Ferroelectrics, 389:95–106, 2009

ISSN: 0015-0193 print / 1563-5112 online
DOI: 10.1080/00150190902988176

Finite Element and Experimental Analysis of the Vibration Response of Radially Poled Piezoceramic Cylinders

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The harmonic response and modal shapes of axially-symmetrical piezoceramic cylinders (tubes) polarised through the wall thickness have been predicted by finite element methods and determined experimentally. Analysis of ceramic cylinders has concentrated on the effects of the variation of diameter to thickness (d/t) ratios, and change in cylinder length (l). Investigation has taken into account material variance and vibration performance with relation to both 'hard' and 'soft' type ceramics. Computational finite element modelling (ANSYS) and numerical techniques has allowed for the prediction of the harmonic response and modal shapes, thus enabling the choice of cylinder geometry and performance. Resonant frequencies of piezoceramic cylinders have been determined experimentally by impedance analysis. The changes in resonant frequencies have been determined for a range of d/t and l/d ratios and for a variety cylinder of lengths. Predictions of harmonic response of the piezoceramic cylinders are shown to agree well with experimental results, with identification of the modal shapes.

Keywords Piezoelectric; cylinder; resonance

Introduction

Piezoelectric ceramic structures are produced in a range of geometries and are extensively used in a wide range of applications due to their ease of configuration and performance. Applications of piezoceramics include sensors, actuators and non-destructive technologies [1–3] and the materials may be excited to produce a vibrational response. The resonant mode and frequency of a piezoceramic structure is a function of its geometry and material properties. The resonant response and the resulting modal shapes of the structure can be used to identify the vibration characteristics of a piezoelectric ceramic.

The aim of this paper is to examine the vibrational response of radially poled piezoelectric cylinders. Knowledge of the vibrational modal shapes and the influence of the material property and geometry are of interest for the further development of novel devices and selection of the appropriate tube geometry for specific applications [3–5]. Radially poled piezoceramic tubes allow for greater length structures, lower driving voltages and lower

Received August 26, 2008.
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polarisation fields and are produced in both 'soft' or 'hard' materials types. Applications of piezoelectric cylinders include micro-positioning devices, fibre optics, hydrophonics and scanning microscopy.

The vibrational response of a piezoceramic device may be characterised by its harmonic response and resulting modal shapes. The former can be experimentally investigated with an impedance analysis over a swept frequency range [3–7]. Both computational modelling and experimental investigations are used in this paper to characterise the vibrational response of piezoelectric structures.

Earlier investigations into the forced vibration response of cylindrical piezoceramic structures have included disks, hollow cylinders or tubes, and rods [3–11]. Characterisation of these structures have predominantly involved a through thickness electrode pattern, thus resulting in a thickness polarisation [3, 5, 7, 8]. Analysis of hollow cylinder piezoceramic devices have been investigated with axis-symmetric models and numerical methods to establish vibration characteristics, of which radial direction polarisation has been included [6, 9–13].

Due to the piezoelectric effect, structures may be excited to produce mechanical vibration and the behaviour of these devices is found to be dictated by the geometry, forced electrical excitation and material properties [5–8]. A forced vibration of an object close to its natural frequency is known as the resonance frequency, and is defined for simple structures as:

$$f_r = \frac{C}{2\pi L} \tag{1}$$

where, f_r is the resonance frequency, L is the length of the device and C is the speed of sound. The speed of sound of a material is found to be dependant on its stiffness, therefore an increase in stiffness is seen to increase the resonance frequency. Similarly, a decrease in length will also increase the resonance frequency, due to resonance occurring when the length equals an integral of a half wavelength [14]. Tubular structures have effectively three lengths (longitudinal, radial and torsional), thus three vibration frequency groups are associated with such structures [12].

This paper investigates the resonant behaviour of radially poled piezoceramic tubes to allow a greater understanding into the dimension ratio concerning the interaction between the three resonating modes. Greater knowledge in the relationship between tube dimensions allow for improved material and structure selection with regards to resonating frequencies and structure stability. As stiffness is found to increase from 'soft' to 'hard' type piezoceramics [15, 16], its effects can be further explored with comparisons of the two materials, and by the anisotropic properties of a radial polarised device. In addition to investigating the resonance frequency, observation of the modal shapes will highlight the fundamental mode and displacement behaviour of the vibrating structure, which can be of interest for specific applications.

Analysis

Investigations using computational finite element methods (FEM) have successfully analysed the behaviour of piezoceramic discs [3, 4, 7]. The versatility of such methods allows for the analysis of a large number of dimensional variations (diameter, length and thickness) and its effects on the vibrational characteristics of the structures. Unsymmetrical modes of a piezoelectric device are observed to be insignificant when symmetrical electrodes allow for symmetrical resonating modes [4, 7]. Therefore, successful harmonic analysis of

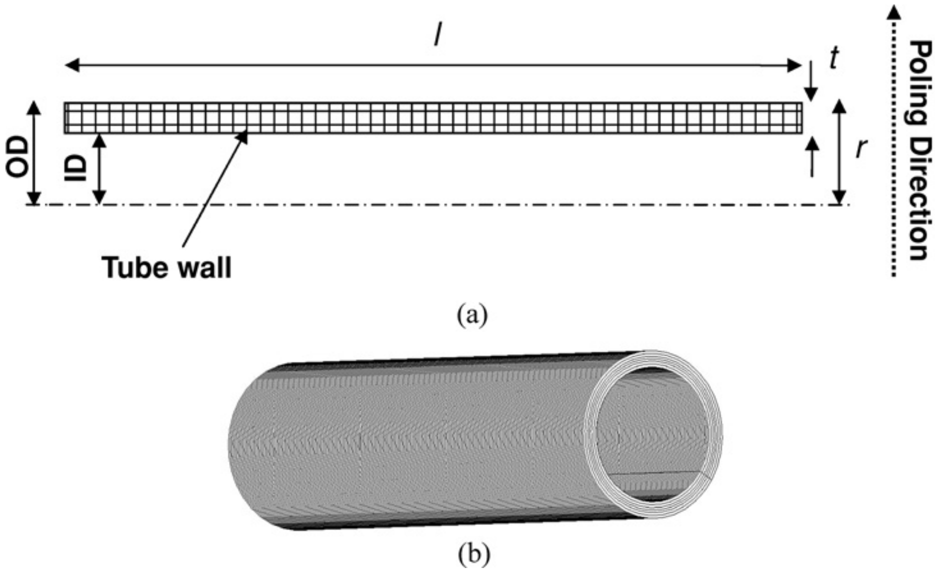


Figure 1. (a) Axis-symmetric model of a piezoceramic tube poled through wall thickness (radial). Diagram illustrate inside diameter (ID), outside diameter (OD), length (l), radius (r), and wall thickness (t). And, (b) fully expanded axis-symmetric model of tube in ANSYS 11.0.

piezoceramic structure can be performed with a simplified 2-D axis-symmetric model in the FEM program ANSYS 11.0. Modelling conducted used a four node 2-D coupled field element (PLANE13), capable of linear piezoelectric effect, with the model geometry and poling direction illustrated in Fig. 1.

Accuracy in defining length, diameter and wall thickness resonant frequency requires a fine mesh. Conversion of material matrices for 2-D analysis in ANSYS allowed for the polarisation characteristics of the tube to be directed through the wall thickness (Fig. 1). Thus giving the following 2-D material matrices:

$$[c_{ij}] = \begin{bmatrix} c_{33} & c_{13} & c_{13} & 0 \\ c_{13} & c_{11} & c_{12} & 0 \\ c_{13} & c_{12} & c_{11} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix}, \tag{2}$$

$$e_{ij} = \begin{bmatrix} 0 & e_{13} \\ 0 & e_{33} \\ 0 & e_{13} \\ e_{15} & 0 \end{bmatrix} \tag{3}$$

where c_{ij} is the piezoelectric stiffness matrix and e_{ij} is the piezoelectric matrix.

Harmonic response analysis of a hollow tube (Fig. 1) was carried out over a frequency (f) range of 1 kHz to 100 MHz with a voltage (V) of 500 mV and 0V applied to the outer and inner electrode respectively. From the nodal charge, Q, on the inner and outer electrodes, impedance (Z) can be determined by the following:

$$Z = \frac{V}{2\pi f Q} \tag{4}$$

Table 1
Material properties of material investigated experimentally (a and b), and material used in ANSYS model (c and d), (Sensor Technology, Canada and Morgans Electroceramics, UK)

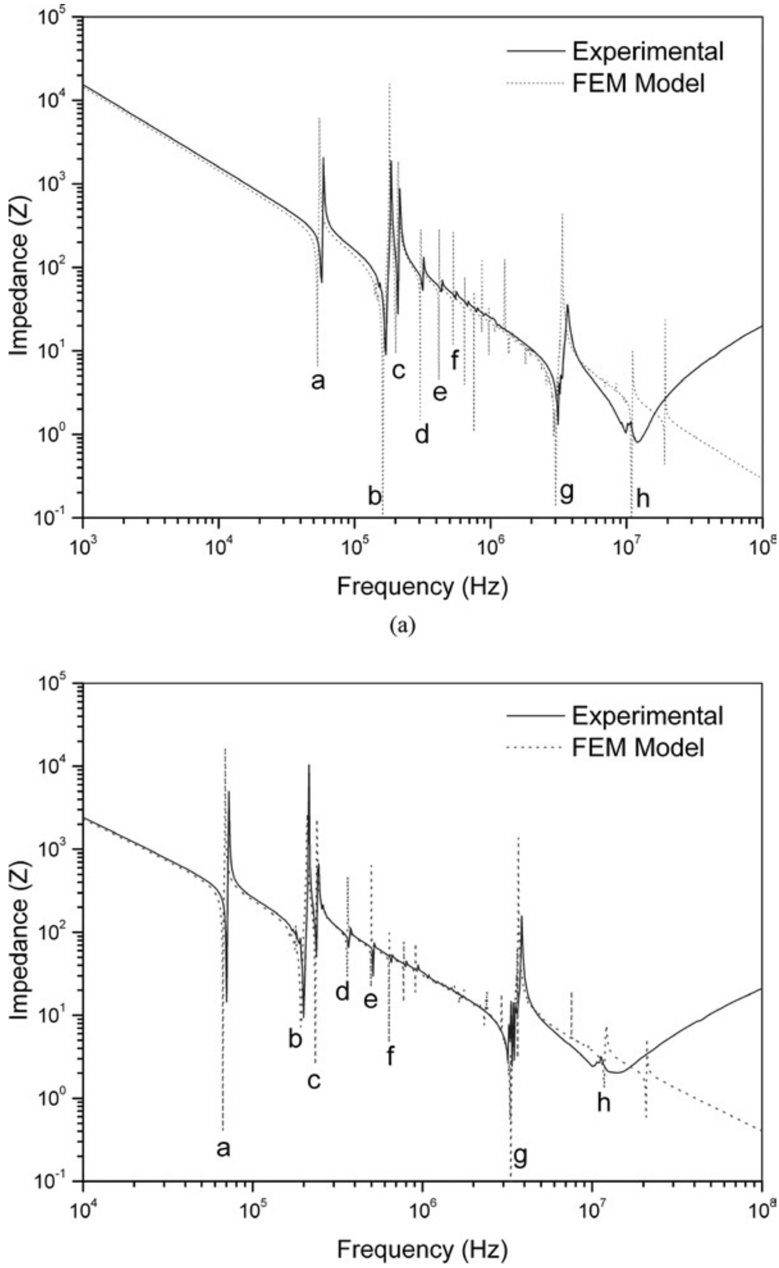
	BM500 (a)	BM800 (b)	PZT-5A (c)	PZT-8 (d)
d_{31} (10^{-12}m/V)	−175	−60	−171	−37
d_{33} (10^{-12}m/V)	365	225	374	225
S_{11}^E	15.5	11.0		
S_{33}^E	19.0	13.5		
C_{11}^E			12.1	14.9
C_{33}^E			11.1	13.2
ρ (10^3 g/m^3)	7.65	7.6	7.75	7.6
Q_M	80	1000	75	1000

Verification of finite element model was performed with comparisons of experimental measurements of both ‘hard’ and ‘soft’ type piezoelectric materials. Finite element analysis investigated the dynamic response of hard PZT-8 and soft PZT-5A (Table 1) and tubes of similar material (BM800 and BM500) were sourced and tested for comparison with the modelling work.

The modelling initially investigated the impedance response of piezoceramic tubes of the industrial standard dimensions of 25.4 mm$\times$ 6.6 mm$\times$ 5.3 mm ($l\times$ $od\times$ id), which were analysed with an Agilent 4294A precision impedance analyser. Such tubes are used in applications such as scanning microscopy, micro-positioning devices and vibration detection. Experimentation also allowed for the investigation into the effects of change of length on the impedance response of ‘hard’ BM800 and ‘soft’ BM500 materials supplied by Sensor Technology, Canada. This accommodated for validation of the FEM model of harmonic response with regards to geometrical variation in piezoceramic tubes. An initial tube length of 25.4 mm to $\sim$ 17 mm was investigated for this study and tubes were gradually reduced in length by a diamond wire saw to provide a range of tube geometries.

Table 2
Evaluation of resonating frequencies of experimental data in comparison to computational FEM model of PZT tubes of similar material properties (PZT-5A and PZT-8).

	Soft PZT		Hard PZT	
	Experiment	FEM	Experiment	FEM
(a)	57304	53200	70211	66700
(b)	170143	161200	198555	193600
(c)	208468	200800	237550	233200
(d)	317534	303400	367120	361000
(e)	436947	416800	512564	497800
(f)	584069	530200	655961	636400
(g)	3143881	3016000	3189832	3322000
(h)	9892446	10900000	10483605	11800000



(a)

(b)

Figure 2. Comparisons of experimental and FEM model of 'soft' and 'hard' type PZT tubes, (i) and (ii) respectively, for tube with dimensions 25.4 mm × 6.6 mm× 5.3 mm ($l\times$ *od* × *id*). Resonant frequencies of interest are highlighted from a to h.

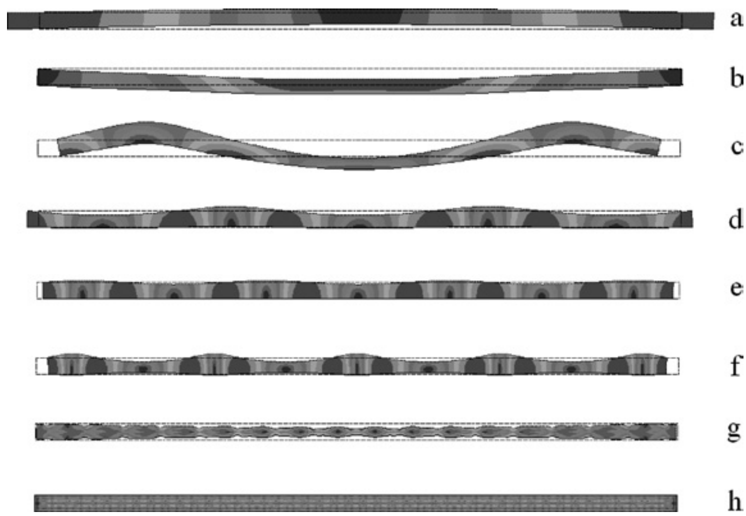


Figure 3. Modal shapes generated from computational analysis of 'hard' PZT type material (PZT-8), as related to analysis in Fig. 3 (ii), not to scale.

The understanding of the resonance characteristics of piezoceramic tubes can be investigated by visualisation the displacement generated in the structure at resonant frequencies which provides the resulting modal shapes [3, 7]. Vibrational modes calculated by the FEM modelling illustrate the modal shape associated with the resonant frequency and clarify the nature of the modes with relation to pure and coupled vibrations. This is of interest to device specific applications.

The three dependant factors to the resonating frequencies, length (l), diameter (d), and wall thickness (t) were investigated in the FEM model. Models were constructed that explored a change in l with constant d and t, change in d with constant an l and t, and change in t with a constant l and d. Three separate models for both the 'soft' and 'hard' materials were performed.

Results and Discussion

Figure 2a and 2b shows a comparison between experimental and FEM impedance analysis for both 'soft' and 'hard' type PZT material respectively. It is firstly seen that 'hard' PZT-8 materials resonate at higher frequency of that of 'soft' PZT-5A, as predicted due to the higher stiffness of the 'hard' material (see Table 1). Secondly, the FEM axis-symmetric model of the same dimensioned tube has produced a good relationship to that of the experimental measurements. This is also shown in Table 2 which compares the experimental and modelled resonant frequencies. The nomenclature of the resonant frequencies (a) to (h) in Fig. 2 and Table 2 will be referred to in subsequent discussions of this paper, since they are found to be resonant frequencies of interest.

Modal shapes of the resonant frequencies (a) to (h) highlighted in Fig. 2b have been visualised in Fig. 3 for the PZT-8 tube which illustrate the sum of displacement generated during vibration. The modal shapes in Fig. 3 allow for the relation between fundamental resonance frequency and its dimensional identity and harmonics. The modes (a) to (h) identified by the model and experimental measurements may be summarised as:

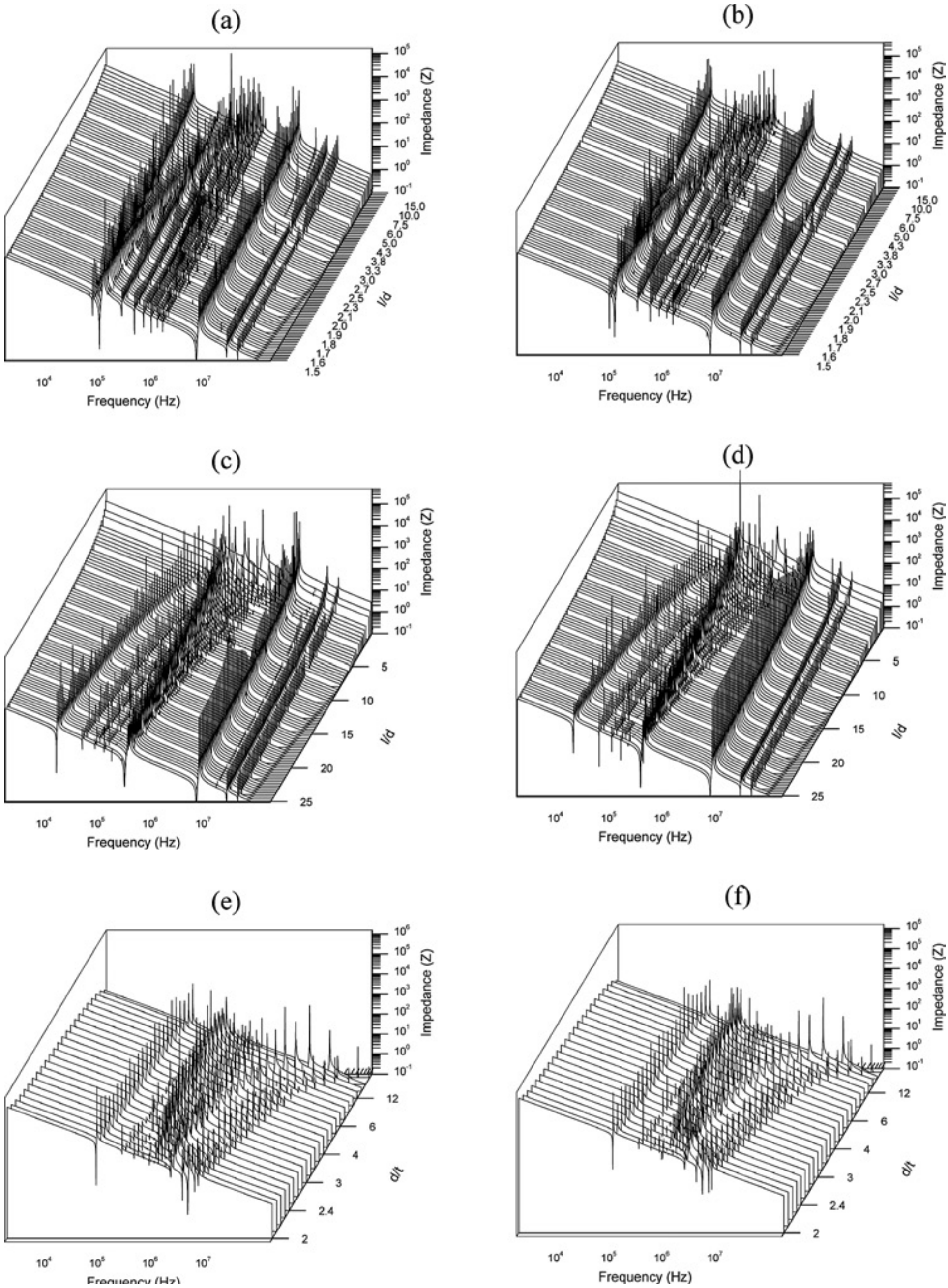


Figure 4. 3-D wall plots for FEM modelled impedance analysis for the change of diameter (a,b), length (c,d), and wall thickness (e,f) for both 'soft' (a,c,e) and hard (b,d,f) type piezoceramic tubes.

(a) First harmonic of the length mode,
(b) First circumferential mode coupling with length mode,
(c), (d), (e) and (f) third, fifth, seventh and ninth length modes,
(g) First harmonic of wall thickness mode, and,

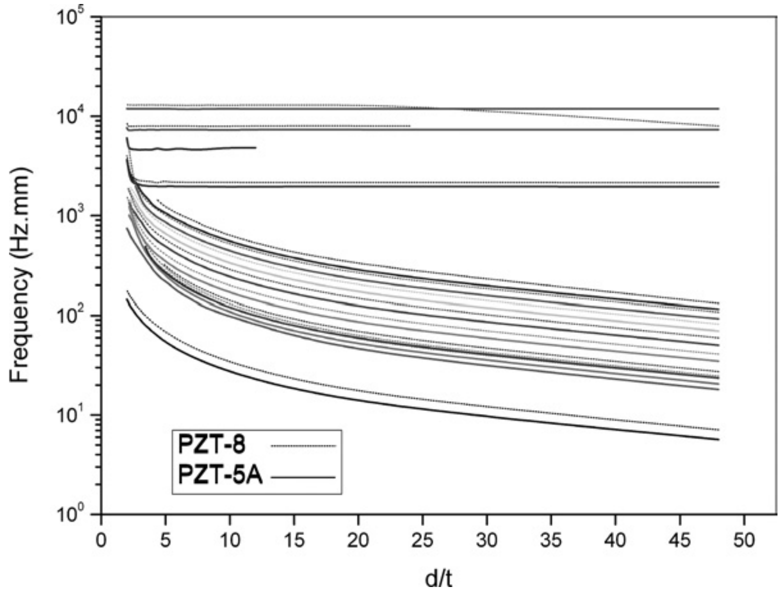


Figure 5. Frequencies of resonance of piezoceramic tube associated with change in wall thickness (t) for 'hard' type (dotted lines) and 'soft' type (solid lines) piezoceramics.

(h) Third harmonic of wall thickness mode.

Only odd harmonics (i.e. 1st, 3rd, 5th, 7th, 9th, etc) are observed from changes in impedance with frequency. Even harmonics are not observed due to their symmetrical displacement, leading to no piezoelectric charge.

Change of Tube Geometry

Further development into the analysis of resonance of piezoceramic tubes investigates the effects of the manipulation of l, d, and t. Analysis of the effects of geometry variation on the resonance is summarised in the three-dimensional charts in Fig. 4, which illustrates the behaviour of resonance with relation to variation in diameter (Fig. 4a,b), wall thickness (Fig. 4c,d) and length (Fig. 4e,f) for both PZT-5A and PZT-8 respectively. Resonant modes associated to the change in dimensions (l, d or t) are seen to change frequency, and as predicted, as the dimension size is decreased the resonant frequencies are increased. This further supports the conclusions established from the modal shape analysis in Fig. 3, as the associated resonance is seen to vary with relation between mode shape and dimension. For example, a decrease in tube length is clearly observed to lead to an increase in the first harmonic of length mode (a), see Figs. 4e, f.

Analysis of Resonant Modes

The data in Fig. 4 may be summarised by plotting the individual resonant modes as a function of the tube geometry (Fig. 5 to Fig. 7). Data from both 'hard' PZT-8 (dotted line) and 'soft' PZT-5A (solid line) are shown. By normalising the frequency data on the y-axis

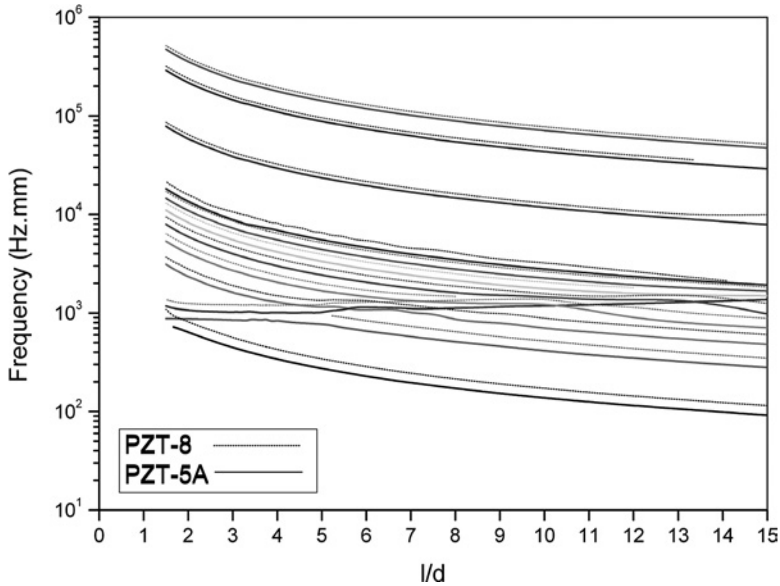


Figure 6. Frequencies of resonance of piezoceramic tube associated with change in diameter (d) for 'hard' type (dotted lines) and 'soft' type (solid lines) piezoceramics.

with respect to a specific tube geometry (e.g. thickness), the resonant mode associated with that geometry should be a horizontal straight line.

The resonant frequencies associated with wall thickness, or through thickness type resonant frequencies (e.g. mode (g) and (h)) are shown in Fig. 5 with the tube dimension ratio, d/t, plotted against frequency (Hz.mm), normalised by the wall thickness (t). The specific modes associated with through thickness resonances are presented as horizontal straight lines. These are seen to be unaffected by the circumference and length modes (i.e. no overlap) when $d/t > 2.5$.

Figure 6 shows the diameter resonant frequencies which are dictated by the diameter of the tube. The aspect ratio, l/d, is plotted against the frequency (Hz.mm) normalised by the length (l). The relatively straight horizontal lines observed correspond to the circumference resonant frequencies. Interaction with the length modes is observed, as the fundamental circumference harmonic is seen to transverse through length harmonics. Coupling with length modes produce the combined modal shapes visualised in Fig. 3(b) to (f).

Analysis of the resonant frequencies with respect to the changing length of the tube (e.g. mode a, c, d, e, f) is illustrated in Fig. 7. The dimension ratio, l/d, is plotted against the normalised frequency (Hz.mm) by its length (l). Resonant frequencies associated with length are presented as near straight horizontal lines. It is seen that the length has the largest collection of significant harmonics in comparison to the circumference and wall thickness. An aspect ratio of $l/d > 10$ is found to have three or more pure length modes without the interaction of circumference modes; in agreement with piezoelectric standards [17]. Resonant frequencies at the aspect ratio (l/d) of the experimental data (Fig. 2), alongside the identification of modes, (a) to (h), linked to modal shapes of Fig. 3 are shown in Fig. 7.

Figure 8 shows a detailed section of Fig. 7 of the aspect ratio range l/d from 2 to 4. Experimental data of a tube of varying length, constant diameter and wall thickness, is presented alongside computational FEM results for both 'hard' and 'soft' type piezoceram-

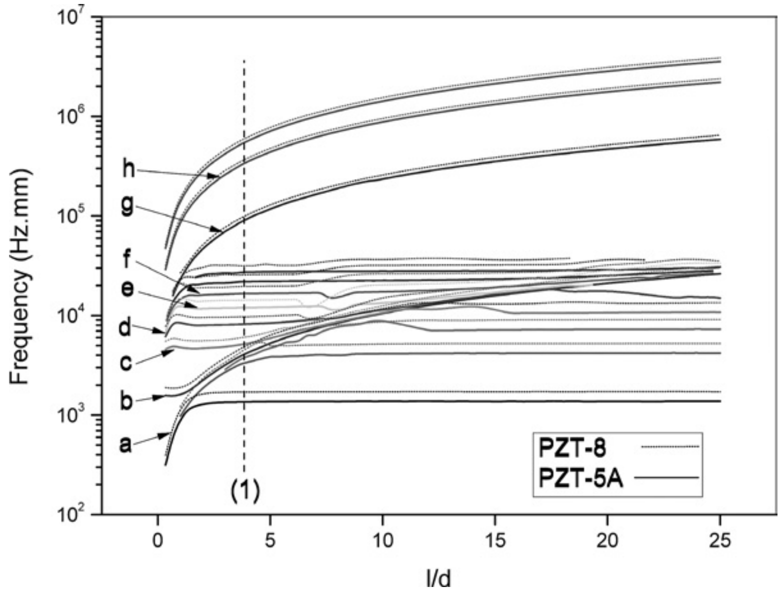


Figure 7. Frequencies of resonance of piezoceramic tube associated with change in length (l) for 'hard' type (dotted lines) and 'soft' type (solid lines) piezoceramics. Aspect ratio for experimental data is shown in (1) with respective modal shapes (a) to (h) illustrated.

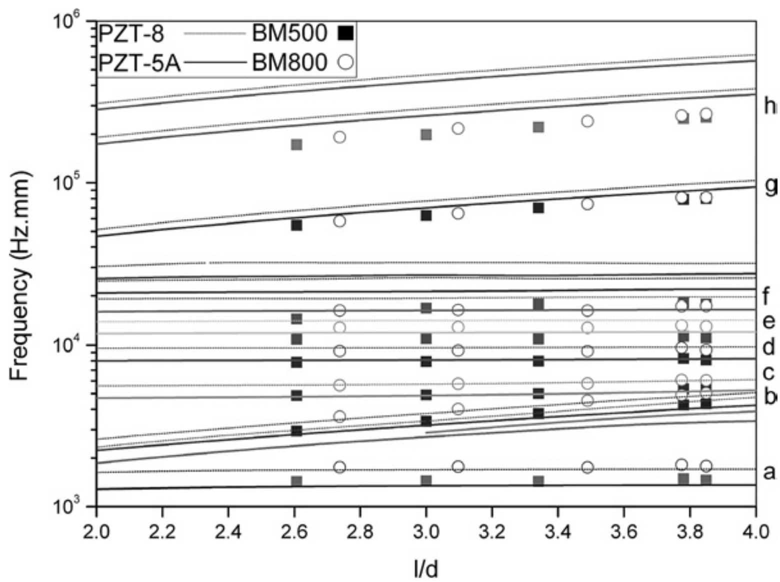


Figure 8. Detailed frequencies of resonance of piezoceramic tube associated with change in length (l) for 'hard' type (dotted lines) and 'soft' type (solid lines) piezoceramics. Comparisons with experimental data for 'hard' (solid square) and 'soft' (open circles) type piezoceramic tube are illustrated.

ics. It is found that the FEM model has excellent agreement to the experimental data of similar material groupings, with trends followed by both analysis approaches. Associated length frequencies are represented by a horizontal straight line, and the stiffer 'hard' type piezoceramics are seen to resonate at higher frequencies.

Conclusions

Computational 2-D axis-symmetric models have been successfully developed to predict the vibration characteristics of radial polarised piezoceramic tubes. Good correlation between model and experimental data has been observed with this modelling approach. The model offers advantages as the analysis of piezoceramic tubes may be conducted for a range of geometric variances and material properties, allowing for the optimisation of vibrating piezoelectric structures. Prediction of resultant modal shapes by the visualisation of displacement results in the understanding of the physical behaviour at selected resonant frequencies. Principally, the three modes associated with piezoceramic tubes [12] are identified by the axis-symmetric model, and are seen to deviate in relation to the affiliated dimension as found in the experimental investigation. In addition, by generating a range of dimension and material variances, the investigation into recognising resonating frequencies analytically allow for improved evaluation of cylindrical structures. Development of the FEM model can be to include the effects of clamping on the free ends and to take into consideration partially electroded piezoceramic cylinders, and its effects on its vibration behaviour. One limitation of the model is that only symmetrical modes can be predicted. Development of a 3-D solid tube would have the capability of predicting non-symmetrical modal shapes, and to investigate complex electrode patterns. This information presented is of interest for determination of the resonant frequencies of piezoelectric tubes of a wide variety of geometries.

Acknowledgments

The authors acknowledge Great Western Research and Ms. Wendy Lambson for their support.

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