Electron Channelling by Microscopic Magnetic Potentials

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Abstract. While conventional electronic devices rely on electrostatic potentials to accelerate and confine charge carriers, a novel type of device has been proposed that uses inhomogeneous magnetic fields to channel and filter electrons. The stray fields emanating from ferromagnetic stripes fabricated at the surface of a two dimensional electron gas were used to deflect the ballistic electrons underneath. Magnetotransport experiments reveal a resistance resonance due to the formation of two types of magnetic edge states that drift in opposite directions perpendicular to the magnetic field gradient. A systematic study of Ni, Fe and Dy devices shows that the peak position increases proportionally to the amplitude of the magnetic modulation. The magnetic origin of the resonance is further evidenced by the collapse of channelling above the Curie temperature of dysprosium. Its helical/ferromagnetic phase transition provides an original means for switching on the magnetic modulation independently of the applied magnetic field by changing the temperature. We use this effect to demonstrate that the magneto-resistance due to snake orbits is one order of magnitude higher that the magneto-resistance due to the stripe being magnetised while the modulation remains uniformly positive.

Experiments in tilted magnetic fields demonstrate that both the resonant peak and the ratio of the Hall resistance upon the Hall resistance of the bare 2D electron gas scale with the perpendicular component of the applied magnetic field. This demonstrates that the Hall resistance measures the trapping of electrons in or out of magnetic edge states rather than the average magnetic field in the Hall junction. Both results were expected from a simple drift-diffusion picture.

Discrete resistance steps obtained when changing the electron density are tentatively assigned to the formation of quantum confined 1D snake subbands. The gate bias dependence of the resonant peak is discussed in relation to the realistic modulation profile.

1. Introduction

The problem of two-dimensional particles in an inhomogeneous magnetic field has received considerable attention over the past decade driven by the need to understand the scattering and localisation of Composite Fermions in the Fractional Quantum Hall regime [1, 2, 3, 4, 5, 6, 7, 8, 9]. The classical transport properties in short range magnetic potentials are well described by the Drude model [9, 10] however, when the correlation radius of magnetic fluctuations becomes larger than the cyclotron radius, the conductivity becomes dominated by a small number of electron orbits meandering about contours of zero magnetic field. These so-called ”snake orbits” were shown to dramatically enhance the magneto-resistance of magnetic superlattices by up to 1700% [11, 12]. Magnetic superlattices are however of limited use for studying the fine structure of snake orbits because (i) the longitudinal resistance averages over several superlattice periods and (ii) the Hall resistance is unaffected by the periodic modulation [13, 14, 15]. The electronic structure of 1D [16, 17, 18] and 2D [19, 20] magnetic

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Minibands have been calculated and observed experimentally [21]. Magnetic potentials have been implemented in devices incorporating non-planar two-dimensional electron gases (2DEG) [22, 23] and by fabricating superconducting elements [24] or micromagnets [25, 26, 11, 27, 28, 12, 29, 30] at the surface of a 2DEG. The devices investigated in this paper belong to the last type since the magnetic modulations produced in this way are non invasive and can be as large as 1T. Recent theoretical works have proposed the implementation of magnetic barriers as tunable momentum filters [31], resonators [32] or magnetic switches [33]. Sharp resonances in the magnetisation [34] and conductance [35] of a magnetic antidot have been associated with the formation of 1D magnetic edge states [36] circulating clockwise and anticlockwise about the antidot.

This paper reports on electron transport perpendicular to an abrupt step of magnetic field. A resistance resonance peak is obtained that marks the transition from a regime dominated by the diffusion of snake states to a regime dominated by cycloid orbits. The peak position, \( B_p \), corresponds to the value of the external magnetic field that cancels the negative modulation. This conclusion is supported by (i) the linear dependence of \( B_p \) on the amplitude of the negative modulation, \( B_m \), in Ni, Fe and Dy devices (ii) the scaling of the peak with the normal component rather than the modulus of the applied field, (iii) the suppression of snake orbit channelling above the Curie temperature of the ferromagnet, (iv) the order of magnitude difference in the size of the magnetoresistance when snake channelling is present or not. The resistance of a magnetic step with zero average field was also measured by varying the Fermi energy of the 2DEG. A periodic structure is obtained whose period is compared to the energy separation of the 1D snake subbands in the magnetic step. A careful investigation of the Hall resistance behaviour in tilted magnetic fields shows that it measures the variation in the number of snake states rather than the average magnetic field in the Hall junction. We justify our use of a drift-diffusion model and show it describes the experimental results remarkably well.

2. Resistance Resonance

Standard Hall devices were fabricated from a \( \delta \)-doped \( Al_{0.3}Ga_{0.7}As/GaAs \) quantum well the centre of which was \( z_0=24nm \) below the surface. Its lowest energy subband hosted a 2DEG with electron density \( n_s = 4 \times 10^{15} m^{-2} \) and mean free path \( l = 4.0 \mu m \) (1.3K). Electron beam lithography and lift-off was used to fabricate a ferromagnetic stripe above the centre of the channel [37, 38] as shown in Fig. 1 (a). Its thickness \( h=140nm \) (Dy), 90nm (Fe), 200(Ni) \( \pm 5nm \) differed according to the ferromagnet used, however all stripes were \( 32\mu m \) long and \( w=400nm \) wide. The Hall channel was finally gated (i) to avoid strain induced piezoelectric potentials [39], (ii) to change the electron density, (iii) to protect the fragile magnetic properties of rare earths from hydrogen contamination [40] or oxydisation. The sample was cooled down to 1.3K in a superconducting magnet equipped with a rotation insert. d.c. resistance measurements were performed in anticipation of possible rectification effects due to the unidirectional drift of snake orbits. Currents of 1 \( \mu A \) or less were applied and the induced voltages were measured across voltage probes with lateral separation ranging from 4\( \mu m \) to 24 \( \mu m \).

Fig.1 (b) shows the stray fields emanating from the stripe for four orientations of the magnetisation [41]. The orbital electron motion in the 2DEG is affected by the z-component of the stray magnetic field only. A perpendicular magnetisation (\( \theta = 0 \)) gives a positive modulation below the stripe and a negative modulation away from it whereas an in-plane magnetisation (\( \theta = 90^\circ \)) divides the 2DEG into two half planes with opposite modulation. At the boundary line between positive and negative regions, the Lorentz force redirects electrons towards the region of opposite magnetic field. These meandering or “snake” orbits dominate
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Figure 1. (a) Micrograph of the 2 µm wide Hall channel. The Dy stripe fabricated above its centre was \( w = 400\text{nm} \) wide and \( h = 140\text{nm} \) thick. The 2DEG lies \( z_0 = 24\text{nm} \) below the surface. (b) The device cross-section (at scale) showing the stray magnetic field threading the stripe when the magnetisation is tilted at various angles with respect to the vertical. The magnetisation was assumed to be uniformly saturated.

the transport properties because they travel at nearly the Fermi velocity whereas the cyclotron centres remote from the boundary remain stationary. The resistivity is therefore reduced below the Drude value \( \rho_0 = \frac{h}{(e^2 k_F l)} \) where \( k_F = \sqrt{\frac{2}{\pi n_s}} \) is the Fermi wave-vector. The external magnetic fi eld \( B_a(\theta = 0) \) reduces the modulation in the regions where it is negative, hence the number of snake orbits decreases and \( \rho_{xx} \) increases. Approximating the magnetic barrier by a square magnetic step with negative amplitude, \( B_m \) [37], \( \rho_{xx} \) is found to peak at \( \rho_{xx} = \rho_0 \) when \( B_a = B_m \). Beyond the peak, cycloid orbits form that acquire a fi nite guiding centre drift due to the different curvature radii at both ends of their trajectory. The number of cycloid orbits fi tting the channel width increases whilst their drift velocity decreases, as a result \( \rho_{xx} \) goes through a minimum. Increasing the magnetic fi eld further fi lls the Fermi surface with cycloid orbits and produces a shoulder in \( \rho_{xx} \) [37]. At \( B_a \gg B_m \), the motion of cycloid orbits is well described by the Alvén drift [42] as they become increasingly stationary.
The magnetoresistance of Ni, Fe, Dy devices is shown in Fig.2 and compared with the magnetoresistance of a test sample without stripe. The negative magnetoresistance of the test sample, at $B_a < 110 mT$, is due size effects [43] involving collisions with diffuse channel boundaries [38]. The magnetic devices exhibit a resistance peak at $B_p=14 mT$ (Ni), $28 mT$ (Fe), $104 mT$ (Dy) which one compares to the value of $B_m$ shown in the inset: $B_m=\pm 44.6 mT$ (Ni), $\pm 132 mT$ (Fe), $\pm 256.8 mT$ (Dy). The modulation profiles were calculated by assuming the ferromagnets to be ideal which allowed us to use the following saturation magnetisations $M_s=0.51 T$ (Ni), $1.74 T$ (Fe), $2.92 T$ (Dy) in the expression:

$$B_c(y) = \frac{M_s}{2\pi} \left[ \cos \theta \left\{ \arctan \frac{4ac[z^2 - c^2 - y^2 + a^2]}{[z^2 - c^2 - y^2 + a^2]^2 + 4ac^2(y^2 - a^2)} + 4a^2(z^2 - c^2) \right\} \right]$$

$$+ \sin \theta \left\{ \ln \left[ \frac{(z+c)^2 + (y-a)^2}{(z-c)^2 + (y-a)^2} \right] + \ln \left[ \frac{(z+c)^2 + (y+a)^2}{(z-c)^2 + (y+a)^2} \right] \right\}$$

Here the origin of coordinates is at the centre of the magnet of Fig.1 (b), $a = w/2$ and...
Figure 3. The shape of the magnetic barrier for different tilt angles in the Dy device.

For all three magnetic devices we find $|B_p/B_m| = 0.3 \pm 0.1$. Since the $M_s$ differs by a factor of 6 between nickel and dysprosium, we conclude that the peak position increases proportionally to $B_m$.

Measuring the resistance over voltage probe separations as large as 24$\mu$m, i.e. distances larger than the mean free path, had no significant effect on the shape of the resonance. The survival of the positive magnetoresistance up to 90K similarly points to a diffusive origin of the resonance peak. The drift velocity of magnetic edge states, $v_d$, enhances the diffusion along the $x$-axis by $\delta D_{xx} = \langle v_d^2 \rangle / \tau$ where $\tau$ is the elastic scattering time and the average is over the Fermi surface. Einstein’s equation $\rho = h/(4\pi m^* e^2) D^{-1}$ allows to calculate the new resistivity components as:

$$\frac{\rho_{xx}}{\rho_0} = 1 - \frac{2 \langle v_d^2 \rangle / v_F^2}{1 + 2 \langle v_d^2 \rangle / v_F^2}$$  \hspace{1cm} (2)

and

$$\rho_{xy} = \frac{B_g \rho_{xx}}{en_s \rho_0}$$  \hspace{1cm} (3)

$\rho_{xx}$ was calculated and plotted in the case of a simple magnetic barrier profile [37].

Unfortunately, the magnetisation curve of the ferromagnet is not known and, as we shall see below, it is not accessible by Hall magnetometry. The formation of regions of negative modulation field indeed requires a ferromagnet sufficiently hard for, at least at low magnetic field, the stray fields are larger than the applied field. In the case of a soft ferromagnet a different resistance peak is expected to occur when the magnetisation saturates [44]. The initial positive magnetoresistance is then explained by the depopulation of quantised conductance channels while the stripe is being magnetised. Our understanding of hybrid structures is further complicated by the many stable magnetic domain structures known to exist when the magnet size decreases and because of the absence of epitaxial growth.
conditions on GaAs. A soft ferromagnet would have its magnetisation depending on the total magnetic field $B_a$ whereas the suppression of snake states would occur at a constant value of its normal component $B_z = B_a \cos \theta$ equal to the amplitude of the negative modulation field. The dependence of the magnetoresistance when the external field is tilted in the plane perpendicular to the stripe, see Fig1 (b), therefore provides a means of distinguishing between the hard and the soft magnetisation scenarios. The corresponding modulation profile was calculated from Eq.1 and shown in Fig.3.

Away from grazing angles, the magnetisation aligns with $B_a$ [45]. A comparison of the angular calibration of the rotation insert with the $\cos \theta$ scaling of both the high field Hall resistance and the Shubnikov de-Haas minima allowed to measure $\theta$ to an accuracy of less than 1 degree of angle. Fig.4 (a) shows the peak position, plotted at a function of $B_a$, that shifts from $B_p = 0.151T$ at $\theta = 0$ to $8.347T$ at $\theta = 89^0$. This variation over almost two orders of magnitude clearly cannot be due to the saturation of the ferromagnet. If instead the same curves are plotted as a function of $B_z = B_a \cos \theta$, as done in Fig.4 (b), both the peak position and the resistance lineshape below 1T are found to be independent of $\theta$. The peak therefore occurs at a constant value of $B_z$ supporting the hard magnet scenario. The quenching of the Shubnikov de Haas oscillations from the $\theta = 0$, $36^0$ traces to the $\theta = 64^0, 74^0$ traces indicates a change in the shape of the magnetic modulation: at $\theta = 0$, Landau levels can possibly form in the relatively square magnetic barrier of Fig.3 whereas at $\theta = 90^0$, its triangular shape lifts the Landau level degeneracy. This could explain the quenching of Shubnikov-de Haas oscillations.

Fig.4 (c) plots the ratio of the Hall resistance upon the resistance of the bare 2DEG
obtained by linear extrapolation of the high magnetic field behaviour. Eq.3 shows that this quantity should be identical to the resistivity normalised by the Drude resistivity. Indeed, a peak is observed at $B_a = 0.21$ in the $\theta = 0$ curve of Fig.4 (a). This represents a shift to slightly higher magnetic field compared to $B_p = 0.15T$. Similar offsets have been reported in all our devices [37] although their origin is not fully understood. Quite remarkably, increasing the tilt angle to $\theta = 81^\circ$ results in a $\cos \theta$ dependence of the peak position similar to that found in $\rho_{xx}$. This invariance of the peak position is demonstrated in Fig.4 (d). Clearly the Hall ratio measures changes in the 2DEG diffusivity due to the inhomogeneous current distribution rather than the average magnetic field in the Hall junction which would peak when the ferromagnet saturates. This result was predicted by Eq.3. Also, electrons trapped in snake states cannot reach the Hall probes. This effect was verified by a sub-linear Hall resistance at $B_a = 0$. As $B_a$ increases from zero, the untrapping of snake states liberates more electrons and increases the Hall resistance up to the peak. The difference between our experimental situation and other magnetically modulated 2DEGs in the diffusive regime [27, 29] is that snake states here form an open subsystem which is independently connected to the current contacts. This result also contrasts with the ballistic regime of magnetic antidots [46] where the Hall resistance was shown to measure the modulation field averaged over the Hall device. We have therefore shown that the diffusion picture interprets the data successfully. X-ray diffraction spectra of thin dysprosium films on GaAs hinted that the hard ferromagnetic behaviour of dysprosium could arise from the hard magnetic axis growing in the plane [45].

Convincing evidence for snake channelling is equally derived from the temperature dependence of the dysprosium device shown in Fig.5. The V-shape magnetoresistance of snake channelling still exist at 50K whilst the valley at $B_a > B_p(\theta = 0)$ has already disappeared at 20K. The relative robustness of snake channelling is due to the small period of snake trajectories compared to the cycloid orbits. The latter temperature dependence is similar to that of Shubnikov de-Haas oscillations. Since the peak position cannot be defined directly, we used the minimum of the second derivative plotted in the inset to Fig.4. We now focus on the temperature dependence near the Curie temperature of Dysprosium $T_c = 85K$. Clearly the V-shaped magnetoresistance persists up to 80K then abruptly collapses in the 90K and 100K curves. Between 50K and 80K the peak position slightly decreases, although this trend is comparable to the magnitude of the experimental error. This is a paradoxical result because one would a priori expect the spontaneous magnetisation to vanish at $T_c$ and both the magnetic modulation and $B_p$ to vanish. The third point to be explained by the model are the two residual peaks in the 90K and 100K curves whose position increases with temperature at a rate of $7mT/K$.

To answer these three points an accurate description of the magnetisation of dysprosium is required [47]. Dysprosium is ferromagnetic up to 85K but instead of a paramagnetic phase above $T_c$, it displays a helical phase in which the magnetic moments wind around the c-axis. Recalling that the c-axis (hard axis) lies randomly in the plane [45], an in-plane magnetic field will tend to reinstate ferromagnetic order once a critical value $B_H$ is reached. This transition is shown in Fig.5 (b). This figure was plotted by inserting the published magnetisation isotherms of dysprosium crystal [47] into Eq.1 to calculate the amplitude of the negative modulation. Raising temperatures make the relatively disordered helical phase more stable, this explains the increase of $B_H$ seen in Fig.5 (b). Because the helical phase magnetisation is vanishingly small compared to the ferromagnetic phase, the temperature can be used in a very original way to switch on the magnetisation at arbitrary values of $B_a$. For example the effect of applying the magnetic barrier at zero average magnetic field ($T < T_c$) can be directly compared with the situation where it is applied while the average magnetic field is larger than the barrier height ($T > T_c$). We therefore have a remarkable tool for differentiating the magnetoresistance arising from snake channelling (sign alternating modulation) and from the magnetisation
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Figure 5. (a) The magnetoresistance of a dysprosium device near the Curie temperature $T_C = 85K$ ($\theta = 0$). The peak position is defined by the minimum of the second derivative shown in the inset. (b) The negative modulation isotherms (full lines) compared to the applied magnetic field (dotted line). At $B_a = B_H$, the applied field destroys the helical phase of Dy and ferromagnetic order is reinstated. (c) The temperature dependence of the peak position (squares) and $B_H$ (open circles).

increase (positive modulation). The applied magnetic field is plotted through the $B_m = B_a$ line in Fig.5 (b). In the 0.2T range between the two intersection points where $B_m > B_a$, regions of negative modulation sustain snake channelling. As temperature increases, both intersections move towards each other. Snake orbit channelling disappears when the $B_m = B_a$ curve is tangential to the knee of the $B_m$ curve. Note that when snake channelling disappears the upper intercept has dropped by 20% but does not vanish. This explains the experimental finding that $B_p$ never reaches zero. At $T=100K$ and above, intersections disappear and the collapse in the magnetoresistance is due to the suppression of snake channelling. At $T=100K$ and 120K the magnetisation switches on ‘too late’ and the modulation is positive throughout the 2DEG.

The origin of the residual peak seen at 90K and 100K in Fig.5 is understood by plotting $B_p$ together with $B_H$ [47] as a function of $B_a$ in Fig.5 (c). Both trends match within 5% both in terms of absolute magnitude and rate of increase 7-10mT/K. The residual peaks are therefore ascribed to the onset of ferromagnetic alignment at $B_H$. Comparing the size of magnetoresistance below $T_c$ (sign alternating modulation) and above $T_c$ (positive modulation) allows us to conclude that changes in the magnetic modulation have an effect ten times greater in the magnetoresistance when snake channelling is involved. Conversely, these experiments show that magnetic edge states can act as local probes of the magnetic phase transitions in micromagnets.
Figure 6. Gate bias dependence of the resonant peak position for the positive \((B_{p+})\) and negative \((B_{p-})\) values of the applied field (with \(\theta = 0\)). The dashed line is the peak position calculated using Eq.4.

3. Realistic Modulation Profiles

One problem complicating the interpretation of data is that, although a square step approximation of the modulation profiles in Fig.3 is sufficient for a qualitative understanding, real magnetic field profiles must generally be considered. One obvious question is then what determines \(B_p\)? One may also ask why is the peak position invariant in Fig.4 (b,d) when in Fig.3 the amplitude of the negative modulation increases from -0.3T \((\theta = 0)\) to -0.8T \((\theta = 90^0)\)? Fig.6 shows the gate voltage dependence of the positive \((B_{p+})\) and negative \((B_{p-})\) peak positions. These are not independent of \(n_s\) as was initially reported [37]. The peak position is largest at low electron densities and decreases to 1/3 of its value when the gate bias increases from -0.2V to +0.3V. This decay is inversely proportional to the mean free path that we calculate from the electron mobility and density measured for each gate bias. Note that magneto-size effects in the Hall channel [43] would instead give an increase in the peak position with increasing electron density. We propose the following simple empirical formula:

\[
B_p = \frac{1}{l} \int_{y(B=0)}^{y(edge)} dy \, B_c(y) \tag{4}
\]

to fit the peak dependence in Fig.6. In addition, the integral of \(B_c(y)\) over the negative barrier is about the same at \(\theta = 0\) and \(\theta = 90^0\): using Eqs.4 and 1 we find \(B_p = 0.15T\) at \(\theta = 0\) and \(B_p = 0.20T\) at \(\theta = 90^0\). Eq.4 thus also accounts reasonably well for the invariance of the peak in Fig.4 (b). A theoretical justification for Eq.4 can be found in the conservation of the electron action that implies each snake orbit encloses a constant magnetic flux over one period [9].
4. One-dimensional Snake Subbands

An exciting prospect is the observation of discrete 1D snake subbands associated with quantum mechanical confinement in the magnetic potential. We have experimentally realised a magnetic step with zero average modulation by magnetising the Dy stripe with a large in-plane magnetic field $(\theta = 90^\circ)$. The in-plane alignment was accurately realised by detecting the zero Hall voltage in an external field of 5T. Fig. 7 (a) shows the dependency of $R_x$ when continuously changing the gate voltage. The +2T, +0.5T, -0.5T curves show a regular structure which is emphasized in the second derivative. The 5 minima in the second derivative are regularly spaced by 120mV, this periodicity remaining unaffected by the applied field between 2T and -0.5T. As the magnetic field reverses to -2T and -5T these oscillations disappear from the resistance curves. Using the correspondence between the electron density and the gate voltage, we estimate that the distance between two consecutive minima translates into a 0.4meV change in the Fermi energy of the 2DEG.

In order to interpret these results, the magnetic subbands [44] were calculated in the case of a simple magnetic step where the field jumps from $B_0$ to $-B_0$ at $y=0$. The eigenenergies $E_n(k_x)$ depend on the position of the oscillator centre $y_c = 7heB/(eB_0)$ which parametrises Schrödinger equation:

$$\left\{-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + \frac{1}{2} m^* \omega_c^2 (|y| - y_c)^2\right\} \phi(y) = E_n(k_x) \phi(y) \tag{5}$$

Eq.5 was solved numerically and gave the energy subbands plotted in Fig. 7 (b) (full lines). The energy states at large positive values of $k_x > \sqrt{2n + \frac{1}{2}}/y_c$ do not feel the magnetic edge. They correspond to twice degenerate Landau oscillators rotating clock-wise and anti-clockwise in the $\pm B_0$ half planes away from the edge. At smaller values of $k_x$, the wavefunctions of these two states start overlapping across the magnetic tunnel barrier and they combine to form bonding and antibonding states shown in Fig. 7 (c). An interesting point to note is that the bonding and antibonding states have opposite group velocities which is a pure quantum effect [44]. A semiclassical interpretation for this is that antibonding electrons circle all the way around cyclotron centres and as a result have their wavefunction localised at $\pm y_c$. By contrast, bonding electrons describe a small arc of trajectory in one cyclotron centre before skipping to describe another small arc on the opposite side; the antibonding wavefunction is therefore localised within the magnetic barrier. When $k_x$ becomes positive the orbit centre $y_c$ belongs to the half plane opposite to that of the ‘trajectory’. The bonding and anti-bonding states become snake states tightly bound to the magnetic edge. The semiclassical description of Fig. 7 (c) is quite satisfactory except near the classical boundary where tunnelling occurs. This can be seen by comparing, in Fig. 7 (b), the exact dispersion curves to the energy subbands calculated using the Wentzel-Kramers-Brillouin approximation [48]. We find the WKB energies are given by the following equation:

$$\begin{cases} E_n(k_x) = (n + 1/2) \hbar \omega \frac{\pi}{2\varphi + \sin 2\varphi} & \text{if } E_n(k_x) > \frac{\hbar k_x^2}{2m^*} \\ E_n = (n + 1/2) \hbar \omega & \text{otherwise} \end{cases} \tag{6}$$

where $\varphi = \arcsin(\sqrt{2m^*E_n(k_x)})$. Eq.6 was solved iteratively.

The energy dispersions in the real magnetic field profile relevant to this experiment are yet to be calculated but their properties can be extrapolated from Fig. 7. The symmetry of the problem at $\theta = 0$ will conserve the clockwise/anti-clockwise degeneracy but the Landau level degeneracy will be lifted by the spatially varying modulation. As a consequence the density of states will exhibit a series of divergent singularities originating at the bottom of the 1D.
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Figure 7. (a) Periodic structure seen in the resistance (arrows) and in its second derivative (lower curve) when the Fermi energy is swept across the magnetic step drawn in the inset. (b) The quantum subbands in a square magnetic step (inset) calculated from Eq.5 (full lines) and from the WKB approximation Eq.6 (dashed lines). The dotted line parabola is the classical boundary between those oscillator centers whose trajectories cross the magnetic step (left) and the pure Landau levels (right). (c) A semi-classical explanation for the opposite group velocities of the bonding and antibonding states ($k_x > 0$) and the snake trajectories ($k_x < 0$).

subbands. These have been calculated in the case of magnetic superlattices [17]. The energy difference between consecutive subband minima, such as those of antibonding subbands in Fig.7, will be of the order of $\hbar \omega_0$ which in the case of our experiment gives a modulation field $B_0 = 0.23T$. This magnetic field is comparable to the 0.8T height of the negative barrier shown in Fig.3 and to the 0.2T equivalent square step calculated using Eq.4. We may therefore conclude to having observed the fingerprints of quantum confined snake subbands.

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References

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