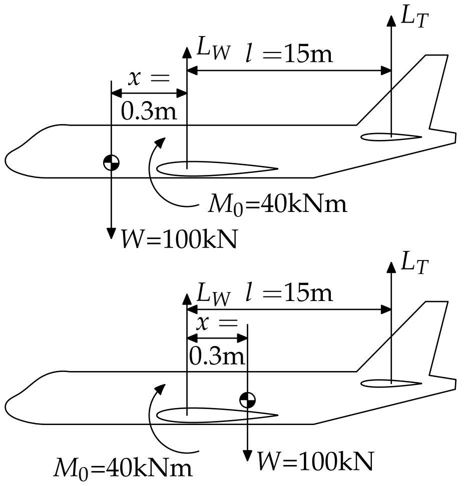


Figure Q1: Aircraft with different centres of gravity

2. Draw the system of forces and moments acting on a conventional aeroplane in steady straight and level flight.

   Show that the pitching moment about the centre of gravity is given by

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T}.$$

   For the sailplane whose details are given below, calculate the value of C_{L_T} required for trim at 50kt EAS with a pilot weighing 0.75kN. The empty weight equipped is 2.5kN, with c.g. on the mean chord $0.45\bar{c}$ aft of the leading edge of $\bar{c}$. The pilot c.g. is assumed to be 0.8m ahead of the leading edge of $\bar{c}$.

   [$C_{L_T} = -0.552$]

   $S = 28\text{m}^2$ $\quad S_T = 1.4\text{m}^2$ $\quad \bar{c} = 1.15\text{m}$
   $l = 5.35\text{m}$ $\quad h_0 = 0.25$ $\quad C_{M_0} = -0.11$

3. Distinguish between stability and trim. Show that for an aircraft to be both stable and able to trim at positive lift coefficient the overall pitching moment about the centre of gravity must be positive at $C_L = 0$ in that configuration.

4. From first principles, show that the portion of the total lift coefficient (C_L) provided by the wing-body-nacelle (WBN) group of a conventional aircraft is given by:

$$C_{L_{WBN}} = C_L\left[1 + (h_0 - h)\frac{\bar{c}}{l}\right] - C_{M_0}\frac{\bar{c}}{l}.$$

   If the aircraft stalls when $C_{L_{WBN}}$ reaches its maximum value, $(C_{L_{WBN}})_{\max}$ say, then obtain an expression relating the stalling speed to the c.g. position at any one given weight.

   Hence calculate the c.g. shift that would increase the stalling speed by 1% if $\bar{c} = 5.6$m, $l = 15.5$m and $(h_0 - h) = 0.05$.

   [$\Delta h = -0.0566$, $\Delta h\bar{c} = -0.317$m]

5. Consider the two situations shown in Figure Q5.

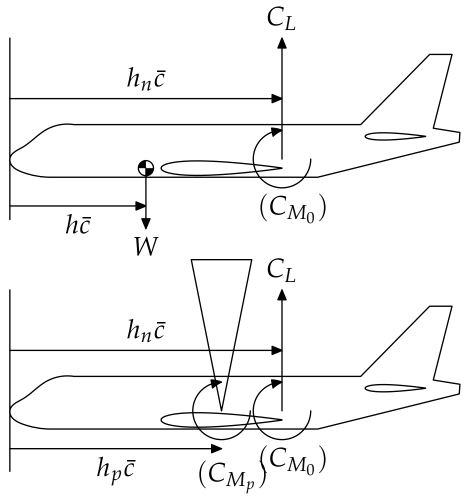


Figure Q5: Full-scale and model aircraft. C_{M_p} is measured by the balance, which restrains the model in pitch.

In (a) the full scale aircraft is in steady free flight with values C_L, h, η for the lift coefficient, c.g. position and elevator angle respectively.

In (b), a model of the same aircraft is suspended from a wind tunnel balance at the same C_L and elevator setting as in (a). The balance measurement gives the pitching moment coefficient C_{MP} about the balance pivot axis which is located at h_p with respect to the same datum line as h.

(a) Write down the moment equations for situations (a) and (b), and hence derive the relationship between the balance reading C_{MP}, equivalent to the steady free flight conditions, and interrelating h_p, h and C_L.

(b) An aircraft model is found to have a zero-lift pitching moment coefficient of 0.027 for a particular elevator angle. The pitching moment is measured about the wind-tunnel axis of rotation P and has a slope:

$$\frac{\mathrm{d}C_{Mp}}{\mathrm{d}\alpha} = 0.15; \quad \text{lift curve slope } a = \ 5.851.$$

Determine the position of the c.g. of the full-scale aircraft relative to P if a stick-fixed margin of 0.11 is required ($\bar{c} = 3.96$m).
If the wing loading is 2.25kN/m^2 in steady level flight with the above elevator angle, what is the airspeed if the air density is 1.030kg/m^3.

[0.537m forward of P, 133.3m/s TAS.]

6. The data shown below apply to an aircraft in steady level flight at 200kt EAS. Calculate the elevator angle required for longitudinal trim. Also obtain the stick-fixed neutral point and static margin.

W = 30kN	S = 23m^2	S_T = 3.5m^2
$\bar{c}$ = 1.96m	l = 5.5m	
h_0 = 0.25	c.g. is 0.61m aft of datum	
C_{M_0} = -0.036	η_T = -1.5°	ϵ = 0.48α
a = 4.58	a_1 = 3.15	a_2 = 1.55

[$\bar{\eta}$ = -1.658°, h_n=0.4027, K_n = 0.0915]

7. The centre of gravity range for an aircraft is found by considering that the

(a) aft c.g. limit (h_{aft}) is determined by the minimum stability condition (minimum K_n);

(b) forward c.g. limit (h_{fwd}) is determined by the maximum elevator angle to trim (while retaining enough elevator movement for manoeuvre).

By considering the static forces and moments on an aircraft in symmetric flight, find an expression for the static margin stick-fixed, K_n, and show that:

$$K_n = -\overline{V}a_2\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right).$$

An aircraft has the following values of the aerodynamic coefficients:

$$h_0\text{=0.25, } a\text{=3.5, } a_1\text{=3.0, } a_2\text{=1.5, } \partial\epsilon/\partial\alpha\text{=0.4.}$$

Find the relationship between the c.g. position and the tail volume ratio:

(a) for a static margin of 0.05 (h_{aft});

(b) for the change in elevator angle to trim to be 10° for a change in C_L of 1.0 (h_{fwd}).

Hence find the minimum tail volume ratio such that with a c.g. shift of 0.15$\bar{c}$ the change in elevator angle to trim is not more than 10° for a change of C_L of 1.0 and the static margin is never less than 0.05.

[$\overline{V} = 0.764$]

8. A transport aircraft with conventional tail is to have zero elevator angle in cruising flight at 560km/h EAS (mass 100,000kg), with the c.g. in the mid position. The landing approach, out of ground effect, is made with flaps down at 210km/h (mass 90,000kg), and the maximum

elevator movement permitted for trimming is $\overline{\eta} = \pm 10°$. Using the data below, calculate the minimum tailplane area suitable for this aircraft, and the tailplane setting η_T relative to the flaps-up wing zero lift line.

Minimum $K_n = 0.05$	c.g. range $\Delta h = 0.50$
$h_0 = 0.075$	$S = 232\text{m}^2$
$\overline{c} = 4.72\text{m}$	$l = 19.5\text{m}$
$a = 5.7$	$a_1 = 2.7$
$a_2 = 2.1$	
$C_{M_0} = -0.14$	$\epsilon = 0.16\alpha$.

The change in C_{M_0} at landing flap setting $\Delta C_{M_0} = -0.10$. Note that the wing zero lift incidence angle changes by 10° when the flaps are lowered to the landing setting.

[$S_T = 68.5\text{m}^2$, $\eta_T = -3.92°$]

9. The static margin, stick-fixed may be obtained in practice from flight tests in which elevator angles to trim are found at certain speeds. In practice, the aeroplane is trimmed at a series of speeds by adjusting the tab setting, and both the elevator angle and tab angle are observed. Since the theory which relates the stick-fixed static margin to the elevator angles to trim implicitly assumes a constant tab angle, show that a correction must be applied to elevator angles obtained in this way such that

$$\overline{\eta}_{\text{corrected}} = \overline{\eta} + \frac{a_3}{a_2}\overline{\beta}$$

where $\overline{\eta}$ and $\overline{\beta}$ are the observed elevator and tab angles to trim at a given speed. Suggest how you would determine a_3/a_2 in flight.

10. A tailless aircraft is controlled in pitch by six elevons. Each elevon is actuated by an independent power control unit. These units are so designed that if a failure occurs the affected elevon is able to move until its hinge moment is zero.

Assuming the failure of one such unit, calculate the elevon angles that will give longitudinal trim of the aircraft whose details are given below:

Weight =	850kN	Speed =	70m/s EAS
Wing area S =	358m^2	$(h_0 - h)$ =	0.15
C_{M_0}=	+0.02	$\partial C_{M_0}/\partial\eta$ =	-0.45
a_1 =	4.0	a_2 =	0.95
b_1 =	-0.7	b_2 =	-1.05

Assume that each elevon contributes equally to a_2 and to $\partial C_{M_0}/\partial\eta$.

[$\eta_{\text{failed}} = -9.61°$, $\eta_{\text{operating}} = -13.15°$]

11. A conventional aircraft flying at low speed has a flexible rear fuselage such that the tailplane setting relative to the wing zero lift line is directly proportional to the tail load. Prove that the reduction in stick fixed static margin compared with that of the rigid aircraft is given by:

$$\Delta K_n = K_n^{\text{rigid}} - K_n^{\text{flexible}},$$
$$= \overline{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[1 - \frac{1}{1 + \frac{1}{2}\rho V^2 S_T a_1 f}\right]$$

For the human-powered aircraft having the characteristics given below, find the fuselage flexibility f (degrees deflection per Newton) that reduces the stick-fixed static margin by 0.05 compared to the rigid case when flying at a speed of 9.2m/s at sea level.

$S = 28\text{m}^2$	$S_T = 1.4\text{m}^2$	$l = 5.34\text{m}$	$\overline{c} = 1.14\text{m}$
$a = 6.0$	$a_1 = 4.5$	$\epsilon = 0.20\alpha$.	

[$f = 0.1°/\text{N}$]

12. The control column of a low-speed aeroplane is connected to the elevator by an arrangement of cables which stretches when a stick force is applied. The stiffness of the circuit is given by $\text{d}H_E/\text{d}\eta = E\text{Nm/rad}$ where H_E is the hinge moment and η is the elevator deflection, the stick being held fixed.

Show that the stick-fixed c.g. margin (as opposed to the "elevator fixed" c.g. margin) is given by:

$$K_n = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[1 - \frac{a_2 b_1}{a_1 b_2}\frac{1}{1-\lambda}\right]$$

where

$$\lambda = \frac{C_L S E}{b_2 S_\eta \overline{c}_\eta W}.$$

It should be assumed that the aircraft is initially in trim with the tab angle adjusted to give zero stick force.

Show how this margin is related to the stick fixed and stick free c.g. margins of a rigid aeroplane. What practical use might you make of this information?

13. Which conditions define the stick-fixed and stick-free manoeuvre points of an aircraft?

From first principles, stating your assumptions, derive an expression for the stick fixed maneouvre point of a low speed aircraft of canard layout. Show whether this is forward or aft of the corresponding neutral point and compare your expression with that for a conventional aircraft.

14. Define the maneouvre point stick-free for a conventional aircraft. How does it differ from the corresponding neutral point?

Find the minimum stick force per g at sea level for the light aircraft whose details are given below. Comment on your result and find the c.g. position required to give 22N/g. Suggest alternative means for increasing the existing value.

$W = 2.7$kN	$S = 7.6\text{m}^2$	$l = 2.9$m
$h_0 = 0.238$	$\bar{c} = 1.2$m	$\overline{V} = 0.34$
$\epsilon = 0.385\alpha$	$a = 3.865$	$a_1 = 2.73$
$a_2 = 2.16$	$b_1 = -0.282$	$b_2 = -0.536$

The permitted c.g. range is from $0.22\bar{c}$ to $0.28\bar{c}$. The stick force per g is given by

$$Q = \frac{P_e}{n} = -m_e S_\eta \bar{c}_\eta \frac{W}{S}\frac{b_2}{a_2\overline{V}} H'_m,$$
$$= 83.2 H'_m \text{N/g} \quad \text{for this aircraft.}$$

[$Q = 5.8$N/g, for $Q = 22$N/g, $h = 0.0853$]

15. The table below shows data for a tailless aircraft. When it performs a steady pullout at $A_N = 2.5$ ($n = 1.5$) at 250kt EAS at a height where the air density $\rho = 1.150\text{kg/m}^3$, the change of elevator setting compared with steady level flight under the same conditions is 3.20°.

Calculate m_q if the static margin is known to be 0.05.

$W = 160$kN	$S = 50\text{m}^2$	$c_0 = 10$m
$\partial C_{M_0}/\partial\eta = -0.5$	$K_n = 0.05$	

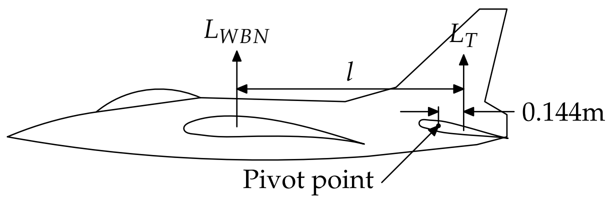
[$m_q = -0.264$]

16. An aircraft of conventional layout is controlled in pitch by an all-moving tailplane, having no separate elevators (see Figure and table). Show that the tail angle per g is given by

$$\frac{\Delta\eta_T}{n} = -\frac{C_L H_m}{\overline{V} a_1}$$

where the symbols have their usual meanings. Hence calculate the tail angle, tail load and pivot moment when the aircraft is flying at 440kt EAS in an 8g ($n = 7$) pullout at a height where the relative density of the air $\sigma = 0.74$. Comment on your results.

W=175kN	V=440kt EAS	S=33.2m^2	S_T=19.1m^2
l=5.25m	$\bar{c} = 2.39$m	a=3.8	a_1=2.7
$C_{M_0} = +0.03$	$\partial\epsilon/\partial\alpha$=0.38	$h_0 = 0.17$	$h = 0.50$
$\sigma = 0.74$			



[$\Delta\eta_T = -4.72°$, $\eta_T = -4.85°$, $L_T = 224.6$kN, $M_P = -32.34$kNm, $C_{L_T} = 0.3739$]

17. For a conventional aircraft show that if the tab setting remains unaltered, the change of elevator hinge moment coefficient-to-trim $\Delta\overline{C_H}$ between two lift coefficients is given by

$$\Delta\overline{C_H} = -\frac{b_2}{a_2\overline{V}}\Delta C_L K'_n.$$

The aircraft described in the table below is making a zero stick force trimmed landing approach at 155kt EAS. Calculate the value to which the speed may be reduced while keeping the stick force within 150N without altering the trim tab setting, indicating clearly whether this is push or a pull force.

	W =785kN		h = 0.26		
	S = 223m^2		$\bar{c}$ = 5.68m		
	S_T = 46.5m^2		l = 15.66m		
	S_η = 11.2m^2		$\bar{c}_\eta$ = 0.908m		
h_0 = 0.16		C_{M_0} = -0.06		ϵ = 0.38α	
a = 4.5		a_1 = 2.75		a_2 = 1.16	
b_0 = 0		b_1 = -0.133		b_2 = -0.16	

The stick–elevator gearing ratio $m_e = 1.0$m/rad.

[118 kt, pull force]

18. Using the approach of §4.2, and the results of §5.1, derive a formula for $\mathrm{d}\overline{P_e}/\mathrm{d}V$, the gradient of stick force with flight speed. What does this tell you about the handling qualities of an aircraft?

19. What are stick-fixed and stick-free manoeuvre points and what is the significance of stick force per g.

    Using the data of question 17, calculate the change of elevator angle required to pull 0.5g flying at 350kt EAS at an altitude where the relative density $\sigma = 0.374$.

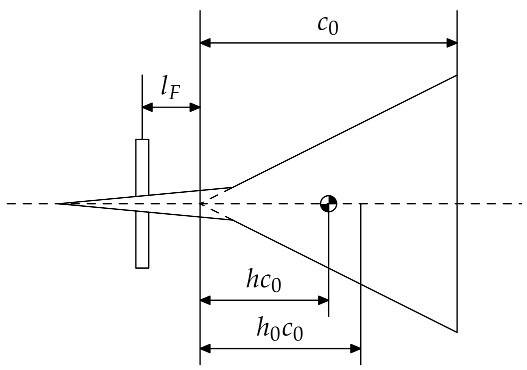
    Explain in physical terms why this change of elevator angle would be greater at a lower altitude when flying at the same lift coefficient.

    $[\Delta\overline{\eta} = -1.005°]$

20. The tailless aircraft shown in the figure has been fitted with a small retractable foreplane. At low speeds this foreplane is extended and, operating in a semi-stalled condition at constant setting, it generates a constant lift coefficient $C_{LF} = 1.2$ (based on S_F). Use of the foreplane enables the aircraft to take off at a higher weight than the original aircraft without the foreplane. Calculate the increment in take-off weight that may be achieved when using the foreplane, by considering the trimmed lift at 200kt EAS, if the incidence is restricted to 12° by ground clearance problems, using the data in the table.

    Calculate the elevon angles to trim of both versions of the aircraft. Comment on your results.

    [With foreplane: $\overline{\eta} = -0.5°$; $L = 1842$kN; without foreplane: $\overline{\eta} = -5.8°$; $L = 1557$kN]



S = 438m^2	S_F = 9.4m^2	C_{M_0} = +0.002
$\partial C_{M_0}/\partial\eta$ = -0.25	a_1 = 3.0	a_2 = 0.80
h_0 = 0.61	c_0 =27.4m	l_F = 13.26m
hc_0 = 15.34m		

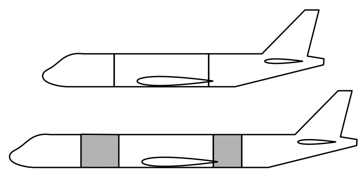
21. The aircraft described in the Figure and table below is to have its capacity increased by lengthening the cylindrical portion of the fuselage by 6m. The centre section (including the wings), the nose and the tail portions are to remain unaltered.

    It is assumed that the c.g. position will be adjusted to remain unchanged with respect to the centre section unit and that, for the lengths considered, $\partial\epsilon/\partial\alpha$ is constant.

    Calculate how the additional fuselage length is to be inserted ahead of and behind the centre section, if the low speed stick-fixed static margin is to be unaltered. The movement of the aerodynamic centre of the aircraft less tail is assumed to be affected only by the change of nose length Δl_N and is given by

$$\Delta h_0 = -0.037\frac{\Delta l_N}{\overline{c}}.$$

    If a variant of the aircraft retains the original fuselage, but has its wing tips extended, how could the longitudinal static stability be affected?



$S = 223$m^2	$\overline{c} = 5.6$m
$S_T = 46$m^2	l=15.5m
$h_0 = 0.25$	$h = 0.20$
$\epsilon = 0.4\alpha$	C_{M_0}=-0.06
$a = 4.5$	$a_1 = 2.75$

[4.1m ahead of wing, 1.9m aft]

22. (a) The 1903 Wright Flyer was a canard configuration of conventional layout, summarized in the table below. Calculate the stick-fixed neutral point, assuming that the wing and canard have approximately equal lift curve slope, and comment on your answer.

    (b) The 1903 Flyer was stabilized in pitch by the addition of ballast to shift the centre of gravity forward. If 30% of the aircraft gross weight can be carried as ballast, where should it be placed to move the centre of gravity to the wing leading edge. What effect would this have on the aircraft performance?

$h_0 \approx 0 \quad \overline{V} = 0.134 \quad C_{M_0} = -0.141$
$h = 0.3\bar{c} \quad W \approx 340\text{kg}$

The 1903 Wright Flyer. The datum is the wing leading edge.

23. The table below contains flight test data for the X-15 spaceplane. Calculate the static margin stick-fixed and estimate the zero-lift pitching moment. Estimate the dimensional and non-dimensional phugoid mode and SPO frequencies.

$S = 18.58\text{m}^2$	$s = 6.82\text{m}$
$\bar{c} = 3.13\text{m}$	$h = 0.22$
$m = 7056\text{kg}$	$B = 10700\text{kgm}^2$
$V = 331\text{kt EAS}$	$a = 3.5/\text{rad}$
$\partial C_M/\partial\alpha = -0.8/\text{rad}$	$Z_u = -332\text{Ns/m}$
$M_w = -40.7\text{Ns}$	$Z_w = -14300\text{Ns/m}$
$M_q = -158600\text{Nms}$	

[$\Omega_{\text{ph}} = 0.0946$ (0.052rad/s); $\Omega_{\text{spo}} = 10.074$ (5.5395rad/s)]

24. NASA CR-2144, Aircraft Handling Qualities Data, contains stability information for ten aircraft. For the Boeing 747:
   (a) calculate the static margin stick-fixed;
   (b) estimate the zero-lift pitching moment;
   (c) estimate the phugoid, SPO and Dutch roll periods;
   (d) estimate the time constants for rolling subsidence and the spiral mode.
25. How would you modify the pitching moment equation (2.3) to give a first approximation to near- and post-stall behaviour?
26. How would you modify the pitching moment equation (2.3) to model the behaviour of an aircraft on the ground, in particular at the point of take-off rotation?
27. Analyze the stability and control characteristics of a Spitfire.

12.1 *Aircraft in the movies*

1. Watch The First Of The Few.
   (a) What is the most unrealistic scene in the film?
   (b) Who was R. J. Mitchell?
   (c) What kind of men are the pilots in the film?
   (d) What is expected of the female characters in the film?
   (e) What impression did you get of Britain, and the Royal Air Force, from the film?
2. Watch The Sound Barrier.
   (a) What is the most unrealistic scene in the film?
   (b) Who was Geoffrey de Havilland Jr?
   (c) What kind of men are the pilots in the film?
   (d) What is expected of the female characters in the film?
   (e) What impression did you get of Britain, and its aircraft industry, from the film?
3. Watch The Right Stuff. Even better, then read the book.
   (a) What is the most unrealistic scene in the film?
   (b) Who was Pancho Barnes?
   (c) What kind of men are the pilots in the film?
   (d) What is expected of the female characters in the film?
   (e) Are modern astronauts chosen to have the same characteristics as the pilots in the film?
   (f) What impression did you get of the US, and its space programme, from the film?

13
Exam questions

These are sample questions taken from previous years' exam papers, intended to show the style of question and give an idea of how to approach the exam. You should also look at the papers themselves to see how they are structured, rather than rely only on the questions here.

1. (a) Show that the static margin stick-fixed of an aircraft is related to its control characteristics via

$$K_n = -\overline{V}a_2\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right).$$

[13 marks]

(b) Data are given in Table Q1 for a transport aircraft with conventional tail. The nominal cruise speed of the aircraft is 300kt. Estimate the minimum tail volume coefficient required if the pilot is to be able to change the aircraft speed by up to 50kt without using the trim tab and with a change in elevator deflection of no more than 5°.

[12 marks]

(c) Discuss qualitatively the aircraft response if the pilot should make an abrupt 5° change in elevator deflection. How will this response vary with centre of gravity position?

[8 marks]

Minimum $K_n =$ 0.05	c.g. range $\Delta h = 0.40$	
$h_0 = 0.075$	$\epsilon = 0.16\alpha$	
$a = 5.7$	$a_1 = 2.7$	$a_2 = 2.1$
$m = 100 \times 10^3$kg	$S = 232$m^2	

Table Q1

2. (a) For a conventional aircraft show that if the tab setting remains unaltered, the change of elevator hinge moment coefficient-to-trim $\Delta\overline{C_H}$ between two lift coefficients is given by

$$\Delta\overline{C_H} = -\frac{b_2}{a_2\overline{\overline{V}}}\Delta C_L K_n'.$$

[12 marks]

(b) The aircraft described in the Table Q2 is making a zero stick force trimmed landing approach at 155kt EAS with flaps up. Calculate the value to which the speed may be reduced while keeping the stick force within 150N without altering the trim tab setting, indicating clearly whether this is push or a pull force. The stick–elevator gearing ratio $m_e = 1.0\text{m/rad}$.

[8 marks]

(c) To a good approximation, flap deployment changes the lift curve slope a by a factor $1.2 \times (1 - d\sin^2\delta_f)$ where δ_f is the flap deflection and $d = 0.25$ for this aircraft. Estimate the stick force required to make the same change in aircraft speed as in part b with 40° flap deflection. Comment on your answer.

[6 marks]

(d) In view of your answer to part c, would you recommend any changes to the operating procedure of the aircraft with regard to control?

[7 marks]

		W	= 785kN	h	= 0.26
		S	= 223m^2	$\bar{c}$	= 5.68m
		S_T	= 46.5m^2	l	= 15.66m
		S_η	= 11.2m^2	$\bar{c}_\eta$	= 0.908m
h_0	= 0.16	C_{M_0}	= -0.06	ϵ	= 0.38α
a	= 4.5	a_1	= 2.75	a_2	= 1.16
b_0	= 0	b_1	= -0.133	b_2	= -0.16

Table Q2

3. (a) Show from first principles that the elevator deflection per gee for a conventional aircraft is given by

$$H_m = -\frac{\overline{V} a_2}{C_L} \frac{\Delta \overline{\eta}}{n},$$

where symbols have their usual meanings.

[12 marks]

(b) The Junkers 87 "Stuka", Figure Q3, was a German dive bomber of the Second World War. Its mode of attack was to enter a vertical dive at an airspeed of 500km/h, drop its bomb, and then enter a 6g pull out ($n = 5$) from low altitude. Using the estimated data in Table Q2, estimate the centre of gravity position which would be required to perform the pull out if the aircraft mass after dropping its bomb is $W = 3800$kg and the change in elevator deflection for the manoeuvre is 15°. Comment on your answer.

[14 marks]

(c) Estimate the minimum height where the pull out could be initiated.

[7 marks]

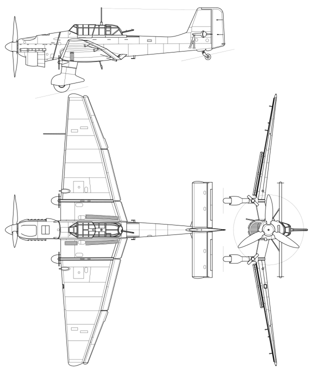


Figure Q3: Ju-87 Stuka (Kaboldy, CC BY-SA 3.0, Wikimedia)

$\epsilon = 0.2\alpha$	$S = 32\text{m}^2$	$\bar{c} = 2.1\text{m}$
$a = 4.5$	$a_1 = 4$	$a_2 = 1.1$
$l = 5.5\text{m}$	$S_T = 4\text{m}^2$	$h_0 = 0.25$

Table Q3

4. (a) Show that the pitch angle oscillation of an aircraft is governed by

$$B\ddot{\alpha} + \frac{\rho V^2 S\bar{c}a}{2} K_n \alpha = 0,$$

where B is the pitching moment of inertia about the centre of gravity and other symbols have their usual meanings. From the equation determine the frequency of short period oscillation (SPO).

[12 marks]

(b) The Hawker Typhoon, Figure Q4, was a successful Second World War ground attack aircraft. Using the approximate data given in Table Q4, estimate the SPO frequency and period for a Typhoon flying at 430km/h with static margin $K_n = 0.05$. Comment on your answer.

[6 marks]

(c) The Hispano–Suiza 20mm cannon fitted in the Typhoon had a muzzle exit velocity of 850m/s. The muzzles lay 1.8m ahead of the centre of gravity. If the short period oscillation had an amplitude of 5°, estimate the angular deflection of the trajectory of the round caused by the SPO and the corresponding error in the trajectory for a target at a distance of 500m. Comment on your answer.

[10 marks]

(d) What implications does SPO have for "pointing accuracy" in aircraft?

[5 marks]

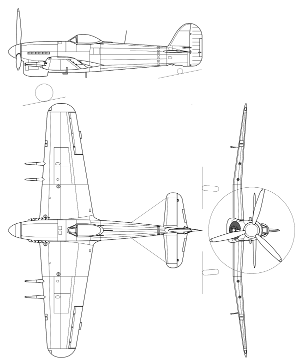


Figure Q4: Hawker Typhoon (Wikipedia Commons)

$S = 29.6\text{m}^2$	$\bar{c} = 2.3\text{m}$	$B = 6000\text{kgm}^2$	$a = 4.5$

Table Q4

5. (a) Show from first principles that the stick-fixed static margin of a canard aircraft is:

$$K_n = h_0 - h - \frac{l_F}{\bar{c}} \frac{S_F}{S} \frac{a_1}{a}$$

where symbols have their usual meanings.

[15 marks]

(b) One way to model the post-stall behaviour of a lifting surface is to treat it as having a negative lift curve slope. Table Q5 gives approximate data for a hypothetical small canard aircraft. Calculate the static margin stick-fixed before and after stall of the foreplane, assuming that post-stall behaviour can be modelled by changing the sign of a_1. Comment on your answer, with particular reference to the handling qualities of the aircraft.

[8 marks]

(c) Canard aircraft can be prone to dynamic stall, where the foreplane incidence is increased by the pitch rate of the aircraft. How can this situation be avoided, through aircraft design or through centre-of-gravity restrictions? What effects will possible solutions have on the usefulness of the aircraft?

[10 marks]

S=5m^2	$S_F = 0.65\text{m}^2$	$\bar{c} = 0.6\text{m}$	$l_F = 4\text{m}$
$a = 4.7$	$a_1 = 2.3$	$h_0\bar{c} = 4.5\text{m}$	$h\bar{c} = 4.2\text{m}$

Table Q5

6. (a) The control column of an aeroplane is connected to the elevator by a flexible cable whose stiffness is given by $\mathrm{d}H_E/\mathrm{d}\eta = E\,\mathrm{Nm/rad}$ where H_E is the hinge moment and η is the elevator deflection, the stick being held fixed.

   Show that the stick-fixed c.g. margin (as opposed to the "elevator fixed" c.g. margin) is given by:

$$K_n = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[1 - \frac{a_2 b_1}{a_1 b_2}\frac{1}{1-\lambda}\right]$$

   where

$$\lambda = \frac{C_L S E}{b_2 S_\eta \bar{c}_\eta W}.$$

   It should be assumed that the aircraft is initially in trim with the tab angle adjusted to give zero stick force.

   Show how this margin is related to the stick fixed and stick free c.g. margins of a rigid aeroplane.

   [15 marks]

   (b) By considering the variation of K_n with flight speed V, show that for a divergent elevator ($b_1 < 0$), the c.g. margin reduces with increased speed.

   [12 marks]

   (c) How could the aircraft design be modified to mitigate the change in margin with speed?

   [6 marks]

7. (a) Show that the pitch angle oscillation of an aircraft is governed by the equation

$$I\ddot{\alpha} + \frac{\rho V^2 S\bar{c}a}{2} K_n \alpha = 0,$$

where I is the pitching moment of inertia about the centre of gravity and other symbols have their usual meanings. From the equation find the frequency of short period oscillation.

[12 marks]

(b) Table Q7 contains basic data for a small electrically-powered UAV used for mapping and agricultural observation. For adequate image quality, it has been found that the aircraft oscillation frequency must be limited so that the distance scanned by the camera as it shoots an image is not more than 0.5m. If the camera exposure time is 1/1000s, estimate the speed at which the image frame sweeps the ground, and if the UAV is to operate at an altitude of 80m, the maximum acceptable pitch oscillation frequency. From this frequency, estimate the required static margin stick-fixed for the UAV. You may neglect the effect of flight speed on the swept area of the image.

$V = 9\text{m/s}$	$S = 0.16\text{m}^2$	$\bar{c} = 0.2\text{m}$
$a = 4.2$	$m = 0.5\text{kg}$	$I = 0.005\text{kgm}^2$

Table Q7

[8 marks]

(c) If the static margin stick-fixed $K_n < 0$, the aircraft is statically unstable. Qualitatively, what would determine the degree to which it could be made statically unstable and still be controllable?

[8 marks]

(d) What are the implications for aircraft handling of the result that SPO frequency is proportional to the square root of static margin stick-fixed?

[5 marks]

8. The C130 Hercules, Figure Q8, is a military aircraft used for transport and parachute dropping of vehicles and other supplies. Estimated aircraft parameters are given in Table Q8. The cargo hold has a capacity of 20,000kg, and extends from 5.3m to 17.6m from the datum, which is the aircraft nose.

   (a) Show that the static margin stick fixed is given by:

$$K_n = -\overline{V} a_2 \frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = (h_0 - h) + \overline{V} \frac{a_1}{a} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right).$$

[10 marks]

   (b) A Hercules normally carries out a drop at a flight speed of 125kt EAS. Size the tailplane such that the minimum static margin, stick fixed, is never less than 0.05, and so that the change in elevator angle to trim during drop of a full payload is no more than 15°. Assume $\Delta h = 0.5$.

[10 marks]

   (c) If the centre of gravity, with payload, is at the aft limit, can the drop be performed safely?

[7 marks]

   (d) What dynamic behaviour would you expect of the aircraft immediately after the drop has been carried out?

[6 marks]

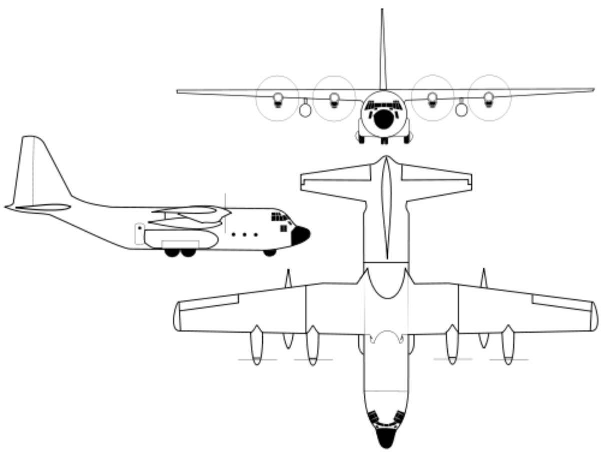


Figure Q8: C130 Hercules (Wikimedia Commons)

$W = 65 \times 10^3$kg (including payload of 20×10^3kg)				$S = 162.1\text{m}^2$	$\epsilon = 0.2\alpha$
$\overline{c} = 4$m	$h_0\overline{c} = 13.6$m	$l = 12.8$m	$a = 5.8$	$a_1 = 4.2$	$a_2 = 2.0$

Table Q8: C130 Hercules data

9. The first variable geometry aircraft, capable of changing its wing sweep in flight, was the Bell X-5, shown in Figure Q9 with estimated parameters given in Table Q9. Lengths are measured from the aircraft nose.

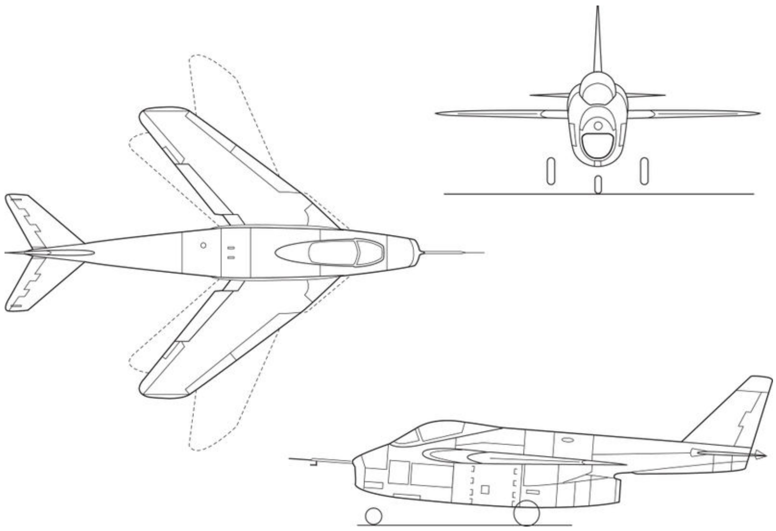


Figure Q9: Bell X-5 variable geometry aircraft (Wikimedia Commons).

Aircraft length $l = 10.1$m	Chord $c_0 = 2.5$m
Wing area $S = 16.26\text{m}^2$	Mass $m = 4500$kg
C_{M_0} = +0.05	$\partial\epsilon/\partial\alpha = 0.4$
Unswept	Swept
$h_0 c_0 = 3.9$m	$h_0 c_0 = 5.2$m
$U = 150$kt EAS	$U = 630$kt EAS
$a = 4.8$	$a = 4$
$a_1 = 3.0$	$a_1 = 2.5$

Table Q9: Bell X-5 parameters.

(a) Given that the centre of gravity range is from 3m to 4m aft of the aircraft nose, find the tail area and setting which give a static margin stick fixed $K_n = 0.05$ in the unswept case and trim with zero elevator deflection in the swept case. Assume that the tailplane lift acts 10.0m from the aircraft nose.

[25 marks]

(b) What are the control issues affecting aircraft which operate at high speed and how can the problems be alleviated?

[8 marks]

10. (a) In the 1952 David Lean film The Sound Barrier, a test pilot experiences "control reversal" at a flight speed near the speed of sound. Describe the control effects of high speed flight, with reference to the pilot's perception of handling, and state what phenomenon actually occurs as an aircraft approaches and exceeds the speed of sound.

    [10 marks]

    (b) In the 1983 film The Right Stuff, based on Tom Wolfe's 1979 book, Chuck Yeager is shown making the first supersonic flight in the X-1. Which principal design feature made the aircraft controllable at high speed and why was it necessary?

    [10 marks]

    (c) The 1942 film The First Of The Few is a fictionalized account of the development of the Spitfire by R. J. Mitchell. It includes a scene of the first flight of the aircraft in which it is shown flying high-g manoeuvres. What are the design considerations relevant to the control of high-performance aerobatic aircraft and how are the flying qualities of such aircraft assessed?

    [14 marks]

11. The Supermarine Spitfire is a very well-known aeroplane which saw extensive service in the 1940s. Table Q11 gives summary data for the Mark 1 variant. The value for ℓ_T is the tailplane distance from the rear centre of gravity limit, h_{aft}. The datum is the leading edge of the mean aerodynamic chord.

(a) Mark 1 Spitfire models were subjected to a series of wind tunnel tests in 1945. Measurements were taken of the pitching moment about h_{aft} on a model with no tail and it was found that $C_{M_0} \approx -0.05$ and $\partial C_M/\partial C_L \approx 0.149$. Estimate the position of the wing-body-nacelle neutral point h_0, the tailplane volume coefficient, and the resulting static margin stick-fixed at the aft centre of gravity limit. Comment on your answer.

[15 marks]

(b) With the centre of gravity at the aft limit, estimate the manoeuvre margin stick-fixed and the elevator deflection required to pull 4g ($n = 4$) at a speed of 200kt. You may use the result

$$\frac{\Delta\overline{\eta}}{n} = -\frac{C_L}{\overline{V}a_2} H_m.$$

Comment on your answer.

[10 marks]

(c) What considerations do you think should be taken into account in designing the control system of the Spitfire, with regard to handling qualities and response to pilot input?

[8 marks]

$S = 22.482\text{m}^2$	$S_T = 3.135\text{m}^2$	
$\overline{c} = 1.99\text{m}$	$\ell_T = 5.462\text{m}$	$h_{\text{aft}} = 0.340\overline{c}$
$a = 4.6$	$a_1 = 2.83$	$a_2 = 2.15$
$\epsilon = 0.037 + 0.3\alpha$		
$W = 3000\text{kg}$		

Table Q11

12. Figure Q12 shows the notation for motion of an aircraft at constant incidence and lift coefficient, acted upon by lift L perpendicular to the trajectory, and by gravity acting vertically downwards.

(a) Show that the trajectory is governed by the equations

$$\cos\theta = \frac{1}{3}\frac{z}{z_1} + C\left(\frac{z_1}{z}\right)^{1/2},$$
$$\frac{z_1}{R} = \frac{1}{3} - \frac{C}{2}\left(\frac{z_1}{z}\right)^{3/2},$$

where R is the radius of curvature of the trajectory, and z is shown on Figure Q12. [12 marks]

(b) Depending on the value of the constant of integration C, the trajectory can have four qualitatively different behaviours. Describe these cases, and sketch the trajectories to which they correspond.

[12 marks]

(c) Describe qualitatively the phugoid response of real aircraft, with reference to dynamic stability, damping, and stick free effects.

[9 marks]

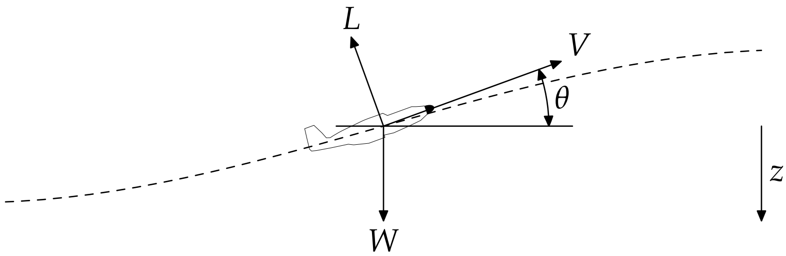


Figure Q12: Motion of a body under lift and gravity

A
Finding out more

A.1 Further reading

- Abzug, Malcolm J. & Larrabee, E. Eugene, *Airplane stability and control: A history of the technologies that made aviation possible*, Cambridge University Press, 2002.
- Brown, Eric, *Wings on My Sleeve: The World's Greatest Test Pilot Tells his Story*, W & N, 2007. Brown moved from naval combat flying to test flying during the Second World War and still holds the record for the greatest number of aircraft types flown by one pilot. You can hear him interviewed on Desert Island Discs, `http://www.bbc.co.uk/programmes/b04nvgq1`.
- Culick, F. E. C., 'The Wright brothers: First aeronautical engineers and test pilots', AIAA Journal, **41**(6):985–1006, 2003.
- Hamilton-Paterson, James, *Empire of the Clouds: When Britain's Aircraft Ruled the World*, Faber & Faber, 2010.
- Hamilton-Paterson, James, *Marked for Death: The First War in the Air*, Pegasus Books, 2016, the human story of how aviation developed during the First World War.
- Langewiesche, William, 'The human factor', *Vanity Fair*, October 2014, `https://bit.ly/2FyE5eB`, an account of the crash of AF447 in 2009, with a lot of detail about the human factors engineering of cockpits.
- Markham, Beryl, *West With The Night*, the memoirs of one of the first female pilots, who made the first non-stop flight from England to North America.
- Mercurio, Jed, *Ascent*, Vintage, 2008, a novel based on a fictitious Soviet lunar mission, with excellent accounts of flying and a physically correct space emergency. The author is an ex-RAF medical doctor who trained as a pilot, which gives him great insight into flying and people who fly.
- de Saint-Exupéry, Antoine, *Courrier sud/Southern Mail*, *Vol de nuit/Night Flight*, *Pilote de guerre/Flight to Arras*, classics of aviation by one of the great pilot-authors.

- Salter, James, *The Hunters*, Penguin Classics, 2007, a novel by one of America's finest writers based on his time as a fast jet pilot in the Korean War.
- United States Air Force Test Pilot School, *Flying Qualities Textbook*, Volume II, Part 1, AF-TPS-CUR-86-02, April 1986, a big book (more than 700 pages) but the chapters on longitudinal stability and flight testing are manageable and well worth reading.
- Vanhoenacker, Mark, *Skyfaring: A Journey With a Pilot*, Vintage, 2015. A commercial pilot on the experience and mechanics of flying. Already a classic, which stands comparison with the finest writing on aviation.
- Vincenti, Walter G., *What Engineers Know and How They Know It*, Johns Hopkins University Press, 1990. This is a collection of studies looking at how aeronautical engineers acquired and acquire knowledge of the systems they work on. You should read all of it, but chapter 3 on how flying quality specifications for aircraft evolved up to 1945 is especially relevant.
- Wolfe, Tom, *The Right Stuff*, regularly republished, an account of the early days of the American space programme: one of the best books written on test flying and the people who do it.
- 'What really happened aboard Air France 447', *Popular Mechanics*, December 2011, `http://bit.ly/1PtPlde`, including a transcript from the cockpit voice recorder.

A.2 Data sources

- Heffley, R. K. & Jewell, W. F., *Aircraft handling qualities data*, NASA CR-2144, 1972, is a collection of flight test data, including aerodynamic derivatives, for a ten aircraft: NT-33A, F-104A, F-4C, X-15, HL-10, Jetstar, CV-880M, B-747, C-5A, and XB-70A.
- Mair, W. A., *High-speed wind-tunnel tests on models of four single-engined fighters (Spitfire, Spiteful, Attacker and Mustang)*, ARC R&M 2535, 1951, is a collection of reports giving detailed aerodynamic data, including tailplane information, for four single-engined aircraft of the 1940s.

A.3 Accident reports

The reports of accident investigations are often used in aeronautical engineering to help understand how things can go wrong and how we can design and operate aircraft safely and reliably.

- The Aviation Safety Network has a database of accidents since 1919, which you can search using various criteria, for example 'Centre of Gravity outside limits'. The network's website is `https://aviation-safety.net/`

- Air Accidents Investigation Branch, *Report No: 2/2000. Report on the incident to Fokker F27-600 Friendship, G-CHNL, near Guernsey Airport, Channel Islands on 12 January 1999*, The Stationery Office, 2000. This is the report of an accident caused directly by incorrect loading leading to a centre-of-gravity position which was too far aft. The analysis section of the report explains very clearly the sequence of events which led to the crash, and how different factors interacted to cause it. The report is available from `http://bit.ly/2bAwr4D`.

A.4 Web sites

- Hush Kit, `http://hushkit.net/`, is an excellent, and well-written, aviation site covering various aspects of historical and modern aircraft, including the top ten best-looking British, French, Swedish, Australian, Soviet, German, Japanese, and Latin-American aircraft.
- Dr Brett Holman of the University of New England in Australia maintains an excellent research blog, `http://airminded.org/`, covering his work on the history of aviation and attitudes to it.
- Mark Vanhoenacker (see above) also has a website associated with his book at `http://www.skyfaring.com/`.

A.5 Further watching

As well as the movies listed on page 82, there are some other films and television programmes worth seeing to develop your knowledge of aviation and its culture.

- Cold War, Hot Jets, part 1 (fighters) `http://bit.ly/1Ha0PuX`; part 2 (bombers) `http://bit.ly/1QBYnSp`.

A.6 Picture credits

All images are the work of the author, except those listed below. The URLs link to the original image with full information on authorship and usage rights.

Figure 2.3

1. Avro Vulcan, James Humphreys, `http://bit.ly/1PxASt2`
2. Saab Gripen, Matthias Kabel, `http://bit.ly/1TRuPjn`
3. Pegasus Quantum, Adrian Pingstone, `http://bit.ly/1E0TkZP`

Figure 7.1

1. Grob 109, public domain, `http://bit.ly/1LhGaal`
2. Jetstream, Dyvroeth, `http://bit.ly/1Jowr4E`
3. Sukhoi 27, Dmitry A. Mottl, `http://bit.ly/1E3jVoO`
4. Vampire, Timothy Swinson, `http://bit.ly/1NhhMsI`
5. Fouga Magister, Tim Felce, `http://bit.ly/1J0pjZ5`
6. Predator UAV, US Air Force/Lt Col. Leslie Pratt, `http://bit.ly/1MwlJbk`
7. Grumman OV-1D, Valder137, `https://bit.ly/2LdlpWc`
8. C2A, US Navy, `https://bit.ly/2uElnw1`

Figure 7.2

1. Rutan VariEze, Stephen Kearney, `http://bit.ly/1Jox3r7`
2. Beechcraft Starship, Ken Mist, `http://bit.ly/1UR0cgA`
3. Piaggio Avanti, Tibboh, `http://bit.ly/1MANkcZ`

TO ME SHE IS ALIVE AND TO ME SHE SPEAKS. I FEEL THROUGH THE SOLES OF MY FEET ON THE RUDDER-BAR THE WILLING STRAIN AND FLEX OF HER MUSCLES. THE RESONANT, GUTTURAL VOICE OF HER EXHAUSTS HAS A TIMBRE MORE ARTICULATE THAN WOOD AND STEEL, MORE VIBRANT THAN WIRES AND SPARKS AND POUNDING PISTONS.

SHE SPEAKS TO ME NOW, SAYING THE WIND IS RIGHT, THE NIGHT IS FAIR, THE EFFORT ASKED OF HER WELL WITHIN HER POWERS.

I FLY SWIFTLY. I FLY HIGH—SOUTH-SOUTHWEST, OVER THE NGONG HILLS. I AM RELAXED. MY RIGHT HAND RESTS UPON THE STICK IN EASY COMMUNICATION WITH THE WILL AND THE WAY OF THE PLANE. I SIT IN THE REAR, THE FRONT COCKPIT FILLED WITH THE HEAVY TANK OF OXYGEN STRAPPED UPRIGHT IN THE SEAT, ITS ROUND STIFF DOME FOOLISHLY REMINDING ME OF THE POISED RIGIDITY OF A PASSENGER ON FIRST FLIGHT.

BERYL MARKHAM, *WEST WITH THE NIGHT*