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Efficiency measures this

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$E_p > 1$ indicate superlinear speedup: we are using more than 100% of the processors!

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Efficiency

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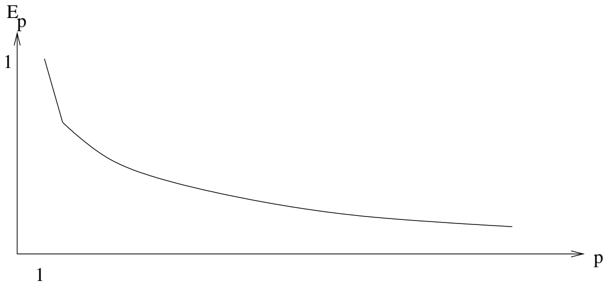
When $E_p < 1$ this indicates that somewhere at some point a processor not working on the computation. Perhaps it is occupied in communication; or possibly just lying idle waiting

Analysis

Efficiency

Typical efficiency graph on a fixed size problem:



Efficiency graph dropoff

Analysis

Speedup and Efficiency

As an example of calculating speedup and efficiency we consider a pipeline (systolic array)

Systolic array

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Data moves from one processor to the next being transformed at each stage: we assume one time step per transform

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This could equally be a CPU instruction pipeline

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A p-stage pipeline will take p time steps to fill; after that it produces one result per time step

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A sequential system will take np time steps to do the p steps on the n computations

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The speedup is

$$S_p = \frac{np}{p+n-1} = \frac{p}{(p-1)/n+1}$$

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A p-stage pipeline has a speedup is less than p, but that gets closer to p as time progresses

Also, the speedup starts low (for $n = 1$, $S_p = p/(p+1-1) = 1$) and increases over time, getting closer and closer to p

Analysis

Speedup and Efficiency

The efficiency is

$$E_p = \frac{S_p}{p} = \frac{n}{p+n-1} = \frac{1}{(p-1)/n+1}$$

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To keep high efficiency we need to avoid this: thus the complications in the designs of modern processors that are aimed at keeping the pipeline full

(Things like speculative evaluation and branch prediction, using many transistors. . .)

Analysis

Other measures

Speedup and Efficiency are simple, but useful measures of a parallel system, as long as you take care over using them

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Exercise Some people use the phrase “negative speedup” rather than “slowdown”. Why is that incorrect?

Analysis

Karp-Flatt

Sometimes people use the *Karp-Flatt metric* as a measure of an implementation to see how well it is doing

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$$e = \frac{\frac{1}{S_p} - \frac{1}{p}}{1 - \frac{1}{p}}$$

where S_p is the measured speedup and p the number of processors

Analysis

Karp-Flatt

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(If we have superlinear speedup, $S_p > p$, and $e < 0$)

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(If we have superlinear speedup, $S_p > p$, and $e < 0$)

Exercise Calculate Karp-Flatt for the pipeline. What does it tell us?

Analysis

Karp-Flatt

Note that Karp-Flatt will give you an estimate for the sequential time *for a given implementation*

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After all, you might just have a poor implementation

A big Karp-Flatt value is often an indication you need to re-think your code

Analysis

Work Efficient

Next: a parallel algorithm is *work efficient* (*cost efficient*) if the number of operations it performs is no more than the sequential algorithm

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The *parallel overhead* is

$$T_o = pT_p - T_s$$

where T_s is the sequential time and T_p is the parallel time

Analysis

Work Efficient

This measures the amount of extra work we are doing to get the parallelism

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Exercise Calculate the parallel overhead for the pipeline. What does it tell us?

Another question is “how scalable is this algorithm?”

Analysis

Isoefficiency

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Here we ask for a relationship between p, the number of processors and n the size of the problem for a given efficiency

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Here we ask for a relationship between p, the number of processors and n the size of the problem for a given efficiency

If we increase p, how much to we have to increase n to maintain a given efficiency?

Increasing p will generally decrease efficiency (Amdahl)

Analysis

Isoefficiency

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A poorly scalable algorithm will need to increase n a lot to maintain efficiency as we increase p

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This relationship is called the *isoefficiency*, and expresses n as a function of p

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A poorly scalable algorithm will need to increase n a lot to maintain efficiency as we increase p

This relationship is called the *isoefficiency*, and expresses n as a function of p

It quantifies the balance between Amdahl and Gustafson

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Isoefficiency

Computing the isoefficiency can be a bit fiddly, but often it is easiest to start by looking at the parallel overhead

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We have efficiency $E = T_s/pT_p$ and overhead $T_o = pT_p - T_s$. Combining these:

$$E = \frac{T_s}{p\left(\frac{T_o+T_s}{p}\right)} = \frac{T_s}{T_o + T_s} = \frac{1}{1 + T_o/T_s}$$

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So to keep E constant, we need to keep T_o/T_s constant

Analysis

Isoefficiency

So we must have

$$T_s = cT_o$$

for some constant c

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As both T_s and T_o depend on n and p, this equation generally gives us enough to solve for n in terms of p

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Isoefficiency

Example. The p-stage pipeline had efficiency

$$E = n/(p + n - 1)$$

on a problem of size n

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$$T_o = pT_p - T_s = p(p + n - 1) - np = p^2 - p$$

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The overhead

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This fixed overhead again tells us it is a good idea to keep the pipeline full!

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We want $T_s = cT_o$ which is

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$$n = c(p - 1)$$

Thus the isoefficiency is

$$n = O(p)$$

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Isoefficiency

This is linear in p: if we double p we need only double n to maintain efficiency

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So this tells us pipelines are very scalable

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Measures Conclusion

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Exercise Compute these measures for summing n numbers using p processors

Analysis

Bandwidth and Latency

While we are thinking about measurement of parallel systems we need to make a quick comment about *bandwidth* and *latency* as they play an important role in the way we regard communications overhead

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Bandwidth is the number of bytes per second transmitted over some medium

Latency is how long we have to wait for the data to arrive

Analysis

Bandwidth and Latency

Bandwidth is often mentioned in descriptions of things as it is easy to visualise (a rate of flow)

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Bandwidths these days are pretty high: Mb and Gb rates are common

Latencies of milliseconds may *seem* small, but relatively speaking they are the big problem

Analysis

Bandwidth and Latency

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Fast, but processors are faster! Data might arrive at a prodigious rate when it does arrive, but a processor could do a lot of work while it was waiting for the first byte to arrive

This is why processors have lots of complex and clever caching to avoid going off-chip

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This is how nodes in a cluster are often connected

Again we are in the range of hundreds of thousands of instructions while waiting

And this does not include the CPU overhead of going through the Operating System to send and receive the packets from the network

The latency affects coding strongly: it may be worthwhile doing duplicate computations if that is faster than fetching a value

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In large parallel systems compute power is cheap and plentiful, but communications are slow and expensive

This is why when we implement parallel code we really need to concentrate on the communications more than the computations

Analysis

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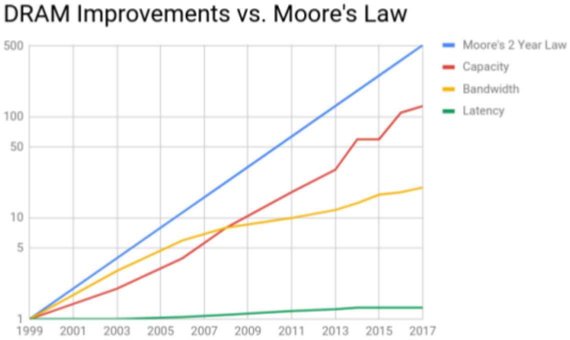
Exercise Read about how data was transmitted to generate the recent (2019) image of a black hole

Analysis

Note: Moore says sizes of RAM are increasing, but latencies are far behind



Sizes of RAM over time

Graph from Kevin K. Chang, PhD., CMU 2017

Analysis

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Particularly on distributed architectures

Shared Memory Systems

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We will look at simple programs that have multiple *threads of control*, i.e., parts of the process are running simultaneously on separate processors

Shared Memory Systems

Note: a single program might use several processes, and each process might contain several threads

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Separate processes have separate (virtual) memory address spaces (my memory location 42 is different from your memory location 42)

Multiple threads in the same process (generally) share the same (virtual) address space (my memory location 42 is the same as your memory location 42)

Here we consider the shared part, i.e., threads within a process