

Mulțumesc din suflet pentru acest titlu  
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SYmbolic and  
NuMeric  
Algorithms for  
Scientific  
CoMputation

**S**ymbolic [Algebra] and  
**N**umeric [Arithmetic]  
**A**lgorithms [Precise Methods] for  
**S**cientific [which includes Engineering]  
**C**omputation

Most people think of numbers [Arithmetic] when they think of computers.

“We may say most aptly that the Analytical Engine weaves algebraical patterns just as the Jacquard loom weaves flowers and leaves” — Ada, Countess Lovelace [Ada43].

The first theses in computer algebra date back to 1953 — a very good year.

The International Mathematical Olympiad, which started in Romania in 1959, separates Algebra from Arithmetic (Number Theory), but real life, and this conference, are not so binary.

The real distinction is that arithmetic deals with particular numbers, and algebra with generalities: so  $3^2 + 4^2 = 5^2$  is a statement about a particular right-angled

Computing, whether symbolic or numeric, requires precision of statement.

**Mathematician** local: “in a suitably chosen open set”

**Computer** global definition



Computers do not have a “choose context” instruction.

This can lead to problems where different open subsets might give incompatible results, and even paradoxes. Hence our Paris work, published in **SYNASC** [CDKS12]. The algebra of contexts is necessary to make sure the arithmetic is correct.

I am a happy computer science professor. When I flew here, I trusted my life to a University of Bath spinout company, and to my former students who work there. This company is responsible for the software of the UK's National Air Traffic Services: over a million hours without a software fault.

- At **SYNASC** 2016, I learned about Line 14 of the Paris Métro, which by now has operated for twenty years without a software error.
- Crash-proof software for heart pacemakers [And17]



But 500,000 Americans had their pacemakers recalled for security reasons [The17]

# Surely this takes a great deal of testing?

Testing is like **arithmetic**: a specific case. “All cases that might arise in 20 years” is a tall order: both numerically, and would you believe such a list?

What is needed is **algebra**, or what people call “formal methods” in software. Very pleased to see (and speak at) FROM 2019, the Working Formal Methods Symposium 2019, just held in conjunction with **SYNASC** 2019.

One of the aims of our recent “**Symbolic Computation** and Satisfiability Checking” (SC<sup>2</sup>) project [ABB<sup>+</sup>16, DEG<sup>+</sup>19] (SC<sup>2</sup> 2016 was with **SYNASC** 2016) was to extend the scope of formal methods from Boolean logic (“There is no train in the next block”) [Coo42] to numerical statements (“the next train is 6km ahead travelling at 150km/h”).

Converting this from pure science to engineering proven software might make our railway tracks 30% more efficient!

**SYNASC** now has a flourishing Artificial Intelligence track. Artificial Intelligence is much in the news currently. But most commentators are referring to one particular type of AI — Machine Learning (really “Pattern Recognition”): multi-layer neural nets trained on big data to recognize patterns. This is the same sort of stimulus/response functionality that our right brain hemispheres carry out. But [Len19], there is also the work that our left brain hemispheres do.



# Artificial Intelligence Compared

(In Mathematics, of course)

**Right Brain** Deepmind (Google) looked at GCSE, an examination taken by all 16-year olds in England, with Machine Learning (ML). It got the worst form of failure, achieved by less than the bottom 10% of English children [New19, SGHK19].

**Left Brain** Todai Robot project [AMIA14] was studying entrance to Japanese universities, especially in mathematics.

- ① Uses ML to read the exam paper,
  - ② ML with linguistics to understand the text,
  - ③ domain-specific reasoners,
  - ④ complex program ML with linguistics to “write” the answer.
- ! Todai Robot couldn't quite get into Tokyo University to study Mathematics, but could get into most other Japanese universities. Roughly Todai Robot was in the top 1%, but not the top 0.1%, of Japanese 17 year olds.

The most successful domain-specific reasoner, in the sense of the one that solved the most problems in the Tokyo University entrance examination, was Real Algebraic Geometry — precisely the subject of several **SYNASC** papers, and Erika Ábrahám's invited talk at **SYNASC** 2017.

There are other domain-specific reasoners in the Todai Robot project, building on various bits of Algebra and Arithmetic such as **SYNASC** represents.

The result of large amounts of computation on massive amounts of data, as the underlying **Numerical Algorithms** try to find distinguishing features. This is basically arithmetic, though one algebraic technique, automatic differentiation, is being used in places [BPRS18].

As such, while it is possible to say what such a Machine Learning system will do on a given piece of input, essentially a test, it is impossible with the current state of technology to make statements in general, essentially the equivalent of formal methods.



For Machine Learning to become truly acceptable, we will have to be able to apply **Symbolic Computation** to it, a topic of active research at Bath and elsewhere, and I hope to see results at future **SYNASCs**.

# No to Machine Learning (Currently)?

Q Does this mean I am glad that the company I mentioned doesn't use machine learning?




A Not at all, for it is used extensively in the process of generating the formal proofs that underpin the safety of their products.



Note that it is *not* used in validating them.

Just as a trained mathematician reaches for the right lemmas, by pattern matching and without examining every lemma ever learned to decide if it is the right one, machine learning has a great rôle to play in equipping formal methods with a similar activity [KBKU13].

Such an interplay between machine learning and algebra has also been part of the **SYNASC** programme [HEDP16].

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