A MULTI-OBJECTIVE ADAPTIVE CONTROLLER FOR MAGNETIC BEARING SYSTEMS

M. Necip Sahinkaya ∗
Department of Mechanical Engineering
University of Bath
Bath, BA2 7AY
United Kingdom
Email: ensmns@bath.ac.uk

Abdul-Hadi G. Abulrub
Clifford R. Burrows
Patrick S. Keogh
Department of Mechanical Engineering
University of Bath
Bath, BA2 7AY
United Kingdom

ABSTRACT
The paper considers three issues in flexible rotor and magnetic bearing systems, namely control of rotor vibration, control of transmitted forces, and prevention of rotor contact with auxiliary bearings. An adaptive multi-objective optimization method is developed to tackle these issues simultaneously using a modified recursive open loop adaptive controller. The proposed method involves automatic tuning of the weighting parameters in accordance with performance specifications.

A two-stage weighting strategy is implemented involving base weightings, calculated from a singular value decomposition of the system’s receptance matrices, and two adjustable weighting parameters to shift the balance between the three objective functions. The receptance matrices are functions of rotational speed and they are estimated in situ. The whole process does not require prior knowledge of the system parameters.

Real-time implementation of the proposed controller is explained and tested by using an experimental flexible rotor magnetic bearing system. The rotor displacements were measured relative to the base frame using four pairs of eddy current displacement transducers. System stability is ensured through local PID controllers. The proposed adaptive controller is implemented in parallel and the effectiveness of the weighting parameters in changing the balance between the transmitted forces and rotor vibrations is demonstrated experimentally.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_Q )</td>
<td>Base scaling for displacement measurements</td>
</tr>
<tr>
<td>( C_T )</td>
<td>Base scaling for transmitted forces</td>
</tr>
<tr>
<td>( J )</td>
<td>Cost function</td>
</tr>
<tr>
<td>( J_A )</td>
<td>Augmented cost function</td>
</tr>
<tr>
<td>( J_M )</td>
<td>Maximum displacement at magnetic bearing locations</td>
</tr>
<tr>
<td>( J_Q )</td>
<td>Overall vibration cost function</td>
</tr>
<tr>
<td>( J_T )</td>
<td>Transmitted force cost function</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
</tr>
<tr>
<td>( F_o )</td>
<td>Vector of unbalance forces in the frequency domain</td>
</tr>
<tr>
<td>( F_T )</td>
<td>Vector of transmitted forces in the frequency domain</td>
</tr>
<tr>
<td>( G )</td>
<td>Transfer functions matrix of local PID controllers</td>
</tr>
<tr>
<td>( H )</td>
<td>Complex control gain matrix</td>
</tr>
<tr>
<td>( H_A )</td>
<td>Augmented complex control gain matrix</td>
</tr>
<tr>
<td>( I )</td>
<td>Unity matrix</td>
</tr>
<tr>
<td>( K_m )</td>
<td>Matrix of magnetic bearing negative stiffness coefficients</td>
</tr>
<tr>
<td>( Q )</td>
<td>Vector of measured displacements in the frequency domain</td>
</tr>
<tr>
<td>( q )</td>
<td>Vector of measured displacements</td>
</tr>
<tr>
<td>( Q_A )</td>
<td>Vector of augmented measurements in the frequency domain</td>
</tr>
<tr>
<td>( Q_m )</td>
<td>Vector of measured displacements at magnetic bearing locations in the frequency domain</td>
</tr>
<tr>
<td>( Q_o )</td>
<td>Vector of measured displacements at other than magnetic bearing locations in the frequency domain</td>
</tr>
</tbody>
</table>

∗Address all correspondence to this author.

© 2009 by ASME
An open-loop strategy for controlling synchronous vibrations uses the receptance matrix $R$ related to control forces. Various closed-loop controllers have been used to control rotor vibrations [8, 9]. Burrows et al. [10] reported pole placement techniques for synchronising vibration control of a rotor-bearing system. An optimisation approach was presented by Keogh et al. [11] to minimise the influence of forcing disturbances, modelling error, and measurement error. The application of multivariable design methodologies, such as $H_\infty$ [11] and $\mu$-synthesis [12], emphasise robustness issues of feedback control under varying operating conditions. Active multi-objective control strategies usually involve design of separate controllers for each objective function, and switching between them in accordance with an algorithm based on speed [21] or base acceleration [22]. This paper discusses a unified adaptive approach, and extends the application of the ROLAC to minimise a multi-objective cost function.

In some applications, e.g. turbomolecular pumps, gas turbines and compressors, it is important to minimise the transmitted forces from the rotor to the support structure. The control of transmitted forces and rotor vibrations imposes conflicting requirements and necessitates the use of multi-objective optimisation.

### EXPERIMENTAL SETUP

The experimental magnetic/bearing system consists of a uniform flexible steel shaft of length 2 m and radius 0.025 m, with four 10 kg disks of radii 0.125 m, as shown in Fig. 1. The rotor of total mass 100 kg is mounted horizontally on two radial magnetic bearings each of which has a radial force capacity of 1.75 kN with a bandwidth of 100 Hz. The magnetic bearings have an radial clearance of 1.2 mm and each is protected by a retainer bearing having 0.75 mm clearance [6].

A schematic view of the experimental setup showing the rotor, magnetic bearings, retainer bearings and sensors is shown in Fig. 2. Critical speeds of the flexible rotors were measured experimentally at 10 Hz, 17 Hz, and 28 Hz. The rotor displacements were measured relative to the base frame with four pairs of eddy current displacement transducers, placed in four planes at $45^\circ$ with the vertical line. At each transducer location, a precision stainless steel collar is mounted on the shaft to minimise the measurement errors due to material imperfections. The rotor-bearing system stability is ensured through local PID controllers. An adaptive controller runs in parallel with the PID controllers.

**Mathematical Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Receptance matrix related to control forces</td>
</tr>
<tr>
<td>$R_A$</td>
<td>Augmented receptance matrix</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Sub-matrix of $R$ corresponding to $Q_m$</td>
</tr>
<tr>
<td>$R_o$</td>
<td>Sub-matrix of $R$ corresponding to $Q_o$</td>
</tr>
<tr>
<td>$W$</td>
<td>Weighting matrix</td>
</tr>
<tr>
<td>$W_A$</td>
<td>Augmented weighting matrix</td>
</tr>
<tr>
<td>$W_f$</td>
<td>Weighting matrix related to $F_T$</td>
</tr>
<tr>
<td>$W_m$</td>
<td>Weighting matrix related to $Q_m$</td>
</tr>
<tr>
<td>$W_o$</td>
<td>Weighting matrix related to $Q_o$</td>
</tr>
<tr>
<td>$Z$</td>
<td>Receptance matrix related to unbalance forces</td>
</tr>
<tr>
<td>$I_1$</td>
<td>Integral constant for ROLAC</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Adjustable weighting parameter</td>
</tr>
<tr>
<td>$\Delta' \gamma$</td>
<td>Change in $\gamma$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Adjustable weighting parameter</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Maximum singular value of $\gamma$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time variable</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Rotational speed</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Fundamental frequency</td>
</tr>
<tr>
<td>$^{(\ast)}$</td>
<td>Optimum</td>
</tr>
<tr>
<td>$^{(\gamma)}$</td>
<td>Predicted</td>
</tr>
<tr>
<td>$^{(\gamma)T}$</td>
<td>Transpose (Hermitian for a complex matrix)</td>
</tr>
</tbody>
</table>

### INTRODUCTION

Magnetic bearings, when used to levitate a rotating shaft, permit relative motion without friction or wear. They are used in many industrial applications such as compressors, turbines, pumps, motors and generators [1, 2]. The future of magnetic bearings in critical applications depends on successfully addressing safety and reliability issues [3]. In addition to vibration control and levitation functions, magnetic bearings can be used to fulfil other functions, such as monitoring, auto-tuning, parameter identification, fault detection and tolerance, e.g. [4–6]. The versatility of magnetic bearings is important in the development of smart rotating machinery [7].

Various closed loop controllers have been used to control the rotor vibrations [8, 9]. Burrows et al. [10] reported pole placement techniques for synchronising vibration control of a rotor-bearing system. An optimisation approach was presented by Keogh et al. [11] to minimise the influence of forcing disturbances, modelling error, and measurement error. The application of multivariable design methodologies, such as $H_\infty$ [11] and $\mu$-synthesis [12], emphasise robustness issues of feedback control of active magnetic bearing systems.

An open-loop strategy for controlling synchronous vibration under varying operating conditions was introduced by Burrows and Sahinkaya [13, 14]. The key features of this open-loop adaptive control (OLAC) strategy, also referred to automatic balancing, are its simplicity and the ability to apply self-tuning in situ with no prior knowledge of the system model or parameter values. This approach has also been adopted in a broader context [15–19]. However, OLAC is not fast enough to respond to sudden changes in operating conditions e.g. due to mass unbalance or rapid transient excitation, because it relies on performing a Fourier transform of the measured steady state response. A recursive open-loop adaptive control (ROLAC) scheme was therefore developed [20]. This utilises a recursive version of the Fourier transform to update optimum control force components at every sampling interval.
The multi-objective controller is an extension of the open loop adaptive controller [13] that has been successfully implemented in various applications. The linearised equations of motion of a flexible rotor and magnetic bearing system can be written in terms of a finite element model of the rotor. The frequency response of the measured displacements \( Q(j\omega) \) can be expressed as follows:

\[
Q(j\omega) = Z(\Omega, j\omega)F_o(j\omega) + R(\Omega, j\omega)U(j\omega)
\]  

(1)

The symbols are defined in the Nomenclature. The two receptance matrices, \( Z \) and \( R \), are functions of the rotational speed due to gyroscopic effects and the dynamic characteristics of the bearings. They also include the effect of the magnetic bearing inherent negative stiffness characteristics and any local PID controllers. If the control force is changed by \( \Delta U \), the resulting frequency response of the measured displacements can be predicted by:

\[
\tilde{Q}(j\omega) = Q(j\omega) + R(\Omega, j\omega)\Delta U(j\omega)
\]  

(2)

or it can be written as:

\[
Q(j\omega) = -R(\Omega, j\omega)\Delta + \tilde{Q}(j\omega)
\]  

(3)

To minimise the weighted sum of squares of the predicted displacements, i.e. the following cost function

\[
J = \tilde{Q}^T W \tilde{Q}
\]  

(4)

the optimum change in the control force \( \Delta \tilde{U} \) can be calculated using a Least Square Estimator [13] (treating the predicted measured frequency response \( Q \) as error):

\[
\Delta \tilde{U}(j\omega) = -R^TWR^{-1}R^TWQ = -HQ
\]  

(5)

This is referred to as the open loop adaptive controller (OLAC). It requires the frequency response of the measured displacements \( Q \), and therefore it can adapt to changes in the unbalance distribution. The partial receptance matrix \( R \) can be estimated on-line [14], and hence the procedure does not require the availability of a system model or any knowledge of system parameters. In essence it is feed forward control that leaves the closed loop properties of the system unchanged and therefore does not have a destabilising effect. A block diagram describing the digital implementation is shown in Fig. 3.

The recursive version of the controller can be implemented using a recursive Fourier transform algorithm and an integrator [23]:

\[
\tilde{U}(j\omega, t) = -\alpha_1 \int_0^t H(\Omega, j\omega)Q(j\omega, \tau)d\tau
\]  

(6)

where \( Q(j\omega, t) \) represents the recursive Fourier transform of the measurement vector at time \( t \), defined as:

\[
Q(j\omega, t) = \frac{\omega_0}{\pi} \int_{-2\pi/\omega_0}^{2\pi/\omega_0} q(\tau)e^{-j\omega_\tau}d\tau
\]  

(7)

In the case of synchronous vibration control, \( \omega = \Omega = \omega_0 \), where \( \omega_0 \) is the fundamental frequency. Parallel controllers can be designed for the control of multi-frequency vibration of the rotor. If control of sub harmonic frequencies is required, then...
ω₀ has to be chosen such that all the frequencies of interest are integer multiples of the fundamental frequency. For synchronous and multi-frequency controllers, the control force in the time domain can be obtained by performing an inverse Fourier transform, i.e. using sine wave generators with controllable amplitude and phase.

The same methodology can be used to include any other measurable variables that are relevant to the optimisation process. For example, in order to consider transmitted forces, the predicted frequency response of the transmitted forces must be expressed in terms of a change in the control force. To obtain an expression for the transmitted forces, the displacement measurement vector is sub-divided into the displacements local to magnetic bearings \( Q_m \), and other residual locations \( Q_o \). The frequency response of the transmitted forces \( F_T \) is equal to the forces applied by the magnetic bearings to the rotating shaft, and includes the PID control \((GQ_m)\), adaptive control \((U)\) and the inherent negative stiffness \((K_mQ_m)\) contributions:

\[
F_T(j\omega) = [G(j\omega) + K_m]Q_m(j\omega) + U(j\omega) \tag{8}
\]

When the control force is changed by \( \Delta U \), an estimation of the resulting transmitted forces can be formulated, with the help of Eq. (2), as follows:

\[
F_T = F_T + \{ (G + K_m)R_m + I\} \Delta U \tag{9}
\]

By combining Eqs. (2) and (9), and using the partitioned measurement vector, the following augmented equation can be obtained:

\[
\begin{bmatrix}
\tilde{Q}_m \\
\tilde{Q}_o \\
\tilde{F}_T
\end{bmatrix}
= \begin{bmatrix}
Q_m \\
Q_o \\
F_T
\end{bmatrix} + \begin{bmatrix}
R_m \\
R_o \\
(G + K_m)R_m + I
\end{bmatrix} \Delta U \tag{10}
\]

or

\[
\tilde{Q}_A = Q_A + R_A \Delta U \tag{11}
\]

The control force receptance matrix is portioned in accordance with the measurement vector as follows:

\[
R(j\omega) = \begin{bmatrix}
R_m \\
R_o
\end{bmatrix} \tag{12}
\]

Therefore the optimum change in the force to minimise the weighted sum of squares can be written as:

\[
\Delta \tilde{U} = -(R_T^TW_A)^{-1}R_T^TW_AQ_A = -H_AQ_A \tag{13}
\]

The augmented weight matrix are partitioned as follows:

\[
W_A = \begin{bmatrix}
W_m & 0 & 0 \\
0 & W_o & 0 \\
0 & 0 & W_f
\end{bmatrix} \tag{14}
\]

Equation (13) minimises a multi-objective cost function including vibrations at the magnetic bearing locations, vibrations at other sensor locations, and transmitted forces in the following form:

\[
J_A = \tilde{Q}_m^TW_m\tilde{Q}_m + \tilde{Q}_o^TW_o\tilde{Q}_o + \tilde{F}_T^TW_f\tilde{F}_T \tag{15}
\]

Setting \( W_m = W_o \) minimises the overall vibration levels in a balanced manner. The recursive version of the multi-objective open loop adaptive controller (MO-ROLAC) can be implemented in a similar manner to ROLAC as discussed earlier.

**SELECTION OF WEIGHTS**

The purpose of the weight selection procedure is to provide a mechanism to shift the relative contributions of individual objective functions in accordance with pre-specified performance requirements. Considering the trade-off between the vibration and transmitted force control, the following parameterisation of weightings is applied:

\[
W_m = W_o = \frac{1}{C_{\tilde{Q}}} , \quad W_f = (1-\beta)\frac{1}{C_{\tilde{T}}} \tag{16}
\]
The base weighting coefficients, $C_Q$ and $C_T$, balance the displacement and transmitted forces such a way that the parameter $0 \leq \beta \leq 1$ can be used to shift the balance effectively. The conventional method to balance weightings is to use the maximum values, known as Bryson’s rule [24]. For example $C_Q$ may be set to the radial clearance of the auxiliary bearing, and $C_T$ to the dynamic force capacity of the magnetic bearings. However, it has been shown in [25] that Bryson’s rule does not provide satisfactory results. Therefore, the following method, which is based on the singular value decomposition of the respective receptance matrices, is proposed to achieve better sensitivity to $\beta$ through the full range from 0 to 1.

$$C_Q = \overline{\sigma}[\mathbf{R}] \quad , \quad C_T = \overline{\sigma}[(\mathbf{G} + \mathbf{K}_m)\mathbf{R}_m + \mathbf{I}]$$  \hspace{1cm} (17)

To demonstrate the benefit of using Eq. (17), Fig. 4 is included from [25], which shows the normalised vibration and transmitted force cost functions in terms of $\beta$. The proposed SVD based scaling enables the $\beta$ parameter to provide effective control of individual objective functions all through its range, whereas the selection based on Bryson’s rule (referred as conventional in the figure) provides control within a very limited range. This is significant when designing an adaptive controller to automatically vary $\beta$ in accordance with system performance.

With this choice of base weighting, the parameter $\beta$ can be used to shift the weight between displacement and transmitted force control. The maximum value of $\beta = 1$ corresponds to RO-LAC, i.e. there is no consideration of the transmitted forces. The minimum value of $\beta = 0$ corresponds to control of only the transmitted force, and theoretically it should result in zero transmitted forces because the degree of freedom of the optimisation process is zero in this application (4 transmitted forces to be controlled by 4 control inputs). Intermediate values of $\beta$ provides approximately proportional changes in both the displacement and transmitted force cost functions [25]. Therefore, the transmitted force is expected to decrease gradually with decreasing $\beta$ from its maximum at $\beta = 1$ to zero at $\beta = 0$. The displacement cost function is expected to move in the opposite direction, i.e. it will increase with decreasing $\beta$ values.

The second adjustable parameter $\gamma$ is introduced to distinguish vibrations between the magnetic bearing locations (i.e. the auxiliary bearing locations where contact may occur) and other locations when $\beta = 1$.

$$\mathbf{W}_m = \frac{1}{C_Q^2} \mathbf{I} \quad , \quad \mathbf{W}_o = \frac{\gamma}{C_Q^2} \mathbf{I}$$ \hspace{1cm} (18)

This parameter is set to unity by default. It is to be used only in cases where $\beta = 1$ (vibration only control) and the displacements at the magnetic bearing locations reach a critical level. Decreasing $\gamma$ should reduce vibrations at the magnetic bearing locations at the expense of higher vibrations elsewhere, hence preventing possible contact with the retainer bearings. In theory, $\gamma = 0$ should provide zero vibration at the magnetic bearing locations as there are only four displacements to control with four control forces.

The above two adjustment procedures can be combined as follows, where only one of the parameters is allowed to deviate from its default unity value:

$$\mathbf{W}_m = \frac{1}{C_Q^2} \mathbf{I} \quad , \quad \mathbf{W}_o = \frac{\gamma}{C_Q^2} \mathbf{I} \quad , \quad \mathbf{W}_f = (1 - \beta) \frac{1}{C_T^2} \mathbf{I}$$ \hspace{1cm} (19)

The effectiveness of the adjustable parameters $\beta$ and $\gamma$ on the system performance is assessed experimentally in the next section.

**EXPERIMENTAL RESULTS**

The multi-objective open loop controller strategy was implemented by dSPACE digital signal processing hardware and software, coupled with the Matlab/Simulink real time programming environment. A sampling rate of 4 kHz was used. The local PID gains were set to provide an effective stiffness of $1 \times 10^6$ N/m,
damping of $5 \times 10^3$ Ns/m, and integral action of $2 \times 10^5$ N/(ms). The negative stiffness coefficient of the magnetic bearings was $-2 \times 10^6$ N/m. The MO-ROLAC is implemented in parallel with the local PID controllers as shown in Fig. 5.

The system comprises of 8 sensors and 4 control force inputs, hence the size of the complex receptance matrix $R$ is $8 \times 4$. The rotor speed was set to 10 Hz. To identify the $R$ matrix, synchronous sine signals were injected from each force channel sequentially. The changes of the frequency response of the displacements were calculated and divided by the frequency response of the test signal to construct the corresponding column of the $R$ matrix. The estimated $R$ matrix and the corresponding base weightings, $C_Q$ and $C_T$, were stored in a file for subsequent use. These are functions of the rotational speed, but not functions of the rotor unbalance distribution nor of the external forces. The $R$ matrix needs to be re-estimated only if there are changes in the rotor/bearing internal dynamics [5], such as cracks in the rotor, changes in magnetic bearing negative stiffness coefficients or the PID parameters.

Figure 6 shows the benefits achieved by switching on the ROLAC ($\beta = 1, \gamma = 1$). Three cost functions are used to measure the performance of the system in terms of the three objective functions, namely the average rotor vibration ($J_Q$), maximum rotor vibration at the magnetic bearing locations ($J_M$), and the transmitted forces ($J_T$). They are defined as follows:

$$J_Q = \sqrt{\frac{1}{8}Q^TQ} \quad J_M = \max(|Q_m|) \quad J_T = \sqrt{F_T^TF_T}$$

As can be seen from Fig. 6, the overall vibration level $J_Q$ is reduced by 84%. The maximum displacement at the magnetic bearing locations $J_M$ is reduced by 82%. Although the transmitted forces are not included in ROLAC, it resulted in a 48% reduction in the transmitted forces. ROLAC forces to achieve this were measured to be 70 N and 47 N at magnetic bearing 1 (MB-1) in the $x$ and $y$ directions, respectively, and 62 N and 56 N at magnetic bearing 2 (MB-2) in the $x$ and $y$ directions, respectively.

**Effect of weight shifting parameter $\beta$**

The first set of experiments involved changing the $\beta$ parameter manually to assess its effect on the individual cost functions. The cost functions were calculated from the steady state data by averaging 10 periods. Figure 7 shows the experimental results when $\beta$ was reduced from unity (vibration only control) to zero (transmitted only control) with increments of 0.1. The influence of $\beta$ can clearly be seen when inspecting $J_Q$ and $J_T$ in Fig. 7(a). As expected transmitted forces are reduced at the expense of the overall vibration index with decreasing $\beta$. What is significant is that, due to the choice of the base scalings, $\beta$ provides effective control over the range. Choice of base weights by using Bryson’s rule does not provide the same functionality of $\beta$. The experimentally determined behavior of vibration and transmitted force cost functions agrees with the simulated predictions [25]. The near linear characteristics of $J_Q$ and $J_T$ with $\beta$ make it possible to automatically tune $\beta$ in accordance with performance specifications. For example, if the objective is to minimise transmitted forces subject to a vibration index limit of 50 $\mu$m, then from Fig. 7(a) the optimum $\beta$ value is 0.2. This would result in a reduction of transmitted forces from 180 N to 28 N (i.e. about 85% reduction). If the acceptable vibration level is selected to be
above 60 \(\mu m\), the optimum \(\beta\) value is zero, resulting negligible transmitted forces. Figure 7(b) shows the maximum displacement at the magnetic bearing locations. Since \(\gamma = 1\) in these experiments, the index \(J_M\) is not used in the MO-ROLAC, but the overall vibration index \(J_Q\) is considered. Therefore the behavior of \(J_M\) is not relevant to the case studied in here.

Figure 8 shows the total synchronous controller effort, i.e. from MO-ROLAC plus PID, as a function of \(\beta\). It is important to note that due to integral action in ROLAC, the results would be the same even for different PID controller settings. Alternative interpretation is that MO-ROLAC achieves the minimum cost function by compensating for the PID contribution at the synchronous frequency. It can be observed that the total force amplitude increases with increasing vibration levels. At the extreme case of \(\beta = 0\), the total control effort should be equal and opposite to the inherent negative stiffness effect of the magnetic bearings to achieve zero transmitted force.

**Effect of weight shifting parameter \(\gamma\)**

The second set of experiments involved changing the \(\gamma\) parameter manually to assess its effect on vibrations at the magnetic bearing locations when \(\beta = 1\). This is important to prevent rotor contact with the auxiliary bearings if the rotor vibrations at the magnetic bearing locations exceed safe levels. If the vibration index cannot be reduced to an acceptable vibration level even with the upper limit of \(\beta = 1\), it is important to observe the index \(J_M\). If the maximum vibration level at the magnetic bearing (hence auxiliary bearing) locations reach a safety limit, then \(\gamma\) should be decreased from its default value of unity to keep the index \(J_M\) within the safety limits.

The experimental results shown in Fig. 9 demonstrate the effectiveness of \(\gamma\) in controlling \(J_M\). As shown in Fig. 9(b), a decrease of \(\gamma\) results in a gradual decrease in \(J_M\). For example, if the safety level at the magnetic bearings was 20 \(\mu m\), then the optimum value for \(\gamma\), from Fig. 9(b), is 0.5. Decreasing \(\gamma\) results in an increase of the overall vibration index. Therefore, although the vibration at the magnetic bearing locations is reduced, the vibration at other sensor locations is increased significantly. It is even possible to pin the rotor at the magnetic bearings, as predicted in simulations [25], with the lower limit of \(\gamma = 0\). The total control force levels are shown in Fig. 10.

The results presented here confirm the theoretical predictions presented in [25], and provide a basis for real-time implementation of the adaptive tuning of the two weight adjustment parameters \(\beta\) and \(\gamma\).

**CONCLUSIONS**

The application of magnetic bearings to control rotor vibrations has been reported in numerous publications. In some situations however it is important to limit the force transmitted to the foundations: this case has not been given such extensive consideration.

The authors have developed a multi-objective optimisation method, based upon a recursive open-loop adaptive control strat-
ergy, to simultaneously account for rotor vibrations and transmitted forces. This includes consideration of the need to prevent rotor contact with auxiliary bearings. The experimental results presented demonstrate the effectiveness of the two-stage weighting strategy employed in the controller, in particular it is shown that a single adjustable parameter $\beta$, is effective in changing the balance between rotor vibration and transmitted forces in accordance with design specifications. It is also shown that a second adjustable parameter $\gamma$, can be used to reduce rotor vibrations at auxiliary bearing locations at the expense of vibrations at other locations. This is significant in preventing potential rotor contact with the auxiliary bearings.

ACKNOWLEDGMENT

The authors acknowledge with thanks the support of the Engineering and Physical Sciences Research Council of the UK under the Platform Grant GR/S64448/01.

REFERENCES


Copyright © 2009 by ASME