Four-wave mixing of linear waves ans solitons in fibers with higher-order dispersion

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We derive phase-matching conditions for four-wave mixing between solitons and linear waves in optical fibers with arbitrary dispersion and demonstrate resonant excitation of new spectral components via this process.

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The use of any optical system, including fibers, for frequency conversion relies on the ability to satisfy so-called phase-matching conditions, which depend critically on the dispersive properties of the system, and on strong enough nonlinearity to allow a reduction in the threshold pump power. The recent surge of interest in theoretical and experimental studies of optical parametric processes in photonic crystal fibers (PCFs) is related to their high nonlinearities, achieved by reduction in core size, and to the possibility of dispersion control by suitable design of the fiber core and photonic crystal cladding.

Here we study parametric generation of new frequencies resulting from four-wave mixing (FWM) of solitons and continuous wave (cw’s) in optical fibers under conditions in which the effects of higher-order dispersion are important, i.e., when pulses are short and (or) the frequency dependence of the group-velocity dispersion (GVD) is steep. Interest in this problem arises from the fact that the Fourier components of a soliton are dispersionless, while freely propagating cw’s are strongly dispersive. Therefore the phase-matching conditions are expected to be satisfied at frequencies different from those generated by the mixing of cw’s. Addressing this issue is timely because of the availability of strongly nonlinear small-core PCFs, which decrease the threshold for observing parametric processes by 1 to 2 orders of magnitude compared with that for conventional fibers: γ_{conventional} \sim 10^{-3} \text{W}^{-1} \text{m}^{-1}, γ_{pcf} \sim 10^{-2}-10^{-1} \text{W}^{-1} \text{m}^{-1}, where γ is the nonlinear fiber parameter. Strongly nonlinear PCFs have already been used to demonstrate the coupling of solitons and cw radiation in supercontinuum generation, and strong red and blue resonant, or Cherenkov, radiation from solitons and cancellation of the soliton self-frequency shift by the spectral recoil. Note that it is natural to expect that FWM of solitons and cw radiation is one of the many nonlinear processes contributing to the shape of the supercontinuum spectra.

The problem of mixing of solitons and cw’s using the idealized nonlinear Schrödinger equation, i.e., with higher-order dispersion disregarded, was analyzed in a number of papers in the past. In these cases several exact analytical solutions for a soliton sitting on the cw background were found and different perturbation techniques suggested. However, none of these studies addressed the issue of generation of new frequencies by mixing of solitons and cw light, which is the central focus of this Letter.

We assume that the dynamics of the dimensionless amplitude A(t, z) of the fundamental fiber mode is governed by the generalized nonlinear Schrödinger equation:

\[ \partial_z A = iD(i\partial_t)A + iA \int_{-\infty}^{+\infty} R(t') |A(t - t', z)|^2 \, dt'. \]  

The dispersion operator in Eq. (1) is given by

\[ D(i\partial_t) = \sum_{m=2}^{M} \frac{Z_{2m-2} \partial_{\alpha_{2m-2}} \beta_{2m}(\omega_0)}{m! |\beta_{2m}(\omega_0)|^m} (i\partial_t)^m, \]  

where τ is the pulse duration and ω_0 is the reference frequency. To avoid any ambiguity in the analytical expressions we adopt the convention of using parentheses (…) to indicate the arguments of functions or operators and […], {…} for all other purposes. R(t) is the response function of the material, which includes instantaneous Kerr and delayed Raman nonlinearities:

\[ R(t) = [1 - \theta] \Delta(t) + \theta a \Theta(t) \exp(-t/\tau_2) \sin(t/\tau_1). \]  

Here Δ(t) and Θ(t) are, respectively, delta and Heaviside functions, a = [τ_1/τ_2 + τ_2/τ_1]/τ_2, θ = 0.18, τ_1 = 12.2 fs/τ, and τ_2 = 32 fs/τ. t is the time in the reference frame moving with group velocity v_0 = v(ω_0) and measured in units of τ: \[ t = [T - z/v_0] / τ, \]  

where T is the physical time. z = Z/L_{gvd}, where Z is the distance along the fiber and L_{gvd} = τ^2/|\beta_{2m}(\omega_0)| is the GVD length. Field amplitude A is measured in units of [γL_{gvd}]^{-1/2}, where N^2 is the ratio of the peak power of the pump pulse to the peak power of a fundamental soliton with duration τ.

The dispersive properties of the soliton and the linear cw are crucial for the following, so we now discuss them in detail. Looking for a linear wave solution of Eq. (1) in the form A \sim \exp(iD_x z - iδ_x t), we find D_x = D(δ_x). In what follows subscript x can take any convenient notation. For instance, δ_x and δ_cw correspond to the frequency shifts of the soliton and the cw pump, respectively. The physical wave number of a linear wave with frequency ω_0 + δ_x/τ is given by k_x = k(ω_0) + D_x/L_{gvd}. Plotting k_x - k(ω_0) versus δ_x one simply recovers the dispersion profile of the fiber. The single-soliton solution...
A = F(ξ)\exp(-iδ_s t + i[D_s + q]z),
F(ξ) = [2q]^{1/2}\text{sech}(ξ/r)
(4)

satisfies Eq. (1) if all derivatives of function $F(ξ)$ higher than second are disregarded, $θ = 0$ and $D_s'' < 0$, i.e., the GVD at the soliton frequency is anomalous. Here $ξ = t - D_s/z$, $r = (-D_s''/(2q))^{1/2}$, $D_s''$ denotes the derivative with respect to $δ_s$, and $q > 0$ is the additional shift of the soliton wave number. Representing $F(ξ)$ through the inverse transform of its Fourier image $\hat{F}(δ)$, i.e., $F = \int dδ \hat{F}(δ) \exp(iξ[δ_s - δ])$, one finds that the wave number of a Fourier component of the soliton with frequency $ω_0 + δ_s/z$ is given by $k_{sξ} = k(ω_0) + D_s + q - [δ_s - δ_s']L_gvd$. The linear dependence of $k_{sξ}$ on $δ_s$ reflects the fact that GVD is suppressed for solitonic pulses. A line representing the dispersion characteristic of the solitons, i.e., a graph of $k_{sξ} = k(ω_0)$ versus $δ_s$, is obtained by taking the tangent to the curve $k_ξ = k(ω_0)$ at the point $δ_s = δ_s$ and by implementing a parallel shift of this tangent up by $q$.

We seek solutions of Eq. (1) in the form

$$A = \{F(ξ) + g(z, ξ)\exp(i[zD_s + q] - iδ_s t),$$

$$ξ = t - zD_s'. \quad (5)$$

Assuming that $g$ is a quasi-linear wave, we derive

$$ip = δ_s g - i\hat{D}(iδ_ξ)g - i2F^2 g - iF^2 g^*, \quad (6)$$

where $\hat{D}(iδ_ξ) = -q - D_s - iD_s'\delta_ξ - D(iδ_ξ + δ_s)$ and $p = [D(iδ_ξ + δ_s) - D_s - iD_s'\delta_ξ + 1/2D_s''(δ_ξ)^2]F$. Now we split $g$ into two parts:

$$g = w \exp(i\phi) + ψ,$$

$$\phi = z\hat{D}(δ_s - δ_s) + ξ[δ_s - δ_{cw}]. \quad (7)$$

The $w$ term in Eqs. (7) is the weak cw pump, which obeys Eq. (6) with $p = F = 0$. The $ψ$ term is the field generated through the mixing of the soliton and cw pump and by the soliton itself. Seeking $ψ$ in the form $ψ = ψ_p + ψ_\pm \exp[i\hat{D}(δ_{cw} - δ_s)] + ψ_\mp \exp[-i\hat{D}(δ_{cw} - δ_s)]$, we find that $ψ_p$ obeys

$$i\begin{bmatrix} p \\ -p^* \end{bmatrix} = \begin{bmatrix} δ_s + i\hat{D} \\ \psi_p \end{bmatrix}, \quad (8)$$

and $ψ_\pm$ are governed by the system of coupled equations

$$iF^2w \exp(iξ[δ_s - δ_{cw}]) \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} δ_s + i\hat{D} + i\hat{W} \\ ψ_\pm \end{bmatrix}, \quad (9)$$

where

$$\hat{L} = \begin{bmatrix} -\hat{D}(iδ_ξ) + 2F^2 \\ \hat{D}(iδ_ξ) + 2F^2 \end{bmatrix},$$

$$\hat{W} = \hat{D}(δ_{cw} - δ_s) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
are observed simultaneously. By changing $\delta_{\text{cw},s}$, we have been able to observe all the newly predicted resonances. Theoretical analysis of this problem is possible within the framework of Eq. (7), and we leave it for future study. The cw powers required for observation of the new FWM resonances are of the order or less than 1 W (see the caption for Fig. 1).

We have analyzed FWM between solitons and cw pump in fibers with higher-order dispersion and predicted the generation of new frequencies, which can be controlled by tuning the cw pump. Frequency range and power levels required for observing the effects described above are within the easy experimental reach. Therefore, in addition to their fundamental significance, our findings can have important practical implications in generation of new frequencies and for understanding of fine features of broadband supercontinuum.

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References