

$$G3 := [a + b + c, a \cdot b + b \cdot c + c \cdot a, a \cdot b \cdot c - 1];$$

$$[a + b + c, a b + a c + b c, a b c - 1] \quad (1)$$

with(Groebner);

$$[\text{Basis}, \text{FGLM}, \text{HilbertDimension}, \text{HilbertPolynomial}, \text{HilbertSeries}, \text{Homogenize}, \text{InitialForm},$$

$$\text{InterReduce}, \text{IsBasis}, \text{IsProper}, \text{IsZeroDimensional}, \text{LeadingCoefficient}, \text{LeadingMonomial},$$

$$\text{LeadingTerm}, \text{MatrixOrder}, \text{MaximalIndependentSet}, \text{MonomialOrder},$$

$$\text{MultiplicationMatrix}, \text{MultivariateCyclicVector}, \text{NormalForm}, \text{NormalSet},$$

$$\text{RationalUnivariateRepresentation}, \text{Reduce}, \text{RememberBasis}, \text{SPolynomial}, \text{Solve},$$

$$\text{SuggestVariableOrder}, \text{Support}, \text{TestOrder}, \text{ToricIdealBasis}, \text{TrailingTerm},$$

$$\text{UnivariatePolynomial}, \text{Walk}, \text{WeightedDegree}] \quad (2)$$

$$\text{Basis}(G3, \text{tdeg}(a, b, c));$$

$$[a + b + c, b^2 + b c + c^2, c^3 - 1] \quad (3)$$

$$\text{Basis}(G3, \text{plex}(a, b, c));$$

$$[c^3 - 1, b^2 + b c + c^2, a + b + c] \quad (4)$$

# From which the solutions are obvious:  $c$  satisfies a cubic,  $b$  satisfies a quadratic (in  $c$ ) and  $a$  is then determined