ATLAS: Interactive and Educational Linear Algebra System Containing Non-Standard Methods

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Plan of Talk

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Introduction

- Numerous linear algebra teaching tools currently exist such as Maple, MATLAB and CoCalc
- But these tools primarily focus on the basic methods
- More advanced methods are absent from these tools
- Most of these tools also do not provide step-by-step solutions for problems
- This project aims to fill that gap, focusing specifically on methods like Strassen's fast matrix multiplication and other less common methods
- ATLAS aims to also provide the ability to compare the step-by-step solutions of methods simultaneously
- By providing non-standard methods and step-by-step solutions for all methods, users have a greater choice and they can see exactly how problems can be solved using the methods

Prior Work: see [PD21]

No work on on-standard methods.

Maple Maple has a feature called Tutor, allowing users to view the step-by-step solutions to a problem for linear algebra functions, such as calculation of eigenvalues [Map21].

MATLAB [Han09] studied whether MATLAB can supplement the learning of university students. Most students liked using MATLAB, with specific preference amongst lower achieving students as they liked getting an answer without working it out, which is not very helpful for their learning. Some students, however, did not like it because it did not show how it got to the answer and they wanted to see how the problem was solved step-by-step, otherwise it wouldn't help them.

CoCalc CoCalc [Ste18] is web-based linear algebra system, which makes it more portable than Maple or MATLAB. Cocal allows teachers to conduct their lessons entirely on CoCalc through an interactive platform. CoCalc acts more as an e-learning platform such as Moodle.

Pedagogical Example: Strassen–Winograd

- Very hard to motivate these.
- But matrix multiplication is bilinear, and these expressions are also bilinear (verification easy).
- Hence it suffices to verify

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$$

  (and three analogues)
- Tedious by hand, but easy with this tool.

Current State (1)

- ATLAS supports the calculation of determinants, inverses, eigenvectors and eigenvalues, in addition to matrix multiplication and solving systems of linear equations.
- For calculating determinants, Laplace expansion, Sarrus' method and LU decomposition are supported, allowing users to compare all 3 methods step-by-step simultaneously.
- This is also true of matrix multiplication, which supports the standard method, Strassen's method and the Laderman method, and calculation of inverses using both the Cramer's rule and the Cayley-Hamilton theorem.
- Systems of linear equations can be solved by Gaussian elimination, Cramer's rule and Cholesky decomposition, whilst being compared with each other simultaneously

Current State (2)

- Unit testing completed.
- Integration and user testing currently in progress.
- The next stage of development is the implementation of symbolic inputs, as ATLAS is currently limited to only numerical inputs.
- There is a desire to extend to comparisons of methods that are considered numerically good or bad to understand the effect of different methods on problems with floating point numbers.
- Also, there is an aspiration to improve the portability of ATLAS by creating a web-based equivalent, similar to CoCalc.

Demonstration

Demo Now we shall demonstrate some of the functionality of ATLAS

Q&A

? Any questions?

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