University of Bath

DEPARTMENT OF COMPUTER SCIENCE EXAMINATION

CM30070: Computer Algebra

No calculators may be brought in and used.

Full marks will be given for correct answers to THREE questions.
Only the best three answers will contribute towards the assessment.

Examiners will attach importance to the number of
well-answered questions.

1. What are meant by ‘normal’ and ‘canonical’ representations? [2]

What is meant by ‘simplification’ in computer algebra (e.g. in Maple), and how does this relate to normal and canonical representations? [4]

For each of the following polynomial representations, outline a representation of $(x^2y^2+1)(xy-1)-x^3y^3$ (which may of course look nothing like this) and explain what choices need to be made to make this expression format normal and/or canonical (if possible). [14]

(i) Recursive sparse.

(ii) Distributed sparse.

(iii) Factored (you may choose any representation for the factors).

(iv) Straight-Line Program.

2. Give a computationally testable definition of what it means for a set of polynomials (with coefficients from a field) to be a Gröbner base, defining any terms you use which are specific to Gröbner base theory. [8]

Explain how to convert your test into an algorithm (proof of termination is *not* required) to produce a Gröbner base, generating the same ideal, from a set of polynomials. [8]

Given a Gröbner base of an ideal with respect to *some* ordering, what can you deduce from the leading monomials of this base? [4]

3. Suppose you have a procedure which, given two univariate polynomials f and g over the integers (i.e. in $\mathbf{Z}[x]$) and a *small* prime p, returns the greatest common divisor of $f \pmod p$ and $g \pmod p$. i.e. f and g viewed as polynomials modulo p. Outline **two** different methods you could use to compute the greatest common divisor of f and g over the integers, identifying any major common components of these solutions. Which would you choose, and why? [20]

4. Define the term "elementary function", as used in integration in finite terms. [2]

State Liouville's Theorem on integration in finite terms. [4]

Why is Liouville's Theorem so important for the theory of integration in computer algebra? [4]

Restricting ourselves to the integration of *transcendental* algebraic functions, state two developments of Liouville's Theorem that further help the construction of algorithms for integration in computer algebra. [4]

If we were to go beyond elementary functions, in the sense of wishing to integrate elementary functions but with a wider class of answers, what sort of extra theorems would we need in order to have a satisfactory integration theory? [6]

You may use the dilogarithm function, defined by $\mathrm{dilog}(x) = \int \frac{\log x}{1-x}$, as your example of a wider class of functions, but you are *not* expected to know any dilog-specific results.