Modeling and Characterization of Piezoelectrically Actuated Bistable Composites

Christopher Rhys Bowen, Peter F. Giddings, Aki I. T. Salo, and Hyunsun Alicia Kim

***Abstract*—This paper develops and validates a finite-element model to predict both the cured shape and snap-through of asymmetric bistable laminates actuated by piezoelectric macro fiber composites attached to the laminate. To fully describe piezoelectric actuation, the three-dimensional compliance $[s_{ij}]$, piezoelectric $[d_{ij}]$, and relative permittivity $[\varepsilon_{ij}]$ matrices were formulated for the macro fiber actuator. The deflection of an actuated isotropic aluminum beam was then modeled and compared with experimental measurements to validate the data. The model was then extended to bistable laminates actuated using macro fiber composites. Model results were compared with experimental measurements of laminate profile (shape) and snap-through voltage. The modeling approach is an important intermediate step toward enabling design of shape-changing structures based on bistable laminates.**

I. Introduction

For aerospace applications, the use of morphing surfaces and smart materials can reduce drag [1], provide load alleviation, and enable aerodynamic control [2]. Asymmetric bistable laminates have been proposed as a materials solution for morphing and shape changing components [3], [4]. Bistable composites can maintain two significantly different shapes without a continuous energy input, requiring only actuation to initiate a transition between states. Piezoelectric materials, such as macro fiber composites (MFCs) [3], have been used to induce snap-through bistable composites from one stable state to another. Fig. 1 shows the two stable states, State A and State B, of a bistable carbon fiber reinforced plastic (CFRP) combined with a piezoelectric MFC. The CFRP is an asymmetric $[0/90]_{\mathrm{T}}$ laminate for which an anisotropy of the coefficient of thermal expansion leads to a residual stress on cooling from the cure temperature and induces a curvature and the existence two stable equilibrium states.

The cured shape and snap-through behavior of bistable laminates without integrated piezoelectric materials have been investigated with analytical [5] and finite-element (FE) techniques [6]. Attempts to predict piezoelectric-induced snap-through of a combined CFRP-MFC composite from one stable state to another have proven more challenging because of the multi-physics nature of the problem. This paper presents a homogenized coupled multi-physics model of MFC-based piezoelectric actuation and integrates it with a bistable CFRP laminate model. After an initial review of existing work to date on bistable composite and piezoelectric actuation in Section II, the paper will develop the compliance $[s^T{}_{ij}]$, piezoelectric $[d_{ij}]$ and relative permittivity $[\varepsilon^T{}_{ij}]$ matrices of the MFC in Section III. The approach is then validated by modeling a simple isotropic beam under open and closed circuit conditions in Section IV. Finally, the model will be extended to include an MFC attached to a bistable laminate in Section V. Both the cured shape and snap-through actuation of the piezoelectrically actuated CFRP-MFC combination will be modeled and compared with experimental measurements.

II. Background

A. MFC Construction

The actuators used were from Smart Materials GmbH (Dresden, Germany) and consist of polycrystalline piezoelectric ceramic rods. Copper interdigitated electrodes are attached to the upper and lower surfaces to apply an electric field parallel to the rod length. The piezoelectric ceramic is a lead zirconate titanate (PZT) material, in this case PZT-5A because this is a soft PZT which exhibits high piezoelectric d_{33} and d_{31} coefficients, i.e., a high strain per unit electric field [7]. To maximize the strain per unit electric field, the poling direction of the PZT is aligned along the rod length using the interdigitated electrodes. This ensures that the actuation strain is generated via the d_{33} coefficient, which is typically twice the d_{31} coefficient [8], [9]. The electrode arrangement also improves damage tolerance; because the electric field is applied at regular intervals along the rod length, any damage or fracture of the rod or electrode merely reduces the functionality of a small region surrounding the defect and does not significantly reduce global actuator performance [9].

B. MFC Modeling Approaches and Bistable Actuation

Efforts to model the microscopic behavior of the constituent materials of the MFC are ongoing, with several workers using both FE [10], [11] and analytical techniques

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C. R. Bowen, P. F. Giddings, and H. A. Kim are with the Materials Research Centre, Department of Mechanical Engineering, University of Bath, Bath, UK.

A. I. T. Salo is with Sport and Exercise Science, University of Bath, Bath, UK.

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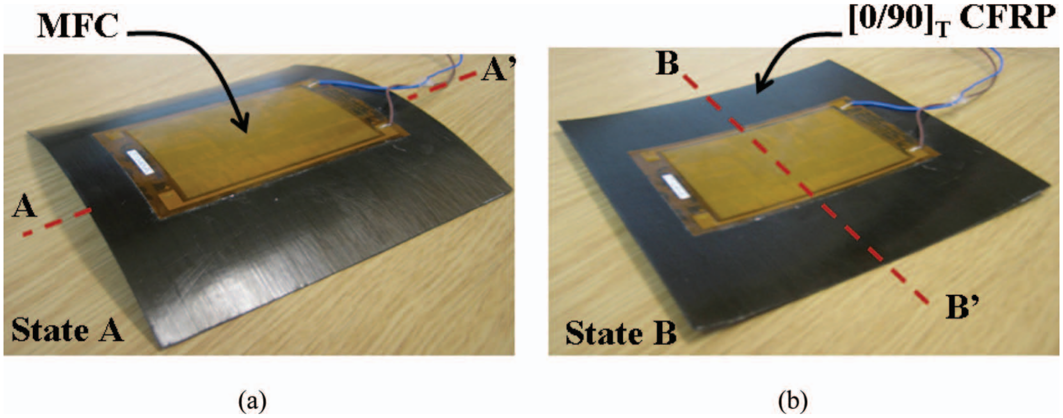

(a) (b)

Fig. 1. $[0/90/0_{\mathrm{MFC}}]_T$ laminate in (a) State A and (b) State B, with axis of curvature shown as a dashed line.

[12]–[14]. Although these efforts bring insight into the underlying mechanisms of MFC function and provide tools to aid in actuator design (optimizing electrode placement, fiber/rod shape, etc.) the level of complexity in this micromechanical modeling is too computationally expensive for integration within a larger macro-scale structural model, such as a bistable laminate. Despite interest in MFC actuators for use in structural actuation schemes [16]–[18], little work adequately captures the change in actuator response with electrical boundary conditions during actuation of bistable laminates. Several investigators [16], [18] have chosen to approximate the piezoelectric strain by altering the coefficient of thermal expansion of the macro fiber composite model to create actuation strains. Although such research efforts provide approximations of actuator behavior, to achieve accurate representation of the electromechanical coupling inherent in piezoelectric devices, the elastic and electrical conditions within a device must be coupled. In addition, thermal approximations would not allow design of combined control and actuation systems utilizing MFC sensor capabilities, which requires prediction of voltage generated within the MFC in response to applied stress. To highlight this, (1) shows the relationship between the closed circuit or constant field compliance ($s_{ij}{}^{E}$) and open circuit or constant electric displacement compliance ($s_{ij}{}^{D}$) [19]. These two quantities are related via the square of the electromechanical coupling coefficient (k^2), defined as the ratio of stored electrical energy to supplied mechanical energy.

$$1 - k^2 = s_{ij}^{D} / s_{ij}^{E} \tag{1}$$

Because k^2 is always positive and less than unity, the closed-circuit compliance is greater than the open-circuit compliance. Based on a typical value for piezoelectric coupling coefficient k for PZT-5A of ~0.7 [20], the open-circuit stiffness of the MFC is approximately twice the closed-circuit stiffness.

To integrate the MFC model with a structural FE model, coupled-field finite elements that provide electromechanical coupling should be used. Although MFCs have received attention as structural actuators [15], [21], less work has been done to investigate their suitability for inducing snap-through in bistable laminates. Analytical techniques based on the Rayleigh-Ritz minimization techniques of Hyer [5] have met with some success for the prediction of snap-through for MFC-actuated bistable laminates [16], [18], [22]. Analytically predicted values for snap-through voltage often do not agree with experimental measurements, although the reduced computational cost has allowed investigators to conduct parametric studies relating to moisture absorption [23] and laminate architecture [24]. If bistable mechanisms are to be viable within morphing structures, they must be integrated within host structures where they are subject to elastic boundary conditions imposed by the stiffness of the host. Current analytical models based on Hyer's energy minimization method are not able to predict the cured shapes of bistable composite laminates embedded within host structures. Gude and Hufenbach [25] created a simple homogenized MFC model and investigated the use of MFCs to induce snap-through in bistable laminates. Analytical and FE models were presented to model MFC device behavior, but no validation of the MFC model was presented and no comparison of FE-predicted snap-through voltage was made to experimental data. Dano *et al.* [21] presented an FE analysis of MFCs used to compensate for thermal deformation of asymmetric composite laminates. The actuation performance of the MFC model was validated against experimental measurement of the deflection of an aluminum beam and unidirectional carbon/epoxy plates. Finite-element predictions of beam deflection were in good agreement with experimental data, but no quantitative comparison of prediction error was presented. Recently, Binette *et al.* [17] conducted experimental characterization of laminate deflection for a composite panel subjected to thermal loading. Piezoelectric actuation via two MFC actuators was used to reverse the induced thermal deformation. This experimental work was conducted to validate a coupled-field FE model of an asymmetric laminate under combined piezoelectric and thermal actuation. The coupled-field MFC model was based on Dano *et al.* [21]

and shared the same set of material properties to represent the MFC. System behavior under isothermal piezoelectric actuation and combined thermal-piezo loading was predicted using the FE model developed. In the case of combined thermal and piezoelectric loading prediction, accuracy varies between 4% and 31%.

Following an attempt to model bistable composite laminates using the commercial ANSYS FE analysis software (release V5.5.2, Ansys Inc., Canonsburg, PA), Gude and Hufenbach [25] attempted to model the snap-through of a bistable composite laminate using 8-node layered solid elements (SOLID46) to represent the laminate and 8-node coupled-field brick elements (SOLID5) for the MFC. The $[0/90]_\mathrm{T}$ laminate was manufactured from an unidentified pre-preg material using T300 carbon fiber reinforcement; it measured 150 × 150 × 0.5 mm. A Smart Materials MFC-8557P1 actuator was bonded to its upper surface. The element types used for both bistable composite and MFC volumes were linear solid elements [25]. Both element types approximate the displacement field between nodes using linear interpolation. This first-order approximation to displacement introduces numerical errors in the analysis of highly curved structures [26]. Because no details of mesh density were given in the work, it is not possible to determine if element size was reduced to minimize these errors. Furthermore, the SOLID46 element is unsuitable for modeling curved structures. When the SOLID46 element is deformed, as occurs in highly curved bistable laminates, the element stiffness matrix is formulated assuming the element coordinate system remains parallel to the original coordinate system of the undeformed element [26]. No predicted snap-through voltage was presented and no comparison between the FE solution and either experimental data or analytical predictions was made. The authors simply state that snap-through was predicted. It should be noted that the analytical model presented by Gude *et al.* [25] in addition to the FE model did not agree well with experimental data contained within the work. Analytically predicted values for snap-through voltage deviated from those observed in experiment by 130% (1260 V predicted, 526 V observed). Gude *et al.* [27] very recently presented a highly novel semi-analytical, geometrically non-linear simulation model using the Rayleigh-Ritz method with good agreement with experiments (snap-through voltage). ANSYS FE analysis was also examined by the authors and it was concluded that meshing the laminate and actuator with shell elements and simulating the piezoelectric strain of the MFC by thermal expansion was more appropriate for fast solution times.

Portela *et al.* [16] presented an analytical technique and an FE model using ABAQUS/EXPLICIT (Simulia, Providence, RI) to predict snap-through voltage for an MFC-actuated bistable laminate. The FE model approximated the behavior of the MFC by applying a different thermal load to the MFC elements than the bistable composite elements. By scaling the coefficient of thermal expansion of the MFC elements to match the strain per unit electric field value of the MFC (d_{33}), a correlation between temperature change within the MFC elements and drive voltage was obtained. In addition, Portela *et al.* [16] predicted the effect of moisture on laminate curvature and snap-through voltage for a range of materials and actuator sizes. They suggested that for any given laminate there is an optimum size of actuator which is capable of initiating snap-through without significantly impacting on the cured shape of the composite laminate. This was supported by the FE analyses, although the work did not contain validating experimental data. Only a single experimental measurement of snap-through voltage is presented by Portela *et al.* [16] with no explanation of which particular laminate was tested to achieve the observed snap-through of 390 V. Without laminate descriptions, test conditions, or experimental procedures being clarified, it was not possible to determine the extent to which the presented model agrees with the experimental data. However, the insights gained into possible effects of moisture absorption on bistable laminates and their snap-through behavior are valuable contributions to the field.

Currently, no adequate FE model exists to predict the actuation behavior of MFC-actuated bistable laminates, and therefore this paper describes the formulation and validation of a coupled-field FE model to predict both snap-through voltage and cured shape of MFC-actuated bistable laminates using the commercial FE-software ANSYS V11.0. We compare model predictions with detailed experimental characterization.

III. MFC Model Formulation and Validation

A. *Compliance Matrix* $[s^E{}_{ij}]$ *Formulation*

The three-dimensional compliance matrix of the MFC was populated by converting the four linear elastic engineering constants measured by Williams *et al.* [28] in Table I into (2). This equation was derived from the standard stress-strain relations for orthotropic materials presented in [29]. The resulting compliance matrix is of the standard form for a transversely isotropic material with a single axis of rotational symmetry parallel with the poling direction in PZT fibers:

$$[s^E_{ij}] = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{31}}{E_1} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{31}}{E_1} & \frac{1}{E_1} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{31}}{E_3} & \frac{-\nu_{31}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{31}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu_{31})}{E_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_{31})}{E_3} \end{bmatrix}, \qquad (2)$$

where E is the Young's modulus, G is the shear modulus, ν is the Poisson's ratio of the material, and the subscripts denote the orientation of each property with respect to the material coordinate system (the 3-direction is the pol-

TABLE I. Mechanical Properties of Smart Materials Corp. M8557-P1 MFC Actuator Taken From Williams *et al.* [28] and Manufacturer's Data Sheet [30].

	Williams *et al.*	Smart Materials Corp.
E_{33} (GPa)	29.4	30.336
E_{11} (GPa)	15.2	15.857
G_{31} (GPa)	6.06	5.515
Υ_{31}	0.312	0.31
Υ_{13}	0.16	0.16

TABLE II. Piezoelectric Coupling Coefficients $[d_{IJ}]$ and Relative Permittivity (ε_I) of Smart Materials M8557 MFC Actuator.

	Smart Materials Corp. M8557-P1 MFC actuator
d_{33} (pm/V)	467
d_{32} (pm/V)	−210
d_{31} (pm/V)	−210
ε^s_{11}	712
ε^s_{22}	1.7
ε^s_{33}	737

Note: ANSYS used permittivity at constant strain as an input parameter $[\varepsilon^s{}_{ij}]$.

ing direction). For comparison, the manufacturer's (Smart Materials GmbH) data sheet values [30] are also presented in Table I and are in good agreement with measured values presented in [28]. The final $s^E{}_{ij}$ matrix used to define the compliance of the MFC model is

$$[s^E_{ij}] = \begin{bmatrix} 0.065 & -0.0205 & -0.0106 & 0 & 0 & 0 \\ -0.0205 & 0.065 & -0.0106 & 0 & 0 & 0 \\ -0.0106 & -0.0106 & 0.034 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.165 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.173 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.173 \end{bmatrix} \times 10^{-9}\ \mathrm{m^2N^{-1}}. \tag{3}$$

B. Piezoelectric Matrix $[d_{ij}]$ Formulation

The effective piezoelectric constants for the device were determined to describe the response of the MFC to an applied electric field. As described by [9] and [10], the relationships between the piezoelectric properties of the constituent materials and the complete MFC are highly complex because of non-uniform polarization of PZT fibers and the composite structure of the actuator. Although predictive models for free-strain behavior agree well with experiment [9], [14], experimental measurement of device behavior provides the best available data on which to base the FE model for this work. Williams *et al.* [9] experimentally determined the free-strain behavior of the same MFC actuator used in the present study (M8557-P1, Smart Materials Corp.). The value for d_{33} (strain per unit electric field in poling direction) presented in [9] agrees with data presented by the manufacturer [30], however no value for d_{31} was reported by either source. Williams *et al.* determined d_{33} and d_{31} for an active fiber composite [9] and the construction and mode of operation is sufficiently similar to an MFC to assume that the measured ratio of d_{31}/d_{33} in both devices is equal [28]. The values for both d_{31} and d_{32} shown in Table II were calculated by multiplying the measured piezoelectric d_{33} constant taken from the manufacturer's product specification [30] by the d_{31}/d_{33} ratio of −0.449 measured by Williams *et al.* [9]. The calculated values used to populate the piezoelectric coefficient $[d_{ij}]$ matrix used in the present FE-model of the MFC are shown in Table II, and the final matrix is

$$[d_{ij}] = \begin{bmatrix} -2.1 & -2.1 & 4.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times 10^{-10}\ \mathrm{m \cdot V^{-1}}. \tag{4}$$

No shear piezoelectric coefficients are included in the matrix (e.g., d_{15}). For conventional piezoelectric ceramics, d_{15} is nonzero; however, in many composites $|d_{15}| \ll |d_{3j}|$. In addition, because the applied electric field will always be along the poling direction (fiber/rod axis), no contributions for the piezoelectric shear coefficients are expected.

C. Relative Permittivity Matrix Formulation

To fully specify the electromechanical coupling within the MFC, the relative permittivity of the active layer must be determined. This property is important in providing the relationship between charge Q [C], capacitance C [F], and voltage (V) for the MFC ($Q = CV$), and determines the magnitude of the induced electric field when subjected to a mechanical stress under open-circuit conditions. To determine the relative permittivity of the active layer, a micromechanical mixing rule for $\varepsilon^T{}_{33}$ presented by Deraemaeker [13] was used, along with a standard mixing rule for dielectric volumes in series representing $\varepsilon^T{}_{22}$; namely

$$\varepsilon^T_{33} = \rho\varepsilon^{T,\mathrm{p}}_{33} + (1-\rho)\varepsilon^{T,\mathrm{m}}_{33} \tag{5}$$

$$\varepsilon^T_{22} = \left(\frac{\varepsilon^{T,\mathrm{p}}_{22}\varepsilon^{T,\mathrm{m}}_{22}}{\rho\varepsilon^{T,\mathrm{m}}_{22} + (1-\rho)\varepsilon^{T,\mathrm{p}}_{22}}\right), \tag{6}$$

where $\varepsilon^T{}_{ij}$ is the relative permittivity and ρ is the volume fraction of PZT within the active layer. The superscripts p and m denote the piezoelectric and matrix materials, respectively. The relative permittivity of PZT-5A data used to calculate the permittivity of the MFC device was taken from Jaffe [20] with the value for the epoxy matrix taken from Deraemaeker [13]. The wide variation in the predicted relative permittivity values between the $\varepsilon^T{}_{11}$ and $\varepsilon^T{}_{22}$ is due to the different electrical conditions in the 1 and 2-directions. For an electric field to propagate in the 2-direction, it must permeate through the epoxy layer and the PZT fiber. This means that the low permittivity of the epoxy significantly reduces the effective permittivity of the active layer in this direction.

IV. MFC Validation on Isotropic Beam

To validate the homogenized materials properties describing the MFC actuator, it was necessary to compare

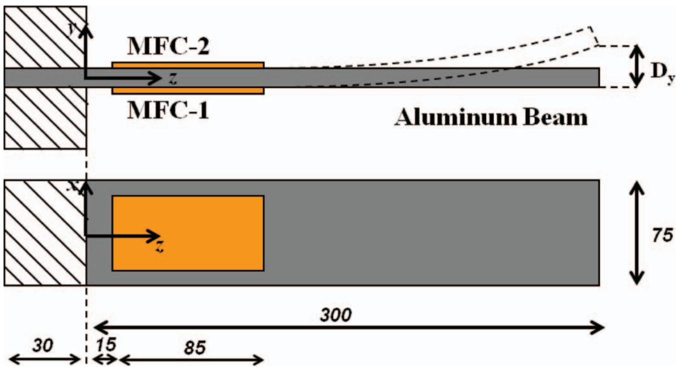

Fig. 2. Experimental setup with aluminum beam, driven actuator (MFC-1), and passive actuator (MFC-2) with dimensions shown in millimeters.

FE model predictions of actuator deflection with experimental data. Two MFC actuators were bonded to a simple aluminum cantilever with one acting as an actuator and the other remaining passive to allow testing of the influence of electrical boundary conditions.

A. Experimental Setup

Two MFC actuators (M-8557P1, Smart Materials Corp.) were bonded to the front and back surfaces of an aluminum beam measuring 330 × 75 × 1.97 mm as shown in Fig. 2. The actuators and aluminum surfaces were cleaned using isopropyl alcohol and a thin coat of a two-part epoxy adhesive applied to both surfaces. The MFCs were then carefully located, while ensuring no air-bubbles were trapped during placement. With the MFCs in place, the assembly was placed under 200 N clamping force for 24 h to allow the epoxy to cure. Both MFCs (labeled MFC-1 and MFC-2 in Fig. 2) were positioned so that the active area was located between $z = 45$ mm and $z = 130$ mm, with the poling direction of the PZT fibers (3-direction) parallel to the z-axis. The beam was clamped so that the 75 mm dimension (x-direction) was aligned vertically. This arrangement isolated beam deflection (D_y) from the influence of gravitational forces. A Nippon LAS5010v laser displacement sensor (Nippon Automation Co. Ltd., Hamakita, Japan) with a resolution of 10 μm was used to measure cantilever deflection as a function of applied voltage. The active MFC (MFC-1) was driven from a signal generator attached to a PZD700 Piezodriver (Trek Inc., Medina, NY). Closed-circuit boundary conditions were imposed on MFC-2 by connecting the positive and ground electrode terminals, whereas for open-circuit conditions, these terminals were insulated from one another. Piezoelectric actuators are subject to slow creep under open loop control [31] because of domain motion. To standardize the piezoelectric creep effects [32], all measurements at each voltage were taken after a 60 s settling period. Beam deflection at $z = 120$ mm as a function of MFC-1 drive voltage was measured from 0 to 400 V, taken at 80 V intervals. Deflection measurements were carried out with MFC-2 under closed-circuit boundary conditions. A datum measurement of beam position was taken before voltage application, and this value subtracted from the actuated beam position to calculate the deflection.

To measure the change in beam deflection caused by changing electric boundary conditions of MFC-2 beam deflection in response to 400 V MFC-1 drive voltage was measured at intervals of 30 mm between $z = 120$ mm and $z = 300$ mm. Closed and open circuit measurements were taken sequentially at each z position. A separate datum measurement was taken for each measurement, and both MFC-1 and MFC-2 were electrically discharged between measurements.

B. FEM Model of MFC-Actuated Isotropic Beam

The aluminum cantilever was modeled using 20-node quadratic brick SOLID186 elements with isotropic mechanical properties (Young's modulus 70.7 GPa and Possion's ratio 0.32). The two MFC actuators were represented by 20-node quadratic brick SOLID226 elements. SOLID226 elements are coupled-field elements which solve the constitutive equations for an elastic piezoelectric solid. In addition to the improved solution accuracy of the 20-node quadratic elements in modeling highly curved structures, when element types are combined within a single model, it is advisable to use 20-node elements throughout [33] to ensure that all nodes on adjacent elements are coincident. Attempting to merge nodes linked to a volume meshed using 8-node elements with 20-node elements connected to an adjacent volume can result in nodes losing connectivity with the model, introducing numerical errors and preventing model solution. Once the model was appropriately meshed, the model consisted of a total of 4680 SOLID186 elements for the beam and 340 SOLID226 elements within the MFC volume. Element density was sufficient to accurately capture beam behavior, because further refinement of the mesh did not significantly affect the predicted beam deflection.

The MFC model volumes were positioned on the front and back surfaces of the cantilever as in Fig. 2 with all coincident nodes on the contact surfaces merged to ensure stress transfer between cantilever and actuator volumes. The clamped mechanical boundary condition was modeled by constraining all three translational degrees of freedom (x, y, and z) for nodes lying in the range -30 mm $< x < 0$ mm on the $y = 0$ mm plane. Nodes in the same range of x-coordinate on the $y = 0.197$ mm plane were constrained in the y-direction only. The FE model used to predict deflection of the experimental beam in response to MFC actuation is shown in Fig. 3. In creating the piezoelectric $[d_{ij}]$ and permittivity $[\varepsilon_{ij}]$ matrices for the MFC actuator, it is assumed that electric field is constant along and aligned with the z-axis. To ensure a constant and well-aligned electric field, the voltage degree of freedom for nodes of equal z-coordinate was coupled to create planes of equal voltage at 5 mm intervals along the z-direction of the MFC volume. To model MFC behavior under closed-circuit boundary conditions (i.e., two electrodes of MFC-2

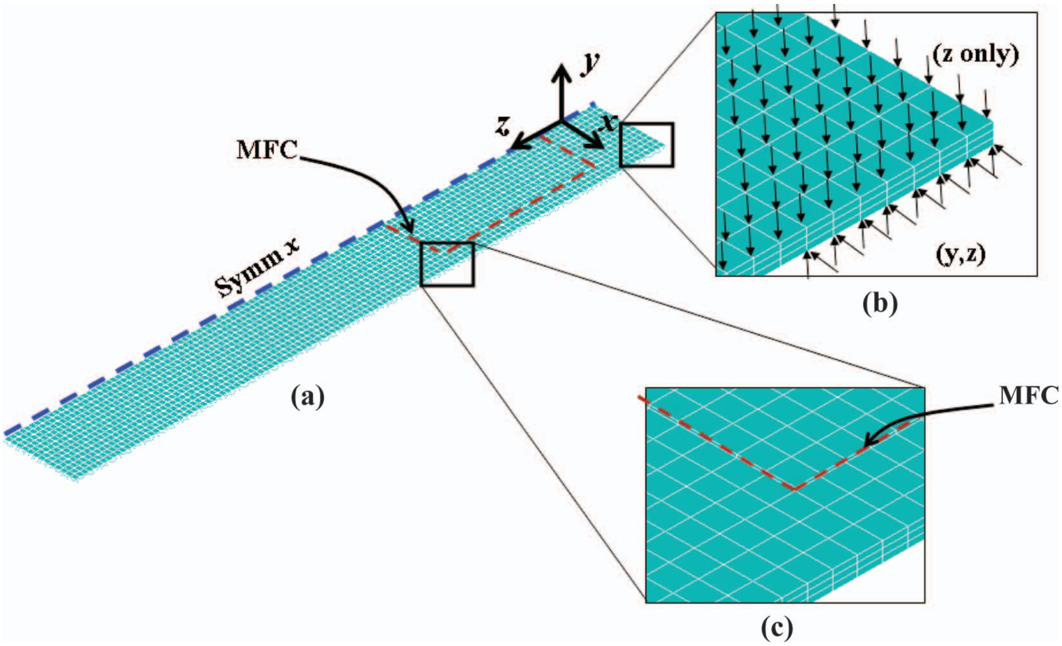

Fig. 3. (a) Finite-element model used to predict beam deflection, showing coordinate system and symmetric boundary constraint, (b) mechanical constraint to model clamped-end condition of experimental setup, and (c) mesh density of both MFC volume and beam.

are electrically connected), the induced potential difference as a result of a stress must be dissipated; however, it is inappropriate to constrain all voltage DOFs within the MFC volume because it reduces solution accuracy [33]. Furthermore, simply altering the voltage constraints at the $x = 35$ mm and $x = 120$ mm faces would not suppress the induced field throughout the volume but only near the extremities. To suppress the induced field throughout the passive MFC volume (MFC-2 in Fig. 2) the piezoelectric coefficients (d_{33} and d_{31}) for that volume were reduced by a factor 1×10^9. This alteration of piezoelectric constants effectively modeled the suppressed induced field which characterizes closed-circuit electrical boundary conditions while maintaining solution accuracy.

C. Comparison of Model and Experimental Results

Beam deflection as a function of MFC-1 drive-voltage is shown in Fig. 4(a) and clearly shows the linear trend predicted by engineering beam theory and observed by other investigators [22], [34]. Predicted FE values of cantilever deflection had excellent agreement with experimental values to within 2%. It should be noted that for all values except the value for 160 V, the error was within measurement uncertainty of ±10 μm. Fig. 4(b) and Fig. 4(c) show beam deflection (D_y) as a function of distance from the clamped region of the cantilever with a drive voltage of 400 V applied to MFC-1 and MFC-2 under open- and closed-circuit boundary conditions, respectively. FE-predictions of D_y were again accurate to within 2% compared with experimental values for both conditions. Prediction of closed-circuit behavior showed excellent quantitative agreement with experiment, with predicted cantilever gradient (dy/dz) over the range $120 \text{ mm} < z < 300 \text{ mm}$ matching the measured value of $3.8\ \mu\text{m}\cdot\text{V}^{-1}$ exactly. In addition, beam tip deflection was predicted to within 1% of the measured value of 0.905 mm. Beam tip deflection decreased under open-circuit boundary conditions compared with closed-circuit values, with the predicted and observed values (0.873 mm and 0.880 mm, respectively) matching to within 1%. However, beam gradient under open-circuit boundary conditions was lower in the model by 2.6%, with FE prediction of $3.7\ \mu\text{m}\cdot\text{V}^{-1}$ compared with the experimental value of $3.6\ \mu\text{m}\cdot\text{V}^{-1}$. This difference in closed- and open-circuit conditions will not be captured by the thermal approximations to piezoelectric actuation [16], [18], [22]. The small discrepancy between the measured and predicted beam deflection under open-circuit conditions indicates that the model generates a larger than expected voltage (via the $V = Q/C$ relation) and electric field in MFC-2 because of the beam deflection. This suggests the relative permittivity used may be too low, leading to increased induced field and hence piezoelectric strain within the MFC-2, increasing the cantilever's effective stiffness. Nevertheless, the excellent agreement for closed-circuit response indicates that both elastic and piezoelectric matrices are appropriately formulated. The next stage is to combine the MFC model with a bistable laminate to predict cured shape and snap-through voltage.

V. MFC-Actuated Bistable Composite Laminates

A. Bistable $[0/90]_T$ and $[-30/60]_T$ Composite Manufacture

Two cross-ply composite laminates were manufactured using carbon fiber-epoxy pre-preg material (Hexcel Corp.,

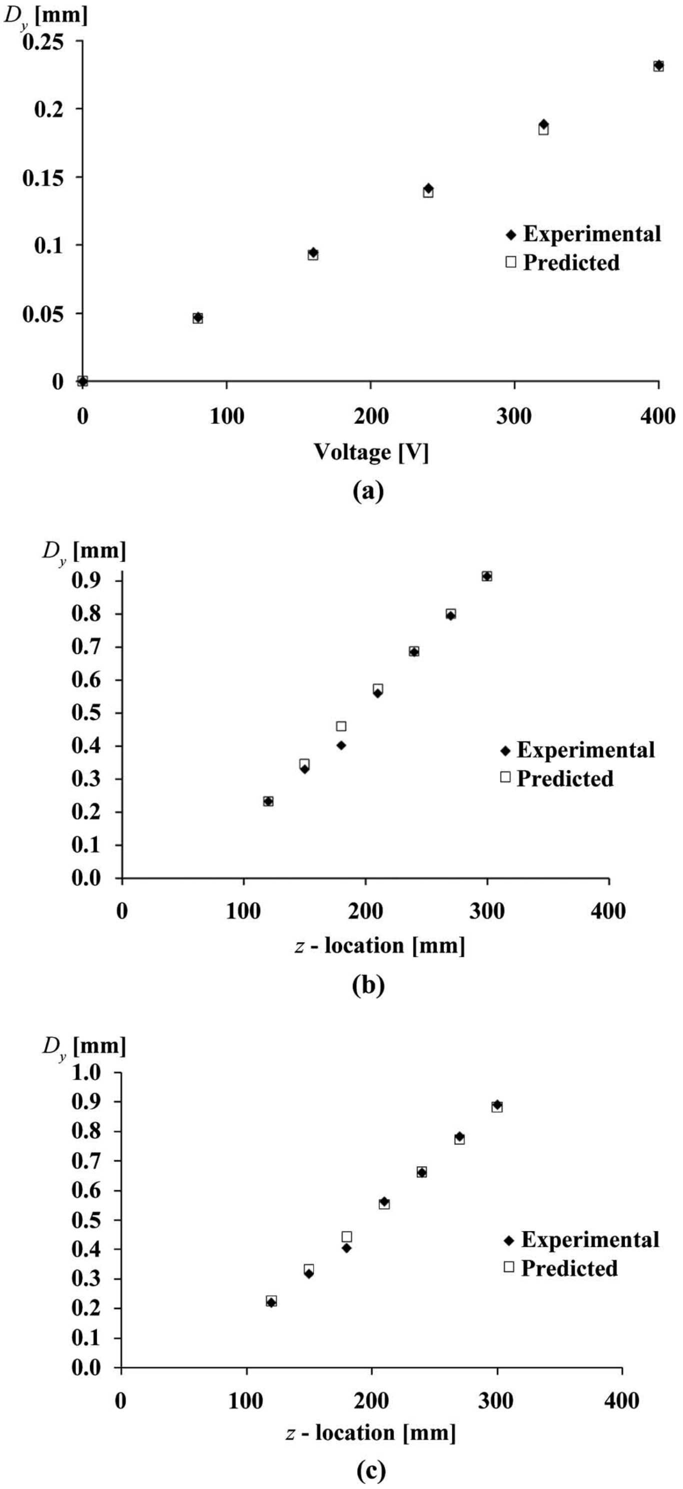

Fig. 4. (a) Beam deflection (D_y) as a function of MFC drive-voltage. (b) D_y as a function of *z*-location with MFC-2 under open-circuit boundary conditions. (c) D_y as a function of *z*-location with MFC-2 under closed-circuit boundary conditions.

Stamford, CT) cut and laid-up by hand and cured at 180°C. A T700/M21 $[0/90]_\mathrm{T}$ laminate and a T800/M21 $[-30/60]_\mathrm{T}$ laminate measuring 150 × 150 mm were manufactured from 268 gram/square meter (gsm) unidirectional pre-preg material. A Smart Materials 8557-P1 MFC actuator was bonded to the smooth surface of the laminate to create $[-30/60/0_\mathrm{MFC}]_\mathrm{T}$ and $[0/90/0_\mathrm{MFC}]_\mathrm{T}$ laminates. To ensure good adhesion, bond surfaces were roughened with emery paper and then cleaned with isopropyl alcohol before a thin layer of two-part epoxy was applied to both MFC and laminate. The MFC was positioned centrally on the laminate surface with PZT-fiber direction aligned with the *y*-axis (Figs. 1 and 5); the active laminate was then placed between two flat aluminum plates under 200 N clamping force for 24 h to allow the epoxy to cure. The two stable states for the $[-30/60/0_\mathrm{MFC}]_\mathrm{T}$ laminate can be seen in Figs. 5(a) and 5(b). Stable states for the $[0/90/0_\mathrm{MFC}]_\mathrm{T}$ laminate can be seen in Figs. 1(a) and 1(b).

B. Snap-Through Actuation Voltage Measurement

A function generator was used to provide a DC step-input from 0 V up to the desired test voltage to a power amplifier (TREK PZD700 piezo driver). Actuation voltage was maintained for 60 s after the step-input to account for the effects of piezoelectric creep [31]. After each test cycle (i.e., snap-through from State A to State B), voltage was reduced to zero and the laminate disconnected from the amplifier. The laminate was manually snapped into each stable state once before being reset to the starting condition and electrically discharged to ensure that no residual charge influenced system characteristics. The lowest snap-through voltage for both laminates was established via binary search in the range 0 to 1500 V with 5 V increments between test voltages. Laminates rested on a polished steel table to ensure laminate deflection was not impeded by frictional forces.

C. Measurement of Cured Shape of Bistable Composites With MFC

The shapes of the two bistable composites, $[0/90/0_\mathrm{MFC}]_\mathrm{T}$ and $[-30/60/0_\mathrm{MFC}]_\mathrm{T}$, were charaterized using standard three-dimensional motion analysis techniques similar to those in Betts *et al.* [24]. To examine the influence of actuator attachment on overall shape, the $[-30/60]_\mathrm{T}$ laminate was also characterized before actuator attachment. Three digital video camera recorders (Sony DCR-TRV 900E or Sony HC9, Sony Corporation, Tokyo, Japan) operating at 50 fields per second were set up in an umbrella configuration around the experimental area as shown in [24]. The three cameras were positioned to achieve the best possible viewing angle for each laminate. One camera was always positioned much higher than the other two cameras, which were further away sideways from the laminates. This ensured that the cameras were not in the same plane in accordance with recommendations by Nigg *et al.* [35]. The distances between the center of lens and the origin of the measurement volume varied from 1.64 to 2.77 m with the height of the cameras ranging from 0.58 to 1.86 m above the measurement surface. A 20 × 20 × 10 mm wire frame was first videotaped on the measurement surface for calibration purposes on both camera set ups. The camera views were restricted to a volume just slightly larger than

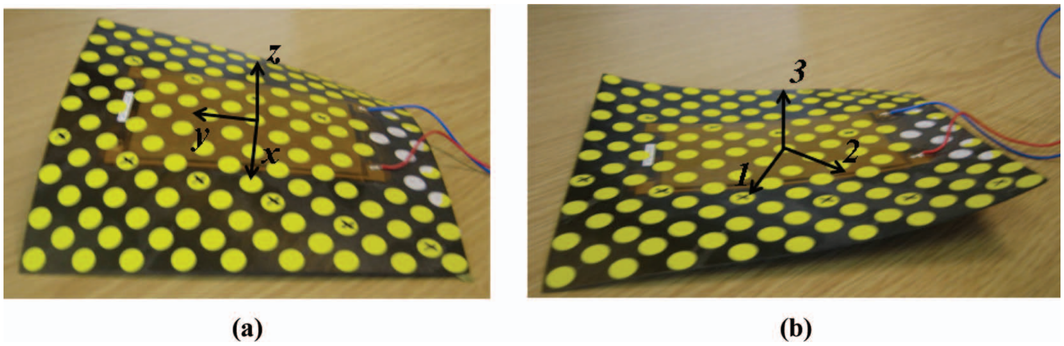

Fig. 5. (a) Cured shape of $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate in State A with global coordinate system and (b) State B showing local material coordinate system for uppermost 60° ply. Circular markers are attached for coordinate mapping of the surface.

the calibration frame. After removing the frame, the laminate was positioned within this measurement volume and videotaped simultaneously with all three cameras.

To map arbitrary coordinates on the surface of the laminates to later create the shape of the laminate, markers were attached on one surface of the laminate. The $[-30/60]_{\mathrm{T}}$ and $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminates had 145 round color labels of 8 mm diameter attached to it, as shown in Fig. 5. The size of the markers were reduced for the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, allowing 279 markers to be put on the surface. The four corner points of each laminate were also used.

The actual mapping of the surface coordinates was carried out using PeakMotus motion analysis system (v. 8.5, Vicon, Centennial, CO) after transferring the calibration and laminate video clips onto the computer. First, the eight corners of the calibration wire frame were manually digitized from each camera view (and for each camera set up). Then, the center of each surface marker and the four corners of the each laminate were manually digitized from all three camera views. The digitized area on the computer screen was 1440 × 1152 pixels. The digitized pixel information from each camera view was combined with the calibration information to transform these to Cartesian coordinates of the laminate surfaces using the direct linear transformation method [36]. The largest directional coordinate RMS error of the different set ups between the known eight calibration coordinates and the respective digitized point was 0.3 mm. The interpolated surface was then constructed from the raw coordinates using the spline-based interpolation method [37] for comparison with FE predictions.

D. Development of MFC-Actuated Bistable Composite Model

A non-linear large deflection FE analysis was conducted to predict both cured shape and snap-through voltage. Model convergence was controlled using the line search convergence control method to improve numerical stability [33], [38]. Formulation of this MFC-bistable composite model was far more complex than the simple isotropic aluminum cantilever beam, because it is necessary to capture:

1) The bistable states of the asymmetrical composite and the corresponding laminate curvature as a result of cooling the asymmetrical composite from the cure temperature,
2) The influence of attaching the MFC actuator to the laminate at room temperature on the curvature of the bistable laminate–MFC combination,
3) The prediction of a snap-through event as a result of the application of a voltage to the MFC.

The $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate was modeled using 20-node quadratic SOLID186 layered brick elements. The laminate was modeled as three volumes shown in Fig 6. A central strip measuring 57 × 150 mm was located underneath the MFC volume (point 1 in Fig. 6) and was meshed with 1360 elements; whereas the two remaining volumes, each measuring 46.5 × 150 mm (points 2 and 3 in Fig. 6) were meshed with 816 elements each to create the mesh shown in Fig. 6(a). Maximum element aspect ratio within the laminate was 9.14. All coincident nodes within the laminate volume were then merged to ensure stress transfer during model solution. To make use of symmetry in the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, a symmetric boundary condition was imposed along the $x = 0$ mm plane. The mean ply-thicknesses for the laminates with ply angles of $[\theta/\theta + 90]_{\mathrm{T}}$ were determined by optical microscopy and used to approximate the laminates' individual ply thicknesses. Table III shows the mean and standard deviation of ply thickness and total laminate thickness for a range of manufactured laminates with ply angles of $[\theta/\theta + 90]_{\mathrm{T}}$. Materials properties for the laminate were determined by batch testing undertaken by Airbus UK [39] for both T700/M21 and T800/M21 pre-preg material, and are shown in Table IV.

A volume measuring 85 × 57 × 0.3 mm was defined and ascribed the homogenized MFC materials properties (Section III) to represent the active area of the Smart Materials MFC8557P1 actuator. The actuator volume was located centrally on the upper surface of the laminate with MFC-fiber orientation aligned with the y-axis as indicated in Fig. 6(b). This volume was meshed with 1200 SOLID226 elements with a maximum element aspect ratio of 9.33. With all volumes meshed, coincident nodes on adjacent surfaces of laminate and MFC were merged

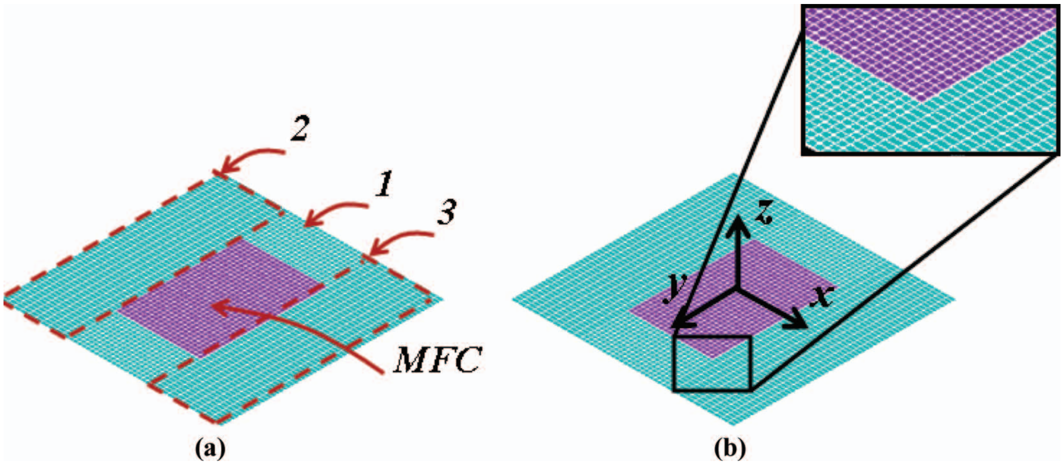

Fig. 6. (a) Meshed FE model of $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate showing three laminate volumes (1, 2, and 3) and centrally located MFC volume, (b) overall mesh density and detail of coincident nodes near corner of MFC volume and global coordinate system.

TABLE III. MEAN AND STANDARD DEVIATION (σ) OF PLY AND TOTAL LAMINATE THICKNESS FOR $[\theta/\theta{+}90]_{\mathrm{T}}$ LAMINATES MADE FROM 268GSM M21/T800.

	Idealized	Measured	
	Thickness (mm)	Thickness (mm)	σ
θ° ply	0.25	0.255	0.013
θ +90° ply	0.25	0.233	0.018
Resin layer	0	0.027	0.021
Total	0.5	0.515	0.045

TABLE IV. ELASTIC PROPERTIES OF 268GSM-1 T800/M21 MATERIAL AND T700/M21.

Property [unit]	T700/M21	T800/M21
E_1 [GPa]	148	172
E_2 & E_3 [GPa]	7.8	8.9
G_{12} & G_{23} [GPa]	3.8	4.2
G_{23}* [GPa]	0.02	0.02
ν_{12} & ν_{13}	0.35	0.35
ν_{23}*	0.01	0.01
α_1 [1×10^{-7} °C^{-1}]	−0.9	−0.9
α_2 & α_3 [1×10^{-5} °C^{-1}]	3	3
E_{r} [GPa]	1.5	1.5
ν_{r}	0.4	0.4
α_{r} [1×10^{-5} °C^{-1}]	9	9
Density [kg·m^{-3}]	1072	1072

*Indicates values calculated using stress-strain relations described in [29].

along with coincident areas to ensure stress transfer between the laminate and MFC volumes. Because of the selection of higher-order solid elements, it is possible to accurately model bending deformation without multiple elements through the thickness [40], and hence a single-element thickness was used. With all volumes meshed and the MFC and laminate volumes merged, the laminate models were mechanically constrained from translation in all three orthogonal directions at the origin of the global coordinate system, shown in Fig. 7. Additionally the node at the point (0, 0, 0.515) was constrained from in-plane translation to ensure the laminate did not rotate about either the x or y-axis (Fig. 7). Because of actuator orientation in the experimental sample, the FE model must converge to stable deformation State B before application of MFC drive-voltage. To force the FE solution to converge to State B, temporary displacement constraints were applied at locations of minimum State B deflection as indicated in Fig. 7(b).

E. Active Laminate Model—Model Solution

With the model mechanically constrained, the cool-down of the bistable composite laminate from the autoclave temperature (180°C) and attachment of the MFC actuator at room temperature was modeled in a four-step process:

1) Application of a temperature change of −160K to composite elements to ensure that the laminate converges to State B.

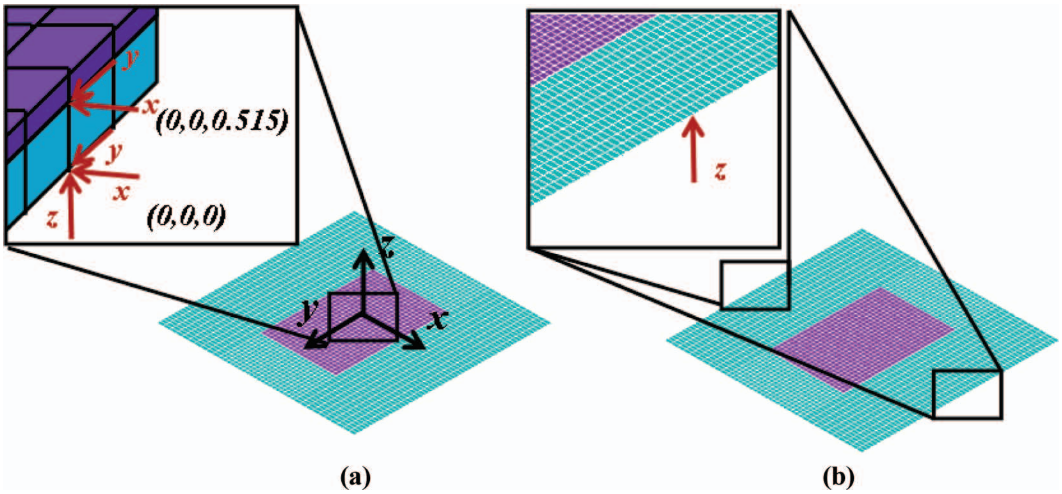

Fig. 7. (a) Meshed FE model, showing mechanical constraint at origin of global coordinate system and (b) temporary mechanical constraint used to force model convergence to deformation State B.

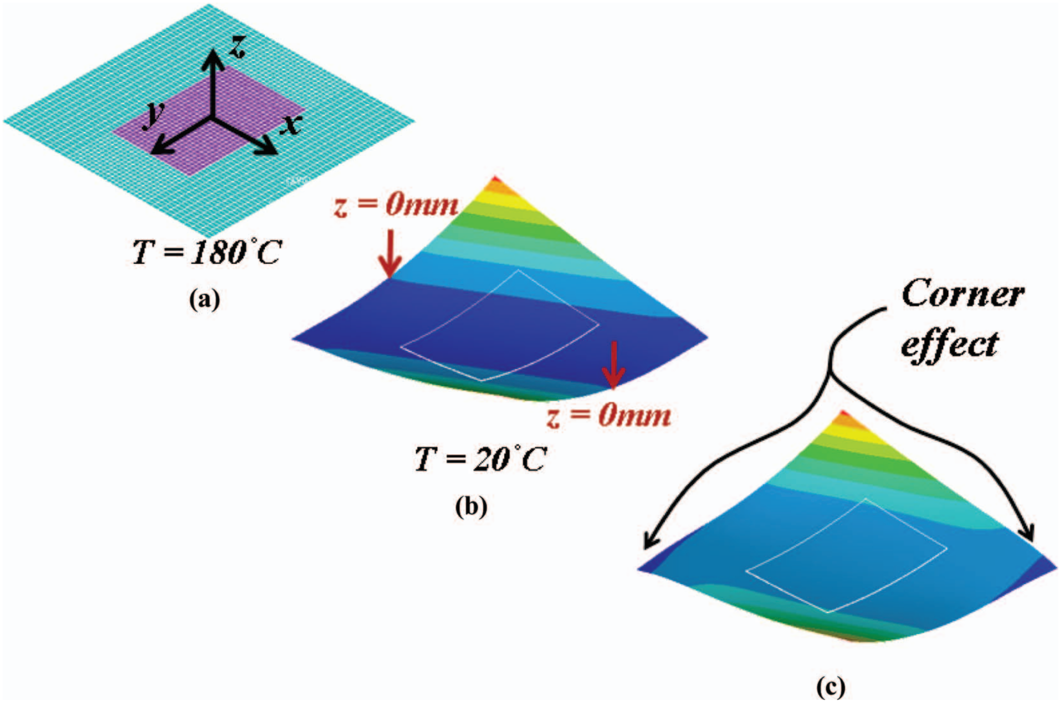

Fig. 8. (a) Finite-element prediction of cured shape for $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate at cure temperature of 180°C, (b) room temperature of 20°C, and (c) in stable deformation State B with offset voltage (V_0) applied.

2) Application of an offset voltage to compensate for thermal stress imposed on the MFC volume.
3) Removal of temporary displacement constraints [shown in Fig. 7(b)].
4) Application of MFC drive-voltage until snap-through of the structure into State A.

Fig. 8(a) shows the laminate and MFC model at the cure temperature of 180°C which is initially flat. Fig. 8(b) shows the highly curved structure at a temperature of 20°C with the temporary displacement constraints still in place. Fig. 8(b) represents stage 1 of the solution process. Fig. 8(c) shows the laminate model after application of the offset voltage (V_0) and removal of the temporary displacement constraints; this represents stages 2 and 3 of the solution process. During application of MFC drive voltage (stage 4 in the solution process), the laminate flattened as MFC drive-voltage and the resulting actuation strain increased before undergoing the sudden transition into deformation State A (snap-through).

To model the cool down from elevated curing temperature of the composite laminate, a temperature difference was applied to the composite elements. The temperature constraint was applied only to composite (SOLID186) elements, whereas the coupled-field (SOLID226) elements of the MFC were not subjected to the imposed temperature constraint because the MFC is attached at room temperature. The electrical degree of freedom for the SOLID226 elements was coupled for nodes of equal z-coordinate at 5 mm intervals along the y-axis. This constraint ensures that applied field remains well aligned with the poling direction of the MFC model and minimizes variation in the field along the y-axis of the MFC volume. During model development, it was noted that solutions of bistable laminates with the MFC model integrated exhibited significantly reduced curvature and did not exhibit a second stable configuration but rather always adopted the State A configuration after application of the thermal load step. This phenomenon was attributed to the interaction of the MFC and laminate under the action of the imposed thermal load at stage 1. Although the composite elements were subjected to the imposed thermal load, the MFC elements experienced an elastic strain comprising the thermal strain of the composite and the mechanical strain caused by laminate deformation deforming the MFC volume. Because nodes within the MFC volume are merged with those on the laminate surface before the temperature change is imposed, both mechanical and thermal strains were imposed upon the MFC model. This introduces an additional mechanical stress within the MFC volume which does not represent the true experimental conditions. To compensate for the superposed thermal stress, an offset voltage was applied to the MFC. Via the converse piezoelectric effect, the offset voltage created a stress field of equal magnitude but of opposite sign to that created by the imposed thermal strain. The total thermal strain in the portion of laminate bonded to the MFC was calculated by considering the MFC as an elastic constraint resisting the thermal contraction. The strain in both laminate and MFC caused by thermal contraction of the composite laminate must be equal, hence force equilibrium leads to:

$$\varepsilon_{\mathrm{M}} = \frac{\varepsilon_{\mathrm{T}} K_1}{K_1 + K_2}, \tag{8}$$

where ε_{M} is the observed strain within the MFC, ε_{T} and K_1 are the unconstrained thermal contraction and transformed reduced stiffness of the layer to which the MFC is bonded, and K_2 is the MFC stiffness; all properties are measured in the y-direction and are aligned with MFC-fiber orientation. The transformed reduced stiffness may be calculated from the orthotropic elastic constants of each layer using [29]

$$\bar{Q}_y = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta, \tag{9}$$

where $\bar{Q}_y$ is the transformed reduced stiffness in the y-direction of the global coordinate system, Q_{ij} are the reduced stiffness values measured in the material coordinate system, and θ is the orientation of the material coordinate system with respect to the global coordinate system. In this case, the global coordinate system whose first principal direction is the y-direction, as shown in Fig. 9(a).

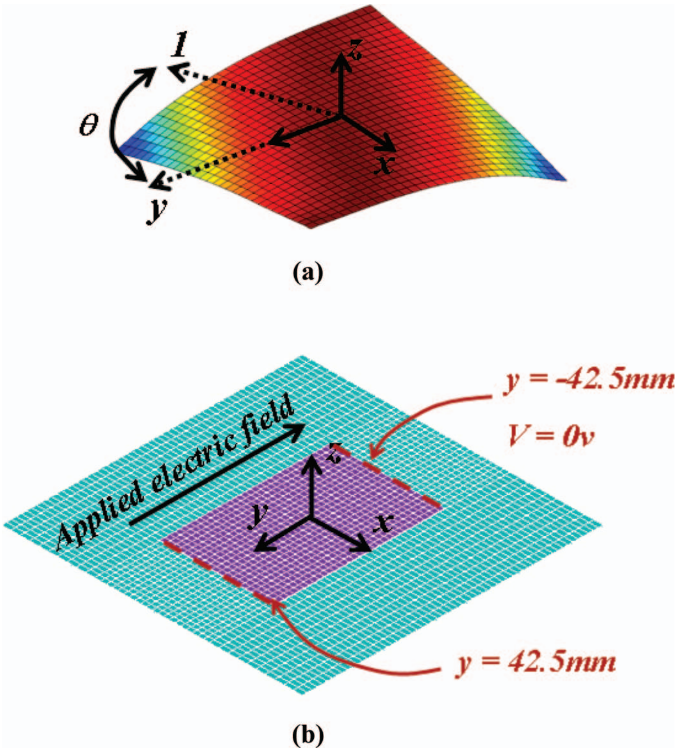

Fig. 9. (a) $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, showing orientation material coordinate system of the uppermost 60° layer, showing angle θ between the 1-direction of the local system and y-direction of the global system. (b) Finite-element model of $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, showing electrical constraint at $y = -42.5$ mm, location of drive-voltage application at $y = 42.5$ mm, and the resulting direction of applied electric field.

The offset voltage required to compensate for the imposed thermal strain may be calculated using

$$V_0 = \varepsilon_{\mathrm{M}} S_{\mathrm{E}} / d_{33}, \tag{10}$$

where V_0 is the offset voltage, ε_{M} is the observed strain in the MFC, and S_{E} is the electrode separation in the FE model. The voltage degree of freedom for nodes on the $y = 42.5$ mm face of the MFC model [indicated in Fig. 9(b)] were coupled and forced to the offset voltage value, whereas nodes on the $y = -42.5$ mm face were constrained to 0 V. This created an effective electrode separation of 85 mm in the FE model.

Piezoelectric actuation was modeled by applying a voltage constraint (V_{c}) on nodes at $y = 42.5$ mm to create a change in the electric field in the MFC. The voltage constraint was applied as a ramp change from the offset voltage (V_0), with several intermediate time steps specified between V_0 and V_{c} to ensure that the model followed the load path accurately. As specified for non-linear buckling analysis in situations in which the arc-length method is not appropriate [26], snap-through was identified as the lowest value of voltage constraint at which the model no longer converged to State B (i.e., snap-through into State A had occurred).

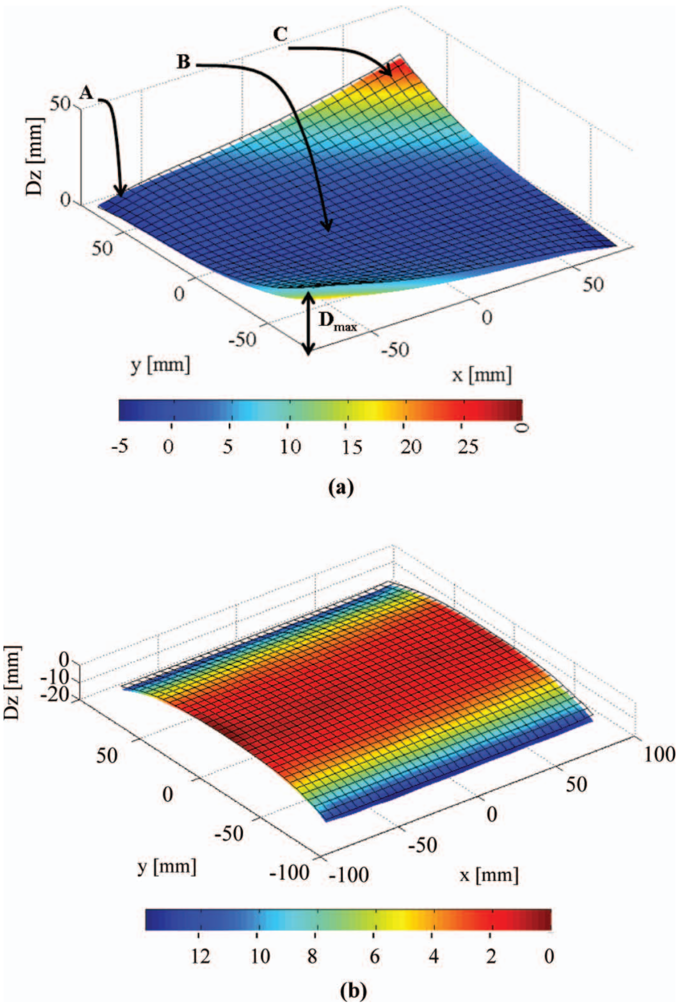

Fig. 10. (a) Interpolated surface plot of 149 measured surface coordinates showing the $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate in State B with the FE-predicted surface overlaid as a mesh. (b) Interpolated surface plot based on 283 measured surface coordinates showing the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate in State B with FE-predicted surface overlaid as a mesh (laminate shown inverted for clarity of presentation).

F. Laminate Shape (Model and Experimental)

The predicted shape of a $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ active laminate in State B is shown in Fig. 10(a) with deviations from simple cylindrical curvature seen at points A and C. The meshed regions in Fig. 10(a) are the FE predictions and the solid-colored sections are the experimental measurement. Overall, the agreement between the two is very good. The laminate adopts a saddle-shape after MFC addition with a significantly flattened section directly underneath the MFC (point B). This reduction in curvature underneath the MFC indicates that the bending stiffness of the actuator has a significant effect on the cured shape of the laminate. When comparing maximum deflections with respect to the laminate geometric center (D_{max}) before and after MFC-bonding, the influence of MFC addition is clear with measured deflections of 38.2 mm for the laminate (without MFC) reduced to 22.5 mm after MFC addition. Maximum deflection for the $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate after MFC addition was predicted to be 25.20 mm for State B, 12.1% higher than the measured maximum deflection of 22.48 mm.

Fig. 10(b) shows an interpolated surface plot of experimental measurement of laminate deflection for the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate in State B with the FE prediction overlaid as a mesh. The results again show close agreement between model and experiment. Maximum deflection for the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate after MFC addition was predicted to be 10.73 mm for State B, 16% lower than experimentally measured maximum deflection of 12.77 mm. Variations in laminate composition and ply orientation commonly seen during hand manufacture can create variations in observed laminate deflection [24]. Because of the high sensitivity of predicted deflection to laminate composition, it is likely that small deviations from the mean MFC thickness and PZT volume fraction would also introduce errors. The combined effect of these unknown variations in laminate and MFC composition could account for the observed errors in predictions of laminate deflection. Despite limitations in quantitative prediction, the model captures the cured shape and local reversals of curvature very well.

G. Snap-Through (Model and Experiment)

Finite-element prediction of snap-through voltage was achieved for both laminates, with non-linear buckling analyses predicting behavior before snap-through. For the $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, the model predicts snap-through at 645 V, whereas experimental observation showed that a drive voltage of 670 V induced snap-through. In the case of the $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminate, the predicted and observed snap-through voltages were 677 V and 700 V, respectively. The predicted snap-through voltages are in excellent agreement with the measured values with errors of less than 4.5% in both cases compared with experimental data. Delayed snap-through was observed at voltages immediately below monotonic snap-through voltage when drive-voltage was applied for a prolonged time period. This was caused by creep of the MFC actuators [31] and could be compensated for in industrial applications by using time-varying input signals [32] and closed-loop control. Because of the discontinuity of the voltage-deflection curve associated with snap-through, the Newton-Raphson solution procedure was not able to predict laminate response throughout the entire load cycle even with line search convergence control enabled.

To fully track voltage-deflection behavior of bistable laminates under MFC actuation, implementation of non-linear stabilization or the arc-length solution methods is necessary. However, neither non-linear stabilization nor the arc-length solution methods are currently implementable with SOLID226 elements. Therefore, the presented model represents the most appropriate formulation within the ANSYS V11.0 FE software and has significantly extended modeling capability and accuracy of coupled-field FE models in the prediction of actuation behavior of bistable composites.

VI. Conclusions

A homogenized FE model of a commercially available MFC actuator was developed and validated to allow prediction of actuator performance under combined electrical and stress fields. Three-dimensional piezoelectric and stiffness matrices for the MFC were calculated using experimentally determined orthotropic constants to create a homogenized material model of the MFC actuator. This data was validated by comparing FE predictions to experimental measurements of tip deflection of an isotropic beam with MFC actuators bonded to both top and bottom surfaces. Predicted values of deflection agreed with experimental data to within 2.5% over this range. With an MFC under closed-circuit boundary conditions, both deflection and gradient of the beam were predicted to within 1% of experimental measurement. Under open-circuit conditions, the developed electrical field within the MFC caused a reduction in beam gradient that remained within 1% of experimental values.

The MFC model was integrated with the model for bistable laminates to predict cured shape and snap-through behavior of two cross-ply bistable laminates. Challenges associated with integration of coupled-field elements with the composite structure have been addressed through application of an offset voltage to compensate for undesired thermal stresses within the MFC volume. Prediction of cured shape after MFC addition is in good agreement with experimental measurement, with maximum error between prediction and measured values of 12 to 16%. The change in cured shape caused by MFC addition and localized variation in curvature were predicted, and quantitative prediction of laminate deflection agrees with experiment sufficiently well to aid prototype design. Snap-through voltage for both $[0/90/0_{\mathrm{MFC}}]_{\mathrm{T}}$ and $[-30/60/0_{\mathrm{MFC}}]_{\mathrm{T}}$ laminates were predicted to within 4.5% of experimental measurements, which is a significant improvement upon previously unvalidated attempts at predicting snap-through. By including correctly formulated homogenized MFC properties and appropriate electrical constraints, the presented model improves and extends the applicability of FE techniques available for mechanism design of morphing structures based on bistable composites.

Acknowledgments

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Christopher Rhys Bowen has a B.Sc. degree in materials science from the University of Bath (1986–1990) and a D.Phil. in ceramics from the University of Oxford (1990–1993). Post-doctoral work has been undertaken at Tecnische Universität Harburg-Hamburg and the University of Leeds (1994–1996). He was Senior Scientist at the Defence Evaluation and Research Agency (DERA) from 1996 to 1998. He joined the University of Bath as a Lecturer in 1998. His research interests include dielectrics, piezoelectric materials, piezoelectric composites, and smart systems. He enjoys playing with his daughter, Rhiannon Bowen.

Peter F. Giddings was born in Norfolk, England in 1982. He began investigating novel multistable composite structures as part of an EPSRC-funded undergraduate research program whilst completing his M.Eng. degree in 2007 at the University of Bath, England. The work he started as an undergraduate formed the basis for a Ph.D. investigation into the use of piezoelectrically actuated bistable structures for morphing structures within aerospace applications that was co-funded by the Great Western Research (GWR and Airbus UK Ltd.) grant which was awarded in 2010 by the Department of Mechanical Engineering at the University of Bath. Since 2008, Dr. Giddings has developed successful businesses in biomechanical assessment and coaching of endurance cyclists and technical communication for specialist consumer publications and business-to-business media from his home in Toulouse, France.

Aki I. T. Salo was born in Mikkeli, Finland, in 1964. He received an M.Sc. degree in sport sciences from the University of Jyväskylä, Finland, in 1991 and a Ph.D. degree from the University of Exeter, England, in 1999. He worked as a research assistant in the Research Institute for Olympic Sports, Jyväskylä, Finland, from 1991 to 1994, after which he moved to England. After taking a few research assistant posts in England, since 1998, he has worked as a lecturer/senior lecturer in sport biomechanics at the University of Bath, England. While analyzing athletes' technique is Dr Salo's primary work, he has developed expertise in using human motion analysis systems innovatively to analyze the shapes and movement of bistable composites and active structures with colleagues at the Department of Mechanical Engineering in Bath. His skills offer a unique capability for three-dimensional shape characterization and provide significant experimental data in comparison to mathematical modeling of these structures.

Hyunsun Alicia Kim has B.Eng. and Ph.D. degrees (1993–2000) in aeronautical engineering from the University of Sydney, Australia, and carried out the post-doctoral research at the University of Warwick, UK, and Virginia Tech. She joined the University of Bath in 2001 and is currently a Senior Lecturer in Aerospace Structures. She is an affiliate at the Los Alamos National Laboratory. Her research interests are in the development and application of computational methods for modeling and optimization. More specifically, they are in the fields of structural and topology optimization, optimization of actuated composites, material modeling of nonlinearity and hysteresis, bone remodeling, and surrogate modeling.