CM30070 Coursework: set 22 October 2009

There are four questions: each worth the same marks except for question 4 which is worth double. The coursework as a whole is worth 25% of the module. I estimate that this should take approximately 16–20 hours of individual study time. I may be consulted if you have any Maple queries. I will provide individual grades and send a collective e-mail describing the answers and common errors after Christmas, but hope to give feedback on the first two questions by the end of November. The deadline for handing in is **Monday 16th November** for questions 1 and 2, and **Monday 14th December** for questions 3 and 4, at 12.00 in both cases. Submission consists of two components:

A. Hand in at the Department of Computer Science Office of paper answers, with the appropriate cover sheet.

B. Electronic submission of four Maple worksheets, one for each question. The sheets should be named `userid-1.mws` etc., so that masjhd's answer to question 3 would be called `masjhd-3.mws`. The worksheets should be verified in a *fresh* instance of Maple 13 before being submitted. Details on the elctronic submission will be provided on Moodle.

When I ask you, say, to run Bareiss's algorithm, it is up to you whether you program it completely, run it by hand, or use a mixture of the two. However, what you do must be comprehensible to an outsider (such as JHD), so programs must be commented/self-explanatory, and sets of commands must have explanations, e.g. against `tmp:=r12:r12:=r13:r13:=tmp:` I would expect a comment such as `#swap 12 and 13 since a(12,12)=0`.

Questions overleaf.

Questions

1. Build (you may use built-in Maple commands to do this) the Sylvester matrix corresponding to the polynumials in the sub-resultant lecture, viz.

$$A(x) = x^8 + x^6 - 3x^4 - 3x^3 + 8x^2 + 2x - 5;$$
$$B(x) = 3x^6 + 5x^4 - 4x^2 - 9x + 21.$$

   Run Bareiss's algorithm on this to compute the determinant. Hand in, on paper, your answer to the determinant. Note that you have to deal with the case of zero elements on the diagonal. **Be careful with the sign.**

2. Run the sub-resultant algorithm on the following two polynomials.

$$f := (y-1)\,x^4 + (y^2-1)\,x^3 + (y^3-1)\,x^2 + (y^4-1)\,x + y^5 - 1;$$
$$g := (y-1)\,x^5 + (y^2-1)\,x^4 + (y^3-1)\,x^3 + (y^4-1)\,x^2 + (y^5-1)\,x + y^6 - 1$$

   They are also stored in the sheet `~ masjhd/CM30070/Q2.mws`, and should be regarded as polynomials in x whose coeffcients are polynomials in y. What do you get as the last term? What is the true greatest common divisor of these two polynomials?

3. Emulate the Buchberger algorithm on cyclic–3: viz: $a + b + c$; $ab + bc + ca$; $abc - 1$. By this I mean that each S-polynomial should be computed and reduced under human control, i.e. the most sophisticated MAPLE commands you can use are of the form `S:=28x*f1-3*y*f2` or S:=S-7*z*f3. Commands from the Groebner package such as `LeadingMonomial` are not allowed. You may use any order you wish, but should state it clearly. How many S-polynomials do you compute? How many reduce to zero?

4. Compute, via the Faugère–Gianni–Lazard–Mora algorithm, a purely lexicographical Gröbner base for the cyclic–5 problem: viz.

$$a + b + c + d + e$$
$$ab + bc + cd + de + ea$$
$$abc + bcd + cde + dea + eab$$
$$abcd + bcde + cdea + deab + eabc$$
$$abcde - 1.$$

   Hence deduce the number of solutions and a description of them. A description might say e is a root of this polynomial. When e is a root of this (smaller) polynomial, then d is given in terms of e by this other polynomial, ...