Chapter 4: Nonlinear fiber optics: part 1

4.1 Linear Schrodinger equation in fibers and Group velocity dispersion
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4.5 Raman and Brillouin effects

Book: Robert Boyd, ‘Nonlinear Optics’
Chapter 13.2 Ultrashort pulse propagation equation
Chapter 10 Stimulated Raman Scattering
Chapter 9 Stimulated Brillouin Scattering

Book: Govind Agrawal, ‘Nonlinear Fiber Optics’ (Chapters are numbered as in 3rd edition)
Chapter 2.3 Pulse propagation equation
Chapter 5.1.1, 5.1.2 Modulational instability
Chapter 6.1.1, 6.1.2, 6.3.1, 6.3.3 Polarization effects
Chapter 8.1 Stimulated Raman Scattering: Basic Concept
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4.1 Linear Schrödinger equation in fibers and group velocity dispersion

\[ E = \int d\omega A(\omega, z) e^{-i\omega t} + c.c. \]

We retain here some spectrum of frequencies

\[ \left( \partial_z^2 + \frac{\omega^2}{c^2} \right) A + \left( \partial_x^2 + \partial_y^2 \right) A + \frac{2i\omega}{c^2} \partial_t A = -\frac{\omega^2}{c^2\epsilon_0} P_0 \]

We select a wavenumber \( k_0 = \omega_0 / c \)
\[ A(\omega, z) = B(\omega, z) e^{ik_0z} \]
\[ P_0 = \epsilon_0 \chi A = \epsilon_0 (\chi_1 + \chi_3 |A|^2 + \chi_5 |A|^4 + \ldots) A \]

\[ 2ik_0 \partial_z B + \partial_z^2 B = (k_0^2 - k^2) B \]
\[ k^2 = \frac{\omega^2}{c^2} (1 + \chi_1) \]
\[ k + k_0 \approx 2k_0 \]
\[ i\partial_z B \approx (k_0 - k) B \]
Now let’s think about \((k_0 - k)\)

\[ k(\omega) = k_0 + \beta_1(\omega - \omega_0) + \frac{1}{2!}\beta_2(\omega - \omega_0)^2 + \frac{1}{3!}\beta_3(\omega - \omega_0)^3 \ldots \]

It can be either Taylor expansion or polynomial fit to the true dispersion

\[ E = \int d\omega B(z, \omega)e^{-i\omega t + ik_0 z} + c.c. = B_0(z, t)e^{-i\omega_0 t + ik_0 z} + c.c. \]

\[ B_0(z, t) = \int B(z, \omega)e^{-i(\omega - \omega_0) t} d\omega \]

Differentiating \(B_0\) in \(z\) we find

\[ \partial_z B_0(z, t) = \int \partial_z B(z, \omega)e^{i(\omega - \omega_0) t} d\omega \]

\[ i\partial_z B = (k_0 - k)B \]
\[ \partial_z B_0(z, t) = \int (\beta_1(\omega - \omega_0) + \frac{1}{2!} \beta_2(\omega - \omega_0)^2 + \frac{1}{3!} \beta_3(\omega - \omega_0)^3 \ldots) e^{-i(\omega - \omega_0)t} B(\omega, z) d\omega \]

\[ i \partial_z B_0 = -\left( \frac{i^1}{1!} \beta_1 \partial_t B_0 + \frac{i^2}{2!} \beta_2 \partial_t^2 B_0 + \frac{i^3}{3!} \beta_3 \partial_t^3 B_0 + \ldots \right) \]

\[ \beta_n \quad \text{are the Dispersion coefficients of different orders} \]

\[ \beta_n = \frac{\partial^n k}{\partial \omega^n} \quad \text{If we’ve done Taylor expansion, not a polynomial fit} \]
Dispersion revisited

One can say that we are talking about **Phase Velocity Dispersion**

\[
\frac{dn}{d\omega} > 0 \quad \text{NORMAL P.V. DISPERSION}
\]

\[
\frac{dn}{d\omega} < 0 \quad \text{ANOMALOUS P.V. DISPERSION}
\]
Normal dispersion at the air glass interface

\[ n = 1 \quad n = f(\lambda) > 1 \]

Fig. 3.2. Refraction at an air–glass interface leads to a spreading of the colors given for normal dispersion
Group velocity dispersion

\[ v_g = \frac{\partial \omega}{\partial k} = \frac{1}{\partial_\omega k} = \frac{1}{\partial_\omega [\omega n(\omega)/c]} = \frac{c}{n(\omega) + \omega n'(\omega)} = \frac{c}{n_g} \]

Group index plot

\[ v_{phase} = \frac{c}{n} \]

\[ \frac{dn_g}{d\omega} > 0 \]

Normal GROUP VELOCITY DISPERSION

\[ \frac{dn_g}{d\omega} < 0 \]

Anomalous G.V.D.

\[ \beta_2 = \frac{\partial^2 k}{\partial \omega^2} = \frac{\partial}{\partial \omega} \frac{n_g}{c} \]

That is why \( \beta_2 \) is called GVD coefficient
4.2 Nonlinear Schrödinger equation in fibers

Continuous wave solution of the linear Schrödinger equation

\[ E = B_0(z, t)e^{-i\omega_0 t + ik_0 z} + c.c., \quad k_0 = k(\omega_0) \]

\[ i\partial_z B_0 = - \left( \frac{i^1}{1!} \beta_1 \partial_t + \frac{i^2}{2!} \beta_2 \partial_t^2 + \frac{i^3}{3!} \beta_3 \partial_t^3 + \ldots \right) B_0 \]

\[ B_0 = A e^{i\kappa z - i\delta t} \quad \delta = \omega - \omega_0 \]

\[ \kappa A = \left( \frac{1}{1!} \beta_1 \delta + \frac{1}{2!} \beta_2 \delta^2 + \frac{1}{3!} \beta_3 \delta^3 + \ldots \right) A \]

\[ k(\omega) = k_0 + \beta_1(\omega - \omega_0) + \frac{1}{2!} \beta_2(\omega - \omega_0)^2 + \frac{1}{3!} \beta_3(\omega - \omega_0)^3 + \ldots \]

Thus \(\delta\) and \(\kappa\) are deviations from the reference frequency and the reference wavenumber.
Continuous wave solution for the NONlinear Schrodinger equation

Nonlinear term is introduced to give a valid(?) expression for nonlinear refractive index

\[ n = n_0(\omega) + n_2|B|^2, \quad k_{\text{nonlin}} = n\frac{\omega}{c} = k(\omega) + \frac{\omega n_2}{c} |B|^2 \]

\[ i\partial_z B_0 = - \left( \frac{i}{1!}\beta_1 \partial_t + \frac{i^2}{2!}\beta_2 \partial_t^2 + \frac{i^3}{3!}\beta_3 \partial_t^3 + \ldots \right) B_0 - \frac{\omega_0 n_2}{c} |B_0|^2 B_0 \]

\[ B_0 = Ae^{i\kappa z - i\delta t} \quad \delta = \omega - \omega_0 \]

\[ \kappa = \left( \frac{1}{1!}\beta_1 \delta + \frac{1}{2!}\beta_2 \delta^2 + \frac{1}{3!}\beta_3 \delta^3 + \ldots \right) + \frac{\omega_0 n_2}{c} |A|^2 \]

\[ \kappa = k(\omega) - k_0 + \frac{\omega_0 n_2}{c} |A|^2 \]

\[ E = Ae^{-i\omega t + ik(\omega)z + i\omega_0 n_2 |A|^2 z/c} + c.c. \]

Note, however, that the nonlinear part of \( \kappa \) is fixed to its value at \( \omega_0 \). It means, that our model correctly reproduces linear dispersion, but approximates nonlinear coefficient to its value at \( \omega_0 \).
Traditional for fiber optics scaling uses units of power for the field squared

$$B'_0 = \sqrt{S} \sqrt{\frac{1}{2} \frac{\varepsilon_0 n_0 c}{\varepsilon_0 n_0 c} B_0}$$

2nd root makes the ampl. measured in the units of Sqrt(intensity) and the area S makes it measured in the units of sqrt(power)

$$i\partial_z B'_0 = -\left(\frac{i^1}{1!} \beta_1 \partial_t + \frac{i^2}{2!} \beta_2 \partial_t^2 + \frac{i^3}{3!} \beta_3 \partial_t^3 + \ldots\right) B'_0 - \gamma |B'_0|^2 B'_0$$

$$\gamma = \frac{\omega_0 2n_2}{cS\varepsilon_0 n_0 c} = \frac{\omega_0 n_2^I}{cS} > 0$$

$$[\gamma] = \left[ \frac{1}{Wm} \right]$$

Nonlinear parameter is positive in glass fibers for all the relevant frequencies
4.3 Degenerate 4WM in fibers (modulational instability)

\[ i \partial_z B = - \left( \frac{i^1}{1!} \beta_1 \partial_t + \frac{i^2}{2!} \beta_2 \partial_t^2 + \frac{i^3}{3!} \beta_3 \partial_t^3 + \ldots \right) B - \gamma |B|^2 B \]

We choose the reference frequency to be the pump frequency \( \omega_p = \omega_0 \)

\[ B = A_p(z) e^{i\kappa_p z} + A_s(z) e^{i\kappa_s z} i\Omega t + A_i(z) e^{i\kappa_i z + i\Omega t} \]

\[ \omega_{s,i} = \omega_0 \pm \Omega \quad \delta \kappa = 2\kappa_p - \kappa_i - \kappa_s \]
SPM is the nonlinearity induced shift of the wavenumber related to the wave itself

\[ e^{i\omega t} \left( k_p - k(\omega_p + \Omega) \right) A_p - \gamma (|A_p|^2 + 2|A_i|^2)^2 A_p \]

XPM is the nonlinearity induced change of the wavenumber related to other waves

\[ e^{i\Omega t} \left( k_p - k(\omega_p - \Omega) \right) A_i - \gamma (|A_i|^2 + 2|A_s|^2)^2 A_i \]

\[ e^{i\delta t} \left( k_p - k(\omega_p - \Omega) \right) A_i - \gamma (|A_i|^2 + 2|A_s|^2)^2 A_i \]
Signal and idler are small. Pump is strong and Z independent

\[ i \partial_z A_p - \kappa_p A_p = -\gamma |A_p|^2 A_p \]

\[ i \partial_z A_s - \kappa_s A_s = \left( k_p - k(\omega_p + \Omega) \right) A_s - 2\gamma |A_p|^2 A_s - \gamma A_p^2 A_i^* e^{i\delta \kappa z} \]

\[ i \partial_z A_i - \kappa_i A_i = \left( k_p - k(\omega_p - \Omega) \right) A_i - 2\gamma |A_p|^2 A_i - \gamma A_p^2 A_s^* e^{i\delta \kappa z} \]

Now we are selecting kappa’s

\[ \kappa_p = \gamma |A_p|^2 \]

\[ \kappa_s = k(\omega_p + \Omega) - k_p + 2\gamma |A_p|^2 \]

\[ \kappa_i = k(\omega_p - \Omega) - k_p + 2\gamma |A_p|^2 \]

\[ i \partial_z A_s = \gamma A_p^2 A_i^* e^{i\delta \kappa z} \]

\[ i \partial_z A_i = -\gamma A_p^2 A_s^* e^{i\delta \kappa z} \]
\[ i \partial_z^2 A_s = i \gamma^2 |A_p|^4 A_s - \delta \kappa \partial_z A_s \]

\[ A_s \sim e^{\lambda z}, \quad \lambda = \frac{i}{2} \delta \kappa \pm \sqrt{\gamma^2 |A_p|^4 - \frac{1}{4} (\delta \kappa)^2} \]

\[ -2\gamma |A_p|^2 < \delta \kappa < 2\gamma |A_p|^2 \]

\[ \delta \kappa = \delta k_{\text{nonl}} \]
\[ = 2[k_p + \gamma |A_p|^2] - [k(\omega_p + \Omega) + 2\gamma |A_p|^2] - [k(\omega_p - \Omega) + 2\gamma |A_p|^2] \]
\[ = \delta k - 2\gamma |A_p|^2 \]

\[ 0 < \delta k < 4\gamma |A_p|^2 \]
Modulational instability growth rate (gain)

\[ \text{Re} \lambda = \text{Re} \sqrt{\gamma^2 |A_p|^4 - \frac{1}{4} (\delta \kappa)^2} = \text{Re} \sqrt{-\frac{1}{4} \delta k (\delta k - 2\gamma |A_p|^2)} \]

Odd vs Even orders of the dispersion

\[ \delta k = 2k_p - k(\omega_p + \Omega) - k(\omega_p - \Omega) \]

\[
k(\omega) = k(\omega_p) + \frac{1}{1!} \beta_1(\omega - \omega_p) + \frac{1}{2!} \beta_2(\omega - \omega_p)^2 + \frac{1}{3!} \beta_3(\omega - \omega_p)^3 + \ldots \]

\[ k_p = k(\omega_p) \]

\[ k(\omega_p \pm \Omega) = k(\omega_p) \pm \frac{1}{1!} \beta_1 \Omega + \frac{1}{2!} \beta_2 \Omega^2 \pm \frac{1}{3!} \beta_3 \Omega^3 + \frac{1}{4!} \beta_4 \Omega^4 + \ldots \]

\[ \delta k = -2 \left[ \frac{1}{2!} \beta_2 \Omega^2 + \frac{1}{4!} \beta_4 \Omega^4 + \ldots \right] \]

Odd order dispersion coefficients are irrelevant for 4WM gain
Modulational instability growth rate, when 2nd order dispersion dominates

\[ \delta k = -\beta_2 \Omega^2 \]

\[ \text{Re} \lambda = \frac{1}{2} \text{Re} \sqrt{-\beta_2 \Omega^2 (\beta_2 \Omega^2 + 2\gamma |A_p|^2)} \]

n2 is positive in fibers, therefore gain can exist only if \( \beta_2 \) is negative, i.e. GVD is anomalous. If GVD is normal, then there is no gain, and signal+idler are not amplified.
4.4 Polarization dependent nonlinear effects in fibres

\[ \vec{E} = (iA_x + jA_y)e^{ik_0z-i\omega_0t} + c.c. \]

\[ (\partial_z^2 + \frac{\omega_0^2}{c^2})A_x = -\frac{\omega_0^2}{c^2\epsilon_0}P_{0x} \]

\[ (\partial_z^2 + \frac{\omega_0^2}{c^2})A_y = -\frac{\omega_0^2}{c^2\epsilon_0}P_{0y} \]

In amorphous glass \( \chi_{1x} = \chi_{1y} \), but \( \chi_3 \) is still a tensor.

Geometric contribution into linear dispersion for x and y polarizations in optical fibres is generally different \( \chi_{1x} \neq \chi_{1y} \)

\[
P_{0x} = \epsilon_0\chi_{1x}A_x + \frac{3}{4}\epsilon_0\chi_{3,xxxx}[\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + \frac{1}{3}A_y^2A_x^*],
\]

\[
P_{0y} = \epsilon_0\chi_{1y}A_y + \frac{3}{4}\epsilon_0\chi_{3,xxxx}[\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + \frac{1}{3}A_x^2A_y^*].
\]
Assuming single frequency propagation

\[ i \partial_z B_x = - (k_x(\omega_0) - k_0) B_x - \gamma(|B_x|^2 + \frac{2}{3}|B_y|^2) B_x - \gamma \frac{1}{3} B_y^2 B_x^* \]

\[ i \partial_z B_y = - (k_y(\omega_0) - k_0) B_y - \gamma(|B_y|^2 + \frac{2}{3}|B_x|^2) B_x - \gamma \frac{1}{3} B_x^2 B_y^* \]

Here we have deviations of \( k_x \) and \( k_y \) from the propagation constant in the perfectly circular fiber. All these are taken for the same frequency. (In the single scalar NLS, we had the deviation due frequency detuning from \( \omega_0 \))
If fibre is made circular, then
B varies randomly along the fiber length

Applying stress, or fabricating elliptical core fibres, one can induce large enough B, so that its random changes are not important, then we are dealing with the

constant modal birefringence
If we send the light polarized along either of the principle axes (in the linear regime)
Then polarization state will not change.
If we make a 45 deg. Angle, then polarization rotation takes place, with the period $L_B$.
It happens because phase velocities for 2 polarizations are different,
Hence the polarization state is going to change in the course of propagation.

$$L_B = \frac{2\pi}{|k_x - k_y|} = \frac{\lambda}{B}$$

$$B_x = be^{ik_x z} = be^{i(k_0 + \frac{1}{2} \delta k) z},$$
$$B_y = be^{ik_y z} = be^{i(k_0 - \frac{1}{2} \delta k) z},$$
$$\delta k = k_x - k_y, \ k_0 = (k_x + k_y)/2$$
$$\frac{1}{2} \delta k L_B = \pi$$
Nonlinearity not only changes the rate of the polarization rotation (through the nonlinear corrections to the wave number), but also can destroy polarization stability, when a fibre is excited along one of its principle axes. Here we again are dealing with the phase matching

We assume, that ‘x’ component is strong and ‘y’ component is weak or at the noise level

\[ i \partial_z B_x = - (k_x(\omega_0) - k_0) B_x - \gamma(|B_x|^2 + \frac{2}{3}|B_y|^2) B_x - \frac{1}{3} B_y B_x^* \]

\[ i \partial_z B_y = - (k_y(\omega_0) - k_0) B_y - \gamma(|B_y|^2 + \frac{2}{3}|B_x|^2) B_y - \frac{1}{3} B_x B_y^* \]

\[ B_x = b_x e^{i\kappa_x z}, \quad B_y = b_y(z) e^{i\kappa_y z} \]

\[ \kappa_x = (k_x(\omega_0) - k_0) + \gamma |b_x|^2 \]

\[ i \partial_z b_y - \kappa_y b_y = -(k_y(\omega_0) - k_0) b_y - \gamma \frac{2}{3} |b_x|^2 b_y - \frac{1}{3} b_x b_y^* e^{i2(\kappa_x \kappa_y) z} \]
\[ \kappa_y = (k_y(\omega_0) - k_0) + \gamma \frac{2}{3} |b_x|^2 \]

\[ i\partial_z b_y = \gamma \frac{1}{3} b_x^2 b_y^* e^{i2(\kappa_x - \kappa_y)z} \]

\[ i\partial_z^2 b_y = i\frac{\gamma^2}{9} |b_x|^4 b_y - 2\delta \kappa \partial_z b_y, \quad b_y \sim e^{\lambda z} \]

\[ \delta \kappa = \kappa_x - \kappa_y = \delta k + \frac{1}{3} \gamma P_x, \quad P_x \equiv |b_x|^2, \quad \delta k = k_x - k_y \]

\[ \lambda = i\delta \kappa \pm \sqrt{\frac{1}{9} \gamma^2 P_x^2 - (\delta \kappa)^2} \]

\( \delta \kappa \) can be made small or 0 only if \( \delta k < 0 \), since the nonlinear shift is always positive. Hence \( k_x < k_y \) (implying \( n_x < n_y \)) is a necessary condition for polarization instability. Hence the instability happens only to the FAST wave!!!
Nonlinear length

\[ E_x = b_x e^{ik_x z + i\gamma P_x z - i\omega_0 t} + c.c. \]

\[ L_B = \frac{2\pi}{|\delta k|} \]

\[ L_N = \frac{2\pi}{\gamma P_x} \]

\[ \delta\kappa = \kappa_x - \kappa_y = \delta k + \frac{1}{3} \gamma P_x \]

Qualitative formulation of the sufficient condition:
For polarization instability of the FAST wave, one needs \( L_N \) shorter than \( L_B \)

\[ \gamma = 10^{-5} \frac{1}{W_m} \text{ to } \gamma = 10^{-2} \frac{1}{W_m} \]

Hence nonlinear changes of the wavenumbers are very small. However interplay happens not with the wavenumbers themselves, but with the various wavenumber mismatches
Raman effect is the frequency conversion of an optical wave due to its energy exchange with vibrational or rotational motion of molecules (centre mass motion of nucleus)

Electronic transitions to virtual levels happen on the time scales less than 1 fs and involve $\hbar \omega_{\text{opt}}$ of energy

Oscillations and vibration are slower (in glass their life time is $\sim 20 \text{fs}$) and corresponding quanta carry $\sim 10$ times less energy than a typical photon
nonlinear part of the ref.ind. \(- n_2 I\).
Nonlinear response is instantaneous.
I.e., nonlinearity is induced as soon as the field is present.

\[ n \sim n_2 \int_{-\infty}^{\infty} \delta(t') I(t - t') dt' \]

This is a very good description of the fast nonlinearities associated with electron transitions

Vibrations and oscillations make their own contribution to \(n_2\), but this contribution is noninstantaneous it depends on values of the fields at the previous times

\[ n_2 (1 - \theta) \int_{-\infty}^{\infty} \delta(t') I(t - t') dt' + n_2 \theta \int_{-\infty}^{\infty} R(t') I(t - t') dt' \]

where \(R(t)\) is the response function

\[ R(t') = H(t') \times f(t') \] – Heviside function \(H\) selects

\[ 0 < t' < +\infty \] and hence \(-\infty < t - t' < t\), i.e.

only past intensities are accounted for
as it should be according to causality
Raman vibrations can be excited only by the modulated intensity, not by the constant intensity of the monochromatic field, which engages only with fast electrons and not with slow nuclea.

\[ n_2(1 - \theta) \int_{-\infty}^{\infty} \delta(t')I(t - t')dt' + n_2\theta \int_{-\infty}^{\infty} R(t')I(t - t')dt' \]

\( R(t) \) is normalised so that \( \int R(t)dt = 1 \)

The if \( I = const. \), we still have \( n = n_0 + n_2I \)

\( \theta \) measures contribution of the noninstantaneous nonlinearity relative to the instantaneous one

\( \theta = 0.18 \) in glass

\[ R(t) = H(t) \frac{\Omega_R^2 + \gamma_R^2}{\Omega_R} e^{-\gamma_R t} \sin t\Omega_R \]

As one can expect, this is just a response function of a harmonic oscillator

\( \Omega_R = \frac{1}{\tau_1} \) is the Raman frequency, i.e. frequency of vibrations:

\( \tau_1 = 12 fs, \Omega_R = 13THz \)

\( \gamma_R = \frac{1}{\tau_2} \) is the decay rate of vibrations: \( \tau_2 = 32 fs \)

Raman vibrations can be excited only by the modulated intensity, not by the constant intensity of the monochromatic field, which engages only with fast electrons and not with slow nuclea.
Typical molecular spectrum

- **Electronic transition** (in optical or uv)
- **Vibrational transition** (in infrared)
- **Rotational transition** (in microwave)
- **Ground state**
- **Excited electronic state**
  - Dissociation energy
Raman gain and theory of frequency conversion by Raman effect

Energy of a quantum of Raman vibrations is between 1 and 2 orders of magnitude less than energy of a photon. Therefore Raman vibrations do not change the number of photons. However, their still absorb some energy from the field. How can this happen?? Through red shift of the photon frequency !!!

\[ \hbar \omega_p = \hbar \omega_{Stokes} + \hbar \Omega_R, \quad \hbar \Omega_R \ll \hbar \omega_p, \quad \hbar \omega_{Stokes} \]

\[ i \partial_z B = - \left( \frac{i^1}{1!} \beta_1 \partial_t + \frac{i^2}{2!} \beta_2 \partial_t^2 + \frac{i^3}{3!} \beta_3 \partial_t^3 + \ldots \right) B - \gamma \theta |B|^2 B - \gamma (1 - \theta) B \int_{-\infty}^{+\infty} R(t') |B(t - t')|^2 dt' \]

\[ B = A_p(z) + A_S(z)e^{i \kappa z + i \delta t}, \quad \delta = (\omega_p - \omega_{Stokes}) > 0 \]
Instantaneous part
\[ |B|^2 B = [|A_p|^2 + 2|A_s|^2] A_p + [|A_s|^2 + 2|A_p|^2] A_s e^{i\kappa_s z + i\delta t} + \ldots \]

Noninstantaneous part
\[ B \int_{-\infty}^{+\infty} R(t')|B(t - t')|^2 dt' = \\
[|A_p|^2 + f(\delta)|A_s|^2] A_p + [|A_s|^2 + f(-\delta)|A_p|^2] A_s e^{i\kappa_s z + i\delta t} + \ldots \]

\[ i\partial_z A_p = -\gamma \{|A_p|^2 + (1 - \theta)|A_s|^2 + \theta f(\delta)|A_s|^2\} A_p \]
\[ i\partial_z A_s = -\gamma \{|A_s|^2 + (1 - \theta)|A_p|^2 + \theta f(-\delta)|A_p|^2\} A_s \]

\[ f(\delta) = \int_{-\infty}^{+\infty} R(t) e^{i\delta t} dt \]
If the Stokes wave is zero initially, then it’ll stay zero. It needs to be seeded to get amplified. Note: NO phase matching requirements !!!!

\[ f(\delta) = \int_{-\infty}^{+\infty} R(t)e^{i\delta t} dt \sim \frac{(\gamma_R^2 - \delta^2 + \Omega_R^2)}{(\gamma_R^2 - \delta^2 + \Omega_R^2)^2 + 4\gamma_R^2\delta^2} \]

\[
\begin{align*}
    i\partial_z A_p &= -\gamma \{ |A_p|^2 + (1 - \theta) |A_s|^2 + \theta f(\delta) |A_s|^2 \} A_p \\
    i\partial_z A_s &= -\gamma \{ |A_s|^2 + (1 - \theta) |A_p|^2 + \theta f(-\delta) |A_p|^2 \} A_s
\end{align*}
\]

\[
\begin{align*}
    A_p &= a_p e^{i\gamma |a_p|^2 z} \\
    A_s &= a_s e^{i\gamma \{ (1-\theta) + \theta f(-\delta) \} |a_p|^2 z}
\end{align*}
\]

\[-Im f(-\delta) \sim \delta: \text{ It implies gain for the Stokes wave at } \delta > 0\]

\[ g_R = -\gamma \theta Im f(-\delta) \text{ is the Raman gain} \]

If the Stokes wave is zero initially, then it’ll stay zero.
It needs to be seeded to get amplified.
Note: NO phase matching requirements !!!!
If we account for loss in fibers
\[ A_s = a_s e^{-\alpha z + i \gamma \{(1-\theta)+\theta f(-\delta)\}|a_p|^2} \]
Then there exists the threshold of the pump power \(|a_p|^2 = P_{th}\) beyond which we have amplification of the Stokes wave
\[ P_{th} = \alpha/g_R \sim 1W \] or below for fibers with larger \(\gamma\)

Stimulated Raman Scattering, Inelastic scattering
The most significant feature of the Raman gain in silica fibers is that its Raman gain profile extends over a large frequency range (~ 30 THz) with a broad peak located near 13 THz away from the pump frequency.

This behavior is due to the noncrystalline nature of silica glass. In amorphous materials such as fused silica, molecular vibrational frequencies spread out into bands that overlap and create a continuum. As a result, in contrast to most molecular gases for which the Raman gain occurs at specific well-defined frequencies, it extends continuously over a broad range in silica fibers.
Electrostriction is compression of glass (material) in the presence of electric field.

Brillouin effect is the frequency conversion of the light waves due to their energy exchange with an acoustic wave created through the process of electrostriction.

The process is similar (qualitatively, but not quantitatively) to the Raman scattering. The Stokes wave in the Brillouin scattering is emitted (most commonly) in the backward direction.
\[
\begin{align*}
\omega_p &= \omega_s + \Omega_B, \quad k_p = k_s - Q_B \\
v &= \Omega_B / Q_B
\end{align*}
\]
Thus Brillouin scattering is very strong, but very 'slow' and is not noticeable for pulse shorter than ~10ns.

\[ P_{th} = \alpha / g_B \sim 1mW \text{ or below for fibers with larger } \gamma \]
Stokes shift is 10GHz (for Raman 13THz)
Line width is 100MHz (for Raman 30THz)

Thus Brillouin scattering is very strong, but very 'slow' and is not noticeable for pulse shorter than ~10ns.