



Modelling of piezoelectrically actuated bistable composites

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ABSTRACT

Bistable asymmetric composites with piezoelectric actuators that induce 'snap-through' from one stable state to another are candidate smart materials for shape-change (morphing) applications. This paper combines a homogenised piezoelectric model of a Macro Fibre Composite with a bistable asymmetric laminate model. Both predicted shape and snap-through voltage of a piezo-actuated $[0/90]_T$ laminate compare favourably with experimental results.

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1. Introduction

For aerospace applications the use of morphing surfaces can reduce drag [1], provide load alleviation and enable aerodynamic control [2]. Asymmetric bistable laminates have been proposed as a solution for shape changing components [3,4]. Bistable composites can maintain two significantly different shapes without a continuous energy input, requiring only actuation to initiate a transition between states. Piezoelectric materials, such as Macro Fibre Composites (MFCs) [3], have been used to induce 'snap-through' of the bistable laminate from one stable state to another. Fig. 1 shows 'State-A' and 'State-B' of a bistable carbon fibre reinforced plastic (CFRP) combined with an MFC. In this case the CFRP is an asymmetric $[0/90]_T$ laminate where an anisotropy of the thermal expansion leads to bistability.

The cured shape and snap-through behaviour of bistable laminates without integrated piezoelectric materials has been successfully investigated with analytical [5] and finite element (FE) techniques [6]. Attempts to predict piezoelectric-induced 'snap-through' of CFRP-MFC laminates have proven more challenging. Investigators [7] have approximated the piezoelectric strain using a thermal expansion; this is not ideal for combined sensing/actuation or for predicting the influence of electrical boundary conditions. To accurately model electromechanical coupling a coupled-field model should be used. Gude [8] created a homogenised MFC model and investigated the use of MFCs to induce snap-through in bistable laminates. Analytical and FE models were presented, but agreement with the experiment was poor. Dano [9] presented FE modelling of MFCs used to compensate for thermal deformation of asymmetric composite laminates. Agree-

ment with the experiment was good; however the work was not extended for 'snap-through' between stable states.

This paper presents a homogenised coupled-field model of an MFC and integrates it with a bistable laminate model. Both the cured shape and snap-through actuation of a piezoelectrically actuated bistable CFRP-MFC combination are modelled and compared to experimental measurements.

2. Macro Fibre Composite

An MFC consists of a series of thin piezoelectric rods with inter-digitated electrodes to apply an electric field along the rod length. Their advantages in this application are flexibility and damage tolerance. Developing a FE model which includes piezoelectric fibres and the complex electric field distribution due to the inter-digitated electrodes does not lend itself to integration with a laminate. Therefore, the active portion of the MFC was modelled as a homogenous coupled-field solid to create a functional representation of the actuator. The stiffness of the MFC was modelled using the four linear elastic engineering constants presented by the manufacturers (Smart Materials GmbH, Dresden) in Table 1 [10,11], derived from standard stress-strain relations [12]. The relationships between the piezoelectric d_{ij} coefficients of the constituent materials and the MFC is complex and is still the subject of research [13]. Williams [13] determined the free-strain of the MFC actuators used here. The d_{33} presented in [13] matches closely with the data of the manufacturer [11]; however no d_{31} is reported. Williams determined the d_{33} and d_{31} for an Active Fibre Composite whose construction is sufficiently similar to an MFC to assume an equal d_{33}/d_{31} ratio. The MFC piezoelectric constants are in Table 2. For the relative permittivity of the MFC layer, mixing rules by Deraemaeker [14] were used. These rules combine the permittivity of the piezoceramic and matrix

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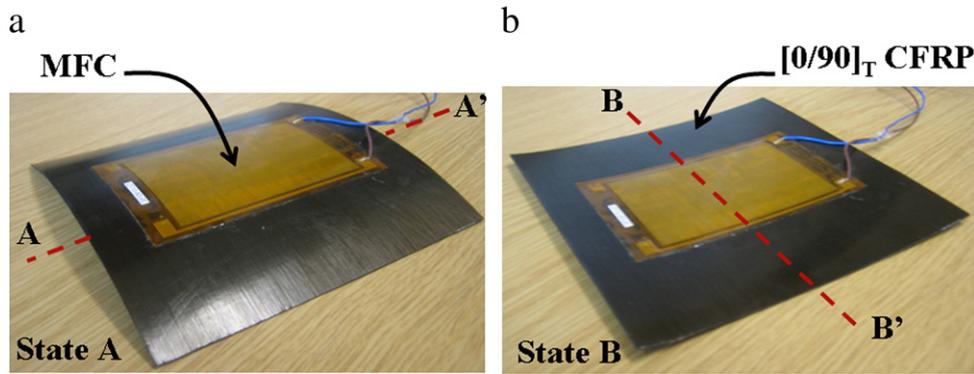


Fig. 1. MFC location on $[0/90]_T$ and curvature axis in (a) State-A and (b) State-B. Sample is 150×150 mm.

according to their volume fraction to determine the effective permittivity. Dielectric constants are in Table 2.

3. Experimental

A CFRP laminate with a $[0/90]_T$ cross-ply layup was manufactured using a T700/M21 pre-preg, cut and laid-up by hand. An autoclave cure cycle for the M21 resin system was used to cure the sample. Once cooled from the 180°C cure temperature the CFRP was bistable with significant curvature. Samples were trimmed to 150×150 mm. To bond the MFC to the CFRP laminate the surfaces were roughened and cleaned with acetone; a thin layer of two-part epoxy was applied to both MFC and laminate. The MFC (Smart Materials Corp M-8557P1) and the laminate were placed between two plates under a 200 N clamping force for 24 h to allow the epoxy to cure. After bonding, the $[0/90/0_{\text{MFC}}]_T$ laminate remained bistable (Fig. 1) but with reduced curvature due to the stiffness of the MFC. The final shape of the MFC-CFRP combination was determined by methods in [15]. To determine the piezoelectric induced snap-through voltage from State-B to State-A, the laminate was placed on a polished steel plate with the MFC on the uppermost surface. The snap-through voltage was established via a binary search in the range 0–1500 V with 5 V increments.

4. Model

Using the MFC (Tables 1 and 2) and CFRP (Table 3) materials properties, the $[0/90/0_{\text{MFC}}]_T$ laminate was modelled using FE to predict both cured shape and snap-through voltage. The 150×150 mm laminate was modelled as described by Giddings [6]

Table 1
Elastic properties of M8557-P1 MFC actuator.

	Ref. [10]	Ref. [11]
E_{33} (GPa)	29.4	30.336
E_{11} (GPa)	15.2	15.857
G_{31} (GPa)	6.06	5.515
ν_{31}	0.312	0.31
ν_{13}	0.16	0.16

Table 2
Piezoelectric constants $[d_{ij}]$ and relative permittivity of MFC.

M8557 MFC actuator	
d_{33} (pm/V)	467
d_{32} (pm/V)	−210
d_{31} (pm/V)	−210
ϵ_1^r	712
ϵ_2^r	12
ϵ_3^r	737

using 20-node layered solid elements with quadratic shape functions for elasticity and thermal responses. An MFC model measuring $85 \times 58 \times 0.3$ mm (the MFC dimensions in Fig. 1) was located centrally on the upper surface of the laminate; modelled by 20-node quadratic coupled field elements. All elements were equally sized quadrilaterals and the nodes on the adjacent surfaces of laminate and MFC were coincident. A 160 K temperature difference was applied to the composite to simulate post-cure cooling; no temperature change was applied to the MFC. With the model in State-B, piezoelectric actuation was modelled by applying a voltage constraint to the MFC. This constraint was applied as a ramp change to determine the ‘snap-through’ voltage from State-B to State-A. A geometric nonlinear analysis was carried out using ANSYS.

5. Results

Fig. 1 shows the two stable states of the $[0/90/0_{\text{MFC}}]_T$ laminate. Fig. 2 shows an interpolated surface plot of the shape measurement of the laminate in State-B. Predicted values by FE are overlaid on the plot as a mesh, showing very good agreement. The model captures the overall shape and the reduction in laminate curvature due to the additional bending stiffness of the MFC. Maximum deflection (out-of-plane displacement of the laminate midpoint) was predicted as 10.73 mm, 16.0% lower than the measured maximum deflection of 12.77 mm. FE prediction of snap-through voltage was achieved for the laminate, with nonlinear analyses predicting behaviour prior to snap-through. Snap-through of the laminate was predicted at 645 V, while experimental demonstrated 670 V induced snap-through.

6. Conclusions

This paper has developed a homogenised model of an MFC piezoelectric which was integrated with an asymmetric cross-ply laminate to predict both cured shape and snap-through behaviour. Overall shape prediction was good and the snap-through voltage for a

Table 3
Elastic properties of T700/M21 CFRP; ^acalculated using stress–strain relations in [12].

Property [unit]	T700/M21
E_1 [GPa]	148
E_2 & E_3 [GPa]	7.8
G_{12} & G_{23} [GPa]	3.8
G_{23}^s [GPa]	0.02
ν_{12} & ν_{13}	0.35
ν_{23}^a	0.01
α_1 [$1 \times 10^{-7} \text{ }^\circ\text{C}^{-1}$]	−0.9
α_2 & α_3 [$1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$]	3
E_r [GPa]	1.5
ν_r	0.4
Density [kgm^{-3}]	1072

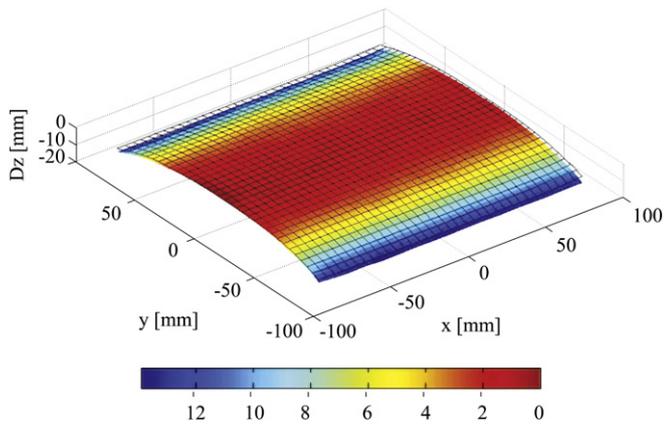


Fig. 2. Surface plot $[0/90/0_{MFC}]_T$ laminate in State-B with FE-predicted surface overlaid as mesh (laminate shown inverted for clarity).

$[0/90/0_{MFC}]_T$ laminate was predicted to within 4% error of experimental measurements. The modelling method is a successful approach for predicting the behaviour of bistable laminates.

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